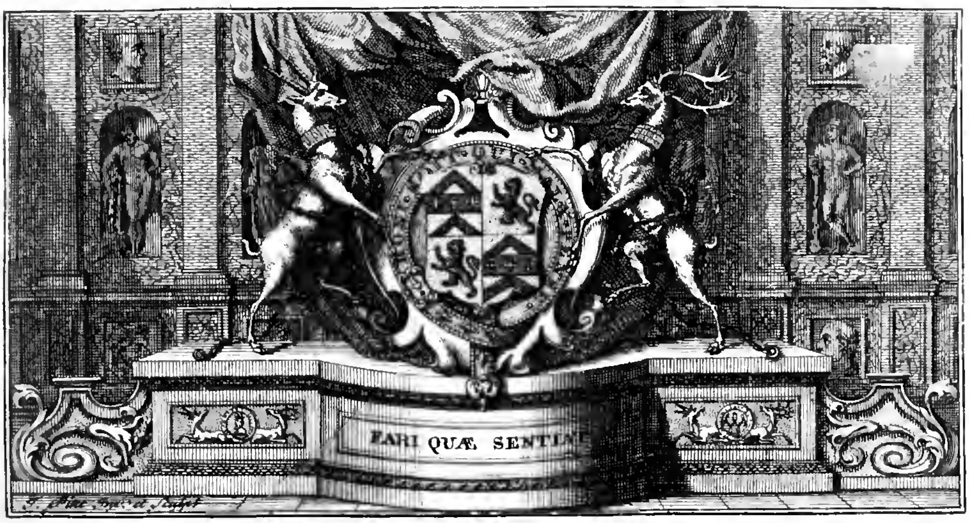

To the Noble and Right Honourable

SIR *ROBERT WALPOLE*.

SIR,

I Take the liberty to send you this view of Sir ISAAC NEWTON's philosophy, which, if it were performed suitable to the dignity of the subject, might not be a present unworthy the acceptance of the greatest person. For his philosophy affords us the only true account of the

operations of nature, which for ſo many ages had imployed the curioſity of mankind; though no one before him was furniſhed with the ſtrength of mind neceſſary to go any depth in this difficult ſearch. However, I am encouraged to hope, that this attempt, imperfect as it is, to give our countrymen in general ſome conception of the labours of a perſon, who ſhall always be the boaſt of this nation, may be received with indulgence by one, under whoſe influence theſe kingdoms enjoy ſo much happineſs. Indeed my admiration at the ſurprizing inventions of this great man, carries me to conceive of him as a perſon, who not only muſt raiſe the glory of the country, which gave him birth; but that he has even done honour to human nature, by having extended the greateſt and moſt noble of our faculties, reaſon, to ſubjects, which, till he attempted them, appeared to be wholly beyond the reach of our limited capacities. And what can give us a more

more pleaſing proſpect of our own condition, than to ſee ſo exalted a proof of the ſtrength of that faculty, whereon the conduct of our lives, and our happineſs depends; our paſſions and all our motives to action being in ſuch manner guided by our opinions, that where theſe are juſt, our whole behaviour will be praiſe-worthy? But why do I preſume to detain you, SIR, with ſuch reflections as theſe, who muſt have the fulleſt experience within your own mind, of the effects of right reaſon? For to what other ſource can be aſcribed that amiable frankneſs and unreſerved condeſcenſion among your friends, or that maſculine perſpicuity and ſtrength of argument, whereby you draw the admiration of the publick, while you are engaged in the moſt important of all cauſes, the liberties of mankind?

I humbly crave leave to make the only acknowledgement within my power, for the benefits, which

which I receive in common with the reſt of my countrymen from theſe high talents, by ſubſcribing my ſelf

SIR,

Your moſt faithful,

and

Moſt humble Servant,

HENRY PEMBERTON.

PREFACE.

I Drew up the following papers many years ago at the desire of some friends, who, upon my taking care of the late edition of Sir ISAAC NEWTON'*s Principia, perswaded me to make them publick. I laid hold of that opportunity, when my thoughts were afresh employed on this subject, to revise what I had formerly written. And I now send it abroad not without some hopes of answering these two ends. My first intention was to convey to such, as are not used to mathematical reasoning, some idea of the philosophy of a person, who has acquired an universal reputation, and rendered our nation famous for these speculations in the learned world. To which purpose I have avoided using terms of art as much as possible, and taken care to define such as I was obliged to use. Though this caution was the less necessary at present, since many of them are become familiar words to our language, from the great number of books wrote in it upon philosophical subjects, and the courses of experiments, that have of late years been given by several ingenious men. The other view I had, was to encourage such young gentlemen as have a turn for the mathematical sciences, to pursue those studies the more chearfully, in order to understand in our author himself the demonstrations of the things I here declare. And to facilitate their progress herein, I intend to proceed still farther in the explanation of Sir* ISAAC NEWTON'*s philosophy. For as I have received very much pleasure from perusing his writings, I hope it is no illaudable ambition to endeavour the rendering them more easily understood, that greater numbers may enjoy the same satisfaction.*

It will perhaps be expected, that I should say something particular of a person, to whom I must always acknowledge my self to be much obliged. What I have to declare on this head will be but short; for it was in the very last years of Sir ISAAC'*s life, that I had the ho-*

nour of his acquaintance. This happened on the following occasion. Mr. Polenus, *a Professor in the University of* Padua, *from a new experiment of his, thought the common opinion about the force of moving bodies was overturned, and the truth of Mr.* Libnitz's *notion in that matter fully proved. The contrary of what* Polenus *had asserted I demonstrated in a paper, which Dr.* MEAD, *who takes all opportunities of obliging his friends, was pleased to shew Sir* ISAAC NEWTON. *This was so well approved of by him, that he did me the honour to become a fellow-writer with me, by annexing to what I had written, a demonstration of his own drawn from another consideration. When I printed my discourse in the philosophical transactions, I put what Sir* ISAAC *had written in a scholium by it self, that I might not seem to usurp what did not belong to me. But I concealed his name, not being then sufficiently acquainted with him to ask whether he was willing I might make use of it or not. In a little time after he engaged me to take care of the new edition he was about making of his Principia. This obliged me to be very frequently with him, and as he lived at some distance from me, a great number of letters passed between us on this account. When I had the honour of his conversation, I endeavoured to learn his thoughts upon mathematical subjects, and something historical concerning his inventions, that I had not been before acquainted with. I found, he had read fewer of the modern mathematicians, than one could have expected; but his own prodigious invention readily supplied him with what he might have an occasion for in the pursuit of any subject he undertook. I have often heard him censure the handling geometrical subjects by algebraic calculations; and his book of Algebra he called by the name of Universal Arithmetic, in opposition to the injudicious title of Geometry, which* Des Cartes *had given to the treatise, wherein he shews, how the geometer may assist his invention by such kind of computations. He frequently praised* Slusius, Barrow *and* Huygens *for not being influenced by the false taste, which then began to prevail. He used to commend the laudable attempt of* Hugo de Omerique *to restore the ancient analysis, and very much esteemed* Apollonius's *book* De sectione rationis *for giving us a clearer notion of that analysis than we had before. Dr.* Barrow *may be esteemed as having*

ing shewn a compass of invention equal, if not superior to any of the moderns, our author only excepted; but Sir ISAAC NEWTON *has several times particularly recommended to me* Huygens's *stile and manner. He thought him the most elegant of any mathematical writer of modern times, and the most just imitator of the antients. Of their taste, and form of demonstration Sir* ISAAC *always professed himself a great admirer: I have heard him even censure himself for not following them yet more closely than he did; and speak with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of* Des Cartes *and other algebraic writers, before he had considered the elements of* Euclide *with that attention, which so excellent a writer deserves. As to the history of his inventions, what relates to his discoveries of the methods of series and fluxions, and of his theory of light and colours, the world has been sufficiently informed of already. The first thoughts, which gave rise to his Principia, he had, when he retired from* Cambridge *in 1666 on account of the plague. As he sat alone in a garden, he fell into a speculation on the power of gravity: that as this power is not found sensibly diminished at the remotest distance from the center of the earth, to which we can rise, neither at the tops of the loftiest buildings, nor even on the summits of the highest mountains; it appeared to him reasonable to conclude, that this power must extend much farther than was usually thought; why not as high as the moon, said he to himself? and if so, her motion must be influenced by it; perhaps she is retained in her orbit thereby. However, though the power of gravity is not sensibly weakened in the little change of distance, at which we can place our selves from the center of the earth; yet it is very possible, that so high as the moon this power may differ much in strength from what it is here. To make an estimate, what might be the degree of this diminution, he considered with himself, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried round the sun by the like power. And by comparing the periods of the several planets with their distances from the sun, he found, that if any power like gravity held them in their courses, its strength must decrease in the duplicate proportion of the increase of distance. This*

he concluded by supposing them to move in perfect circles concentrical to the sun, from which the orbits of the greatest part of them do not much differ. Supposing therefore the power of gravity, when extended to the moon, to decrease in the same manner, he computed whether that force would be sufficient to keep the moon in her orbit. In this computation, being absent from books, he took the common estimate in use among geographers and our seamen, before Norwood *had measured the earth, that 60 English miles were contained in one degree of latitude on the surface of the earth. But as this is a very faulty supposition, each degree containing about 69 ½ of our miles, his computation did not answer expectation; whence he concluded, that some other cause must at least join with the action of the power of gravity on the moon. On this account he laid aside for that time any farther thoughts upon this matter. But some years after, a letter which he received from Dr.* Hook, *put him on inquiring what was the real figure, in which a body let fall from any high place descends, taking the motion of the earth round its axis into consideration. Such a body, having the same motion, which by the revolution of the earth the place has whence it falls, is to be considered as projected forward and at the same time drawn down to the center of the earth. This gave occasion to his resuming his former thoughts concerning the moon; and* Picart *in* France *having lately measured the earth, by using his measures the moon appeared to be kept in her orbit purely by the power of gravity; and consequently, that this power decreases as you recede from the center of the earth in the manner our author had formerly conjectured. Upon this principle he found the line described by a falling body to be an ellipsis, the center of the earth being one focus. And the primary planets moving in such orbits round the sun, he had the satisfaction to see, that this inquiry, which he had undertaken merely out of curiosity, could be applied to the greatest purposes. Hereupon he composed near a dozen propositions relating to the motion of the primary planets about the sun. Several years after this, some discourse he had with Dr.* Halley, *who at* Cambridge *made him a visit, engaged Sir* ISAAC NEWTON *to resume again the consideration of this subject; and gave occasion*

to

to his writing the treatiſe which he publiſhed under the title of mathematical principles of natural philoſophy. This treatiſe, full of ſuch a variety of profound inventions, was compoſed by him from ſcarce any other materials than the few propoſitions before mentioned. in the ſpace of one year and an half.

Though his memory was much decayed, I found he perfectly underſtood his own writings, contrary to what I had frequently heard in diſcourſe from many perſons. This opinion of theirs might ariſe perhaps from his not being always ready at ſpeaking on theſe ſubjects, when it might be expected he ſhould. But as to this, it may be obſerved, that great genius's are frequently liable to be abſent, not only in relation to common life, but with regard to ſome of the parts of ſcience they are the beſt informed of. Inventors ſeem to treaſure up in their minds, what they have found out, after another manner than thoſe do the ſame things, who have not this inventive faculty. The former, when they have occaſion to produce their knowledge, are in ſome meaſure obliged immediately to inveſtigate part of what they want. For this they are not equally fit at all times: ſo it has often happened, that ſuch as retain things chiefly by means of a very ſtrong memory, have appeared off hand more expert than the diſcoverers themſelves.

As to the moral endowments of his mind, they were as much to be admired as his other talents. But this is a field I leave others to exſpatiate in. I only touch upon what I experienced my ſelf during the few years I was happy in his friendſhip. But this I immediately diſcovered in him, which at once both ſurprized and charmed me: Neither his extreme great age, nor his univerſal reputation had rendred him ſtiff in opinion, or in any degree elated. Of this I had occaſion to have almoſt daily experience. The Remarks I continually ſent him by letters on his Principia were received with the utmoſt goodneſs. Theſe were ſo far from being any ways diſpleaſing to him, that on the contrary it occaſioned him to ſpeak many kind things of me to my friends, and to honour me with a publick teſtimony of his good opinion. He alſo approved of the following treatiſe, a great part of which we read together. As many alterations were

 made

made in the late edition of his Principia, ſo there would have been many more if there had been a ſufficient time. But whatever of this kind may be thought wanting, I ſhall endeavour to ſupply in my comment on that book. I had reaſon to believe he expected ſuch a thing from me, and I intended to have publiſhed it in his life time, after I had printed the following diſcourſe, and a mathematical treatiſe Sir ISAAC NEWTON had written a long while ago, containing the firſt principles of fluxions, for I had prevailed on him to let that piece go abroad. I had examined all the calculations, and prepared part of the figures; but as the latter part of the treatiſe had never been finiſhed, he was about letting me have other papers, in order to ſupply what was wanting. But his death put a ſtop to that deſign. As to my comment on the Principia, I intend there to demonſtrate whatever Sir ISAAC NEWTON has ſet down without expreſs proof, and to explain all ſuch expreſſions in his book, as I ſhall judge neceſſary. This comment I ſhall forthwith put to the preſs, joined to an engliſh tranſlation of his Principia, which I have had ſome time by me. A more particular account of my whole deſign has already been publiſhed in the new memoirs of literature for the month of march 1727.

I have preſented my readers with a copy of verſes on Sir ISAAC NEWTON, which I have juſt received from a young Gentleman, whom I am proud to reckon among the number of my deareſt friends. If I had any apprehenſion that this piece of poetry ſtood in need of an apology, I ſhould be deſirous the reader might know, that the author is but ſixteen years old, and was obliged to finiſh his compoſition in a very ſhort ſpace of time. But I ſhall only take the liberty to obſerve, that the boldneſs of the digreſſions will be beſt judged of by thoſe who are acquainted with PINDAR.

A POEM

A POEM ON Sir *ISAAC NEWTON.*

TO NEWTON's genius, and immortal fame
Th' advent'rous muſe with trembling pinion ſoars.
Thou, heav'nly truth, from thy ſeraphick throne
Look favourable down, do thou aſſiſt
My lab'ring thought, do thou inſpire my ſong.
NEWTON, who firſt th' almighty's works diſplay'd,
And ſmooth'd that mirror, in whoſe poliſh'd face
The great creator now conſpicuous ſhines;
Who open'd nature's adamantine gates,
And to our minds her ſecret powers expos'd;
NEWTON demands the muſe; his ſacred hand
Shall guide her infant ſteps; his ſacred hand
Shall raiſe her to the Heliconian height,
Where, on its lofty top inthron'd, her head
Shall mingle with the Stars. Hail nature, hail,
O Goddeſs, handmaid of th' ethereal power,
Now lift thy head, and to th' admiring world
Shew thy long hidden beauty. Thee the wiſe
Of ancient fame, immortal PLATO's ſelf,
The Stagyrite, and Syracuſian ſage,

From

From black obſcurity's abyſs to raiſe,
(Drooping and mourning o'er thy wondrous works)
With vain inquiry ſought. Like meteors theſe
In their dark age bright ſons of wiſdom ſhone:
But at thy Newton all their laurels fade,
They ſhrink from all the honours of their names.
So glimm'ring ſtars contract their feeble rays,
When the ſwift luſtre of Aurora's face
Flows o'er the skies, and wraps the heav'ns in light.

The Deity's omnipotence, the cauſe,
Th' original of things long lay unknown.
Alone the beauties prominent to ſight
(Of the celeſtial power the outward form)
Drew praiſe and wonder from the gazing world.
As when the deluge overſpread the earth,
Whilſt yet the mountains only rear'd their heads
Above the ſurface of the wild expanſe,
Whelm'd deep below the great foundations lay,
Till ſome kind angel at heav'n's high command
Roul'd back the riſing tides, and haughty floods,
And to the ocean thunder'd out his voice:
Quick all the ſwelling and imperious waves,
The foaming billows and obſcuring ſurge,
Back to their channels and their ancient ſeats
Recoil affrighted: from the darkſome main
Earth raiſes ſmiling, as new-born, her head,
And with freſh charms her lovely face arrays.
So his extenſive thought accompliſh'd firſt
The mighty task to drive th' obſtructing miſts
Of ignorance away, beneath whoſe gloom
Th' inſhrouded majeſty of Nature lay.
He drew the veil and ſwell'd the ſpreading ſcene.
How had the moon around th' ethereal void

Rang'd,

Rang'd, and eluded lab'ring mortals care,
Till his invention trac'd her ſecret ſteps,
While ſhe inconſtant with unſteady rein
Through endleſs mazes and meanders guides
In its unequal courſe her changing carr:
Whether behind the ſun's ſuperior light
She hides the beauties of her radiant face,
Or, when conſpicuous, ſmiles upon mankind,
Unveiling all her night-rejoicing charms.
When thus the ſilver-treſſed moon diſpels
The frowning horrors from the brow of night,
And with her ſplendors chears the ſullen gloom,
While ſable-mantled darkneſs with his veil
The viſage of the fair horizon ſhades,
And over nature ſpreads his raven wings;
Let me upon ſome unfrequented green
While ſleep ſits heavy on the drowſy world,
Seek out ſome ſolitary peaceful cell,
Where darkſome woods around their gloomy brows
Bow low, and ev'ry hill's protended ſhade
Obſcures the dusky vale, there ſilent dwell,
Where contemplation holds its ſtill abode,
There trace the wide and pathleſs void of heav'n,
And count the ſtars that ſparkle on its robe.
Or elſe in fancy's wild'ring mazes loſt
Upon the verdure ſee the fairy elves
Dance o'er their magick circles, or behold,
In thought enraptur'd with the ancient bards,
Medea's baleful incantations draw
Down from her orb the paly queen of night.
But chiefly Newton let me ſoar with thee,
And while ſurveying all yon ſtarry vault
With admiration I attentive gaze,
Thou ſhalt deſcend from thy celeſtial ſeat,

And

And waft aloft my high-aſpiring mind,
Shalt ſhew me there how nature has ordain'd
Her fundamental laws, ſhalt lead my thought
Through all the wand'rings of th' uncertain moon,
And teach me all her operating powers.
She and the ſun with influence conjoint
Wield the huge axle of the whirling earth,
And from their juſt direction turn the poles,
Slow urging on the progreſs of the years.
The conſtellations ſeem to leave their ſeats,
And o'er the skies with ſolemn pace to move.
You, ſplendid rulers of the day and night,
The ſeas obey, at your reſiſtleſs ſway
Now they contract their waters, and expoſe
The dreary deſart of old ocean's reign.
The craggy rocks their horrid ſides diſcloſe ;
Trembling the ſailor views the dreadful ſcene,
And cautiouſly the threat'ning ruin ſhuns.
But where the ſhallow waters hide the ſands,
There ravenous deſtruction lurks conceal'd,
There the ill-guided veſſel falls a prey,
And all her numbers gorge his greedy jaws.
But quick returning ſee th' impetuous tides
Back to th' abandon'd ſhores impell the main.
Again the foaming ſeas extend their waves,
Again the rouling floods embrace the ſhoars,
And veil the horrours of the empty deep.
Thus the obſequious ſeas your power confeſs,
While from the ſurface healthful vapours riſe
Plenteous throughout the atmoſphere diffus'd,
Or to ſupply the mountain's heads with ſprings,
Or fill the hanging clouds with needful rains,
That friendly ſtreams, and kind refreſhing ſhow'rs
May gently lave the ſun-burnt thirſty plains,

Or to replenifh all the empty air
With wholfome moifture to increafe the fruits
Of earth, and blefs the labours of mankind.
O NEWTON, whether flies thy mighty foul,
How fhall the feeble mufe purfue through all
The vaft extent of thy unbounded thought,
That even feeks th' unfeen receffes dark
To penetrate of providence immenfe.
And thou the great difpenfer of the world
Propitious, who with infpiration taught'ft
Our greateft bard to fend thy praifes forth ;
Thou, who gav'ft NEWTON thought ; who fmil'dft ferene,
When to its bounds he ftretch'd his fwelling foul ;
Who ftill benignant ever bleft his toil,
And deign'd to his enlight'ned mind t' appear
Confefs'd around th' interminated world :
To me O thy divine infufion grant
(O thou in all fo infinitely good)
That I may fing thy everlafting works,
Thy inexhaufted ftore of providence,
In thought effulgent and refounding verfe.
O could I fpread the wond'rous theme around,
Where the wind cools the oriental world,
To the calm breezes of the Zephir's breath,
To where the frozen hyperborean blafts,
To where the boift'rous tempeft-leading fouth
From their deep hollow caves fend forth their ftorms.
Thou ftill indulgent parent of mankind,
Left humid emanations fhould no more
Flow from the ocean, but diffolve away
Through the long feries of revolving time ;
And left the vital principle decay,
By which the air fupplies the fprings of life ;
Thou haft the fiery vifag'd comets form'd

With vivifying ſpirits all replete,
Which they abundant breathe about the void,
Renewing the prolifick ſoul of things.
No longer now on thee amaz'd we call,
No longer tremble at imagin'd ills,
When comets blaze tremendous from on high,
Or when extending wide their flaming trains
With hideous graſp the skies engirdle round,
And ſpread the terrors of their burning locks.
For theſe through orbits in the length'ning ſpace
Of many tedious rouling years compleat
Around the ſun move regularly on;
And with the planets in harmonious orbs,
And myſtick periods their obeyſance pay
To him majeſtick ruler of the skies
Upon his throne of circled glory fixt.
He or ſome god conſpicuous to the view,
Or elſe the ſubſtitute of nature ſeems,
Guiding the courſes of revolving worlds.
He taught great NEWTON the all-potent laws
Of gravitation, by whoſe ſimple power
The univerſe exiſts. Nor here the ſage
Big with invention ſtill renewing ſtaid.
But O bright angel of the lamp of day,
How ſhall the muſe diſplay his greateſt toil?
Let her plunge deep in Aganippe's waves,
Or in Caſtalia's ever-flowing ſtream,
That re-inſpired ſhe may ſing to thee,
How NEWTON dar'd advent'rous to unbraid
The yellow treſſes of thy ſhining hair.
Or didſt thou gracious leave thy radiant ſphere,
And to his hand thy lucid ſplendours give,
T' unweave the light-diffuſing wreath, and part

The blended glories of thy golden plumes?
He with laborious, and unerring care,
How diff'rent and imbodied colours form
Thy piercing light, with juſt diſtinction found.
He with quick ſight purſu'd thy darting rays,
When penetrating to th' obſcure receſs
Of ſolid matter, there perſpicuous ſaw,
How in the texture of each body lay
The power that ſeparates the diff'rent beams.
Hence over nature's unadorned face
Thy bright diverſifying rays dilate
Their various hues: and hence when vernal rains
Deſcending ſwift have burſt the low'ring clouds,
Thy ſplendors through the diſſipating miſts
In its fair veſture of unnumber'd hues
Array the ſhow'ry bow. At thy approach
The morning riſen from her pearly couch
With roſy bluſhes decks her virgin cheek;
The ev'ning on the frontiſpiece of heav'n
His mantle ſpreads with many colours gay;
The mid-day skies in radiant azure clad,
The ſhining clouds, and ſilver vapours rob'd
In white tranſparent intermixt with gold,
With bright variety of ſplendor cloath
All the illuminated face above.
When hoary-headed winter back retires
To the chill'd pole, there ſolitary ſits
Encompaſs'd round with winds and tempeſts bleak
In caverns of impenetrable ice,
And from behind the diſſipated gloom
Like a new Venus from the parting ſurge
The gay-apparell'd ſpring advances on;
When thou in thy meridian brightneſs ſitt'ſt,
And from thy throne pure emanations flow

 Of

Of glory burſting o'er the radiant skies:
Then let the muſe Olympus' top aſcend,
And o'er Theſſalia's plain extend her view,
And count, O Tempe, all thy beauties o'er.
Mountains, whoſe ſummits graſp the pendant clouds,
Between their wood-invelop'd ſlopes embrace
The green-attired vallies. Every flow'r
Here in the pride of bounteous nature clad
Smiles on the boſom of th' enamell'd meads.
Over the ſmiling lawn the ſilver floods
Of fair Peneus gently roul along,
While the reflected colours from the flow'rs,
And verdant borders pierce the lympid waves,
And paint with all their variegated hue
The yellow ſands beneath. Smooth gliding on
The waters haſten to the neighbouring ſea.
Still the pleas'd eye the floating plain purſues;
At length, in Neptune's wide dominion loſt,
Surveys the ſhining billows, that ariſe
Apparell'd each in Phœbus' bright attire:
Or from a far ſome tall majeſtick ſhip,
Or the long hoſtile lines of threat'ning fleets,
Which o'er the bright uneven mirror ſweep,
In dazling gold and waving purple deckt;
Such as of old, when haughty Athens power
Their hideous front, and terrible array
Againſt Pallene's coaſt extended wide,
And with tremendous war and battel ſtern
The trembling walls of Potidæa ſhook.
Creſted with pendants curling with the breeze
The upright maſts high briſtle in the air,
Aloft exalting proud their gilded heads.
The ſilver waves againſt the painted prows
Raiſe their reſplendent boſoms, and impearl

The

The fair vermillion with their glist'ring drops:
And from on board the iron-cloathed host
Around the main a gleaming horrour casts;
Each flaming buckler like the mid-day sun,
Each plumed helmet like the silver moon,
Each moving gauntlet like the light'ning's blaze,
And like a star each brazen pointed spear.
But lo the sacred high-erected fanes,
Fair citadels, and marble-crowned towers,
And sumptuous palaces of stately towns
Magnificent arise, upon their heads
Bearing on high a wreath of silver light.
But see my muse the high Pierian hill,
Behold its shaggy locks and airy top,
Up to the skies th' imperious mountain heaves
The shining verdure of the nodding woods.
See where the silver Hippocrene flows,
Behold each glitt'ring rivulet, and rill
Through mazes wander down the green descent,
And sparkle through the interwoven trees.
Here rest a while and humble homage pay,
Here, where the sacred genius, that inspir'd
Sublime Mæonides and Pindar's breast,
His habitation once was fam'd to hold.
Here thou, O Homer, offer'dst up thy vows;
Thee, the kind muse Calliopæa heard,
And led thee to the empyrean seats,
There manifested to thy hallow'd eyes
The deeds of gods; thee wise Minerva taught
The wondrous art of knowing human kind;
Harmonious Phoebus tun'd thy heav'nly mind,
And swell'd to rapture each exalted sense;
Even Mars the dreadful battle-ruling god,
Mars taught thee war, and with his bloody hand

Instructed

Inſtructed thine, when in thy ſounding lines
We hear the rattling of Bellona's carr,
The yell of diſcord, and the din of arms.
PINDAR, when mounted on his fiery ſteed,
Soars to the ſun, oppoſing eagle like
His eyes undazled to the fierceſt rays.
He firmly ſeated, not like GLAUCUS' ſon,
Strides his ſwift-winged and fire-breathing horſe,
And born aloft ſtrikes with his ringing hoofs
The brazen vault of heav'n, ſuperior there
Looks down upon the ſtars, whoſe radiant light
Illuminates innumerable worlds,
That through eternal orbits roul beneath.
But thou all hail immortalized ſon
Of harmony, all hail thou Thracian bard,
To whom APOLLO gave his tuneful lyre.
O might'ſt thou, ORPHEUS, now again revive,
And NEWTON ſhould inform thy liſt'ning ear
How the ſoft notes, and ſoul-inchanting ſtrains
Of thy own lyre were on the wind convey'd.
He taught the muſe, how ſound progreſſive floats
Upon the waving particles of air,
When harmony in ever-pleaſing ſtrains,
Melodious melting at each lulling fall,
With ſoft alluring penetration ſteals
Through the enraptur'd ear to inmoſt thought,
And folds the ſenſes in its ſilken bands.
So the ſweet muſick, which from ORPHEUS' touch
And fam'd AMPHION's, on the ſounding ſtring
Aroſe harmonious, gliding on the air,
Pierc'd the tough-bark'd and knotty-ribbed woods,
Into their ſaps ſoft inſpiration breath'd
And taught attention to the ſtubborn oak.
Thus when great HENRY, and brave MARLB'ROUGH led

Th'

A Poem on Sir Isaac Newton.

Th' imbattled numbers of BRITANNIA's ſons,
The trump, that ſwells th' expanded cheek of fame,
That adds new vigour to the gen'rous youth,
And rouzes ſluggiſh cowardize it ſelf,
The trumpet with its Mars-inciting voice,
The winds broad breaſt impetuous ſweeping o'er
Fill'd the big note of war. Th' inſpired hoſt
With new-born ardor preſs the trembling GAUL;
Nor greater throngs had reach'd eternal night,
Not if the fields of Agencourt had yawn'd
Expoſing horrible the gulf of fate;
Or roaring Danube ſpread his arms abroad,
And overwhelm'd their legions with his floods.
But let the wand'ring muſe at length return;
Nor yet, angelick genius of the ſun,
In worthy lays her high-attempting ſong
Has blazon'd forth thy venerated name.
Then let her ſweep the loud-reſounding lyre
Again, again o'er each melodious ſtring
Teach harmony to tremble with thy praiſe.
And ſtill thine ear O favourable grant,
And ſhe ſhall tell thee, that whatever charms,
Whatever beauties bloom on nature's face,
Proceed from thy all-influencing light.
That when ariſing with tempeſtuous rage,
The North impetuous rides upon the clouds
Diſperſing round the heav'ns obſtructive gloom,
And with his dreaded prohibition ſtays
The kind effuſion of thy genial beams;
Pale are the rubies on AURORA's lips,
No more the roſes bluſh upon her cheeks,
Black are Peneus' ſtreams and golden ſands
In Tempe's vale dull melancholy ſits,
And every flower reclines its languid head.

By

A Poem on Sir Isaac Newton.

By what high name ſhall I invoke thee, ſay,
Thou life-infuſing deity, on thee
I call, and look propitious from on high,
While now to thee I offer up my prayer.
O had great Newton, as he found the cauſe,
By which ſound rouls thro' th' undulating air,
O had he, baffling times reſiſtleſs power,
Diſcover'd what that ſubtle ſpirit is,
Or whatſoe'er diffuſive elſe is ſpread
Over the wide-extended univerſe,
Which cauſes bodies to reflect the light,
And from their ſtraight direction to divert
The rapid beams, that through their ſurface pierce.
But ſince embrac'd by th' icy arms of age,
And his quick thought by times cold hand congeal'd,
Ev'n Newton left unknown this hidden power;
Thou from the race of human kind ſelect
Some other worthy of an angel's care,
With inſpiration animate his breaſt,
And him inſtruct in theſe thy ſecret laws.
O let not Newton, to whoſe ſpacious view,
Now unobſtructed, all th' extenſive ſcenes
Of the ethereal ruler's works ariſe;
When he beholds this earth he late adorn'd,
Let him not ſee philoſophy in tears,
Like a fond mother ſolitary ſit,
Lamenting him her dear, and only child.
But as the wiſe Pythagoras, and he,
Whoſe birth with pride the fam'd Abdera boaſts,
With expectation having long ſurvey'd
This ſpot their antient ſeat, with joy beheld
Divine philoſophy at length appear
In all her charms majeſtically fair,
Conducted by immortal Newton's hand:

So

So may he ſee another ſage ariſe,
That ſhall maintain her empire: then no more
Imperious ignorance with haughty ſway
Shall ſtalk rapacious o'er the ravag'd globe:
Then thou, O NEWTON, ſhalt protect theſe lines,
The humble tribute of the grateful muſe;
Ne'er ſhall the ſacrilegious hand deſpoil
Her laurel'd temples, whom his name preſerves:
And were ſhe equal to the mighty theme,
Futurity ſhould wonder at her ſong;
Time ſhould receive her with extended arms,
Seat her conſpicuous in his rouling carr,
And bear her down to his extreameſt bound.

FABLES with wonder tell how Terra's ſons
With iron force unloos'd the ſtubborn nerves
Of hills, and on the cloud-inſhrouded top
Of Pelion Oſſa pil'd. But if the vaſt
Gigantick deeds of ſavage ſtrength demand
Aſtoniſhment from men, what then ſhalt thou,
O what expreſſive rapture of the ſoul,
When thou before us, NEWTON, doſt diſplay
The labours of thy great excelling mind;
When thou unveileſt all the wondrous ſcene,
The vaſt idea of th' eternal king,
Not dreadful bearing in his angry arm
The thunder hanging o'er our trembling heads;
But with th' effulgency of love replete,
And clad with power, which form'd th' extenſive heavens.
O happy he, whoſe enterprizing hand
Unbars the golden and relucid gates
Of th' empyrean dome, where thou enthron'd
Philoſophy art ſeated. Thou ſuſtain'd
By the firm hand of everlaſting truth

Despiseſt all the injuries of time:
Thou never know'ſt decay when all around,
Antiquity obſcures her head. Behold
Th' Egyptian towers, the Babylonian walls,
And Thebes with all her hundred gates of braſs,
Behold them ſcatter'd like the duſt abroad.
Whatever now is flouriſhing and proud,
Whatever ſhall, muſt know devouring age.
Euphrates' ſtream, and ſeven-mouthed Nile,
And Danube, thou that from Germania's ſoil
To the black Euxine's far remoted ſhore,
O'er the wide bounds of mighty nations ſweep'ſt
In thunder loud thy rapid floods along.
Ev'n you ſhall feel inexorable time;
To you the fatal day ſhall come; no more
Your torrents then ſhall ſhake the trembling ground,
No longer then to inundations ſwol'n
Th' imperious waves the fertile paſtures drench,
But ſhrunk within a narrow channel glide;
Or through the year's reiterated courſe
When time himſelf grows old, your wond'rous ſtreams
Loſt ev'n to memory ſhall lie unknown
Beneath obſcurity, and Chaos whelm'd.
But ſtill thou ſun illuminateſt all
The azure regions round, thou guideſt ſtill
The orbits of the planetary ſpheres;
The moon ſtill wanders o'er her changing courſe,
And ſtill, O Newton, ſhall thy name ſurvive:
As long as nature's hand directs the world,
When ev'ry dark obſtruction ſhall retire,
And ev'ry ſecret yield its hidden ſtore,
Which thee dim-ſighted age forbad to ſee
Age that alone could ſtay thy riſing ſoul.
And could mankind among the fixed ſtars,

E'en

A Poem on Sir Isaac Newton.

E'en to th' extremeſt bounds of knowledge reach,
To thoſe unknown innumerable ſuns,
Whoſe light but glimmers from thoſe diſtant worlds,
Ev'n to thoſe utmoſt boundaries, thoſe bars
That ſhut the entrance of th' illumin'd ſpace
Where angels only tread the vaſt unknown,
Thou ever ſhould'ſt be ſeen immortal there:
In each new ſphere, each new-appearing ſun,
In fartheſt regions at the very verge
Of the wide univerſe ſhould'ſt thou be ſeen.
And lo, th' all-potent goddeſs NATURE takes
With her own hand thy great, thy juſt reward
Of immortality; aloft in air
See ſhe diſplays, and with eternal graſp
Uprears the trophies of great NEWTON's fame.

R. GLOVER.

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INTRODUCTION.

THE manner, in which Sir ISAAC NEWTON has published his philosophical discoveries, occasions them to lie very much concealed from all, who have not made the mathematics particularly their study. He once, indeed, intended to deliver, in a more familiar way, that part of his inventions, which relates to the system of the world; but upon farther consideration he altered his design. For as the nature of those discoveries made it impossible to prove them upon any other than geometrical principles; he apprehended, that those, who should not fully perceive the force of his arguments, would hardly be prevailed on to exchange their former sentiments for new opinions, so very different from

what were commonly received [a]. He therefore chofe rather to explain himfelf only to mathematical readers; and declined the attempting to inftruct fuch in any of his principles, who, by not comprehending his method of reafoning, could not, at the firft appearance of his difcoveries, have been perfuaded of their truth. But now, fince Sir ISAAC NEWTON's doctrine has been fully eftablifhed by the unanimous approbation of all, who are qualified to underftand the fame; it is without doubt to be wifhed, that the whole of his improvements in philofophy might be univerfally known. For this purpofe therefore I drew up the following papers, to give a general notion of our great philofopher's inventions to fuch, as are not prepared to read his own works, and yet might defire to be informed of the progrefs, he has made in natural knowledge; not doubting but there were many, befides thofe, whofe turn of mind had led them into a courfe of mathematical ftudies, that would take great pleafure in tafting of this delightful fountain of fcience.

2. IT is a juft remark, which has been made upon the human mind, that nothing is more fuitable to it, than the contemplation of truth; and that all men are moved with a ftrong defire after knowledge; efteeming it honourable to excel therein; and holding it, on the contrary, difgraceful to miftake, err, or be in any way deceived. And this fentiment is by nothing more fully illuftrated, than by the inclination of men to gain an acquaintance with the operations of nature; which difpofition to enquire after the caufes of things is

[a] Philofoph. Nat. princ. math. L. iii. introduct.

fo

ſo general, that all men of letters, I believe, find themſelves influenced by it. Nor is it difficult to aſſign a reaſon for this, if we conſider only, that our deſire after knowledge is an effect of that taſte for the ſublime and the beautiful in things, which chiefly conſtitutes the difference between the human life, and the life of brutes. Theſe inferior animals partake with us of the pleaſures, that immediately flow from the bodily ſenſes and appetites; but our minds are furniſhed with a ſuperior ſenſe, by which we are capable of receiving various degrees of delight, where the creatures below us perceive no difference. Hence ariſes that purſuit of grace and elegance in our thoughts and actions, and in all things belonging to us, which principally creates imployment for the active mind of man. The thoughts of the human mind are too extenſive to be confined only to the providing and enjoying of what is neceſſary for the ſupport of our being. It is this taſte, which has given riſe to poetry, oratory, and every branch of literature and ſcience. From hence we feel great pleaſure in conceiving ſtrongly, and in apprehending clearly, even where the paſſions are not concerned. Perſpicuous reaſoning appears not only beautiful; but, when ſet forth in its full ſtrength and dignity, it partakes of the ſublime, and not only pleaſes, but warms and elevates the ſoul. This is the ſource of our ſtrong deſire of knowledge; and the ſame taſte for the ſublime and the beautiful directs us to chuſe particularly the productions of nature for the ſubject of our contemplation: our creator having ſo adapted our minds to the condition, wherein he has placed us, that all his viſible

works, before we inquire into their make, ſtrike us with the moſt lively ideas of beauty and magnificence.

3. But if there be ſo ſtrong a paſſion in contemplative minds for natural philoſophy; all ſuch muſt certainly receive a particular pleaſure in being informed of Sir Isaac Newton's diſcoveries, who alone has been able to make any great advancements in the true courſe leading to natural knowledge: whereas this important ſubject had before been uſually attempted with that negligence, as cannot be reflected on without ſurprize. Excepting a very few, who, by purſuing a more rational method, had gained a little true knowledge in ſome particular parts of nature; the writers in this ſcience had generally treated of it after ſuch a manner, as if they thought, that no degree of certainty was ever to be hoped for. The cuſtom was to frame conjectures; and if upon comparing them with things, there appeared ſome kind of agreement, though very imperfect, it was held ſufficient. Yet at the ſame time nothing leſs was undertaken than intire ſyſtems, and fathoming at once the greateſt depths of nature; as if the ſecret cauſes of natural effects, contrived and framed by infinite wiſdom, could be ſearched out by the ſlighteſt endeavours of our weak underſtandings. Whereas the only method, that can afford us any proſpect of ſucceſs in this difficult work, is to make our enquiries with the utmoſt caution, and by very ſlow degrees. And after our moſt diligent labour, the greateſt part of nature will, no doubt, for ever remain beyond our reach.

4. This

4. THIS neglect of the proper means to enlarge our knowledge, joined with the presumption to attempt, what was quite out of the power of our limited faculties, the Lord BACON judiciously observes to be the great obstruction to the progress of science [a]. Indeed that excellent person was the first, who expresly writ against this way of philosophizing; and he has laid open at large the absurdity of it in his admirable treatise, intitled NOVUM ORGANON SCIENTIARUM; and has there likewise described the true method, which ought to be followed.

5. THERE are, saith he, but two methods, that can be taken in the pursuit of natural knowledge. One is to make a hasty transition from our first and slight observations on things to general axioms, and then to proceed upon those axioms, as certain and uncontestable principles, without farther examination. The other method; (which he observes to be the only true one, but to his time unattempted;) is to proceed cautiously, to advance step by step, reserving the most general principles for the last result of our inquiries [b]. Concerning the first of these two methods; where objections, which happen to appear against any such axioms taken up in haste, are evaded by some frivolous distinction, when the axiom it self ought rather to be corrected [c]; he affirms, that the united endeavours of all ages cannot make it successful; because this original error in the first digestion of the mind (as he expresses himself) cannot afterwards be remedied [d]: whereby he would signify to us, that if we set out in a

[a] Nov. Org. Scient. L. i. Aphorism. 9.
[b] Nov. Org. L. 1. Aph. 19.
[c] Ibid. Aph. 25.
[d] Aph. 30. Errores radicales & in prima digestione mentis ab excellentia functionum & remediorum sequentium non curantur.

wrong

wrong way; no diligence or art, we can uſe, while we follow ſo erroneous a courſe, will ever bring us to our deſigned end. And doubtleſs it cannot prove otherwiſe; for in this ſpacious field of nature, if once we forſake the true path, we ſhall immediately loſe our ſelves, and muſt for ever wander with uncertainty.

6. THE impoſſibility of ſucceeding in ſo faulty a method of philoſophizing his Lordſhip endeavours to prove from the many falſe notions and prejudices, to which the mind of man is expoſed [a]. And ſince this judicious writer apprehends, that men are ſo exceeding liable to fall into theſe wrong tracts of thinking, as to incur great danger of being miſled by them, even while they enter on the true courſe in purſuit of nature [b]; I truſt, I ſhall be excuſed, if, by inſiſting a little particularly upon this argument, I endeavour to remove whatever prejudice of this kind, might poſſibly entangle the mind of any of my readers.

7. HIS Lordſhip has reduced theſe prejudices and falſe modes of conception under four diſtinct heads [c].

8. THE firſt head contains ſuch, as we are ſubject to from the very condition of humanity, through the weakneſs both of our ſenſes, and of the faculties of the mind [d]; ſeeing, as this author well obſerves, the ſubtilty of nature far exceeds the greateſt ſubtilty of our ſenſes or acuteſt reaſonings [e]. One

[a] Aph. 38.
[b] Ibid.
[c] Aph. 39.
[d] Aph. 41.
[e] Aph. 10, 24.

of

of the falſe modes of conception, which he mentions under this head, is the forming to our ſelves a fanciful ſimplicity and regularity in natural things. This he illuſtrates by the following inſtances; the conceiving the planets to move in perfect circles; the adding an orb of fire to the other three elements, and the ſuppoſing each of theſe to exceed the other in rarity, juſt in a decuple proportion [a]. And of the ſame nature is the aſſertion of DES CARTES, without any proof, that all things are made up of three kinds of matter only [b]. As alſo this opinion of another philoſopher; that light, in paſſing through different mediums, was refracted, ſo as to proceed by that way, through which it would move more ſpeedily, than through any other [c]. The ſecond erroneous turn of mind, taken notice of by his Lordſhip under this head, is, that all men are in ſome degree prone to a fondneſs for any notions, which they have once imbibed; whereby they often wreſt things to reconcile them to thoſe notions, and neglect the conſideration of whatever will not be brought to an agreement with them; juſt as thoſe do, who are addicted to judicial aſtrology, to the obſervation of dreams, and to ſuch-like ſuperſtitions; who carefully preſerve the memory of every incident, which ſerves to confirm their prejudices, and let ſlip out of their minds all inſtances, that make againſt them [d]. There is alſo a farther impediment to true knowledge, mentioned under the ſame head by this noble writer, which is; that whereas, through the weakneſs and imperfection of our ſenſes, many things are concealed

[a] Aph. 45.
[b] Des Cartes Princ. Phil. Part. 3. §. 52.
[c] Fermat, in Oper. pag. 156, &c.
[d] Nov. Org. Aph. 46.

from

from us, which have the greateſt effect in producing natural appearances; our minds are ordinarily moſt affected by that, which makes the ſtrongeſt impreſſion on our organs of ſenſe; whereby we are apt to judge of the real importance of things in nature by a wrong meaſure [a]. So, becauſe the figuration and the motion of bodies ſtrike our ſenſes more immediately than moſt of their other properties, DES CARTES and his followers will not allow any other explication of natural appearances, than from the figure and motion of the parts of matter. By which example we ſee how juſtly his Lordſhip obſerves this cauſe of error to be the greateſt of any [b]; ſince it has given riſe to a fundamental principle in a ſyſtem of philoſophy, that not long ago obtained almoſt an univerſal reputation.

9. THESE are the chief branches of thoſe obſtructions to knowledge, which this author has reduced under his firſt head of falſe conceptions. The ſecond head contains the errors, to which particular perſons are more eſpecially obnoxious [c]. One of theſe is the conſequence of a preceding obſervation: that as we are expoſed to be captivated by any opinions, which have once taken poſſeſſion of our minds; ſo in particular, natural knowledge has been much corrupted by the ſtrong attachment of men to ſome one part of ſcience, of which they reputed themſelves the inventers, or about which they have ſpent much of their time; and hence have been apt to conceive it to be of greater uſe in the ſtudy of na-

[a] Aph. 50.
[b] Ibid.
[c] Aph. 53.

tural

tural philoſophy than it was: like ARISTOTLE, who reduced his phyſics to logical diſputations; and the chymiſts, who thought, that nature could be laid open only by the force of their fires [a]. Some again are wholly carried away by an exceſſive veneration for antiquity; others, by too great fondneſs for the moderns; few having their minds ſo well balanced, as neither to depreciate the merit of the ancients, nor yet to deſpiſe the real improvements of later times [b]. To this is added by his Lordſhip a difference in the genius of men, that ſome are moſt fitted to obſerve the ſimilitude, there is in things, while others are more qualified to diſcern the particulars, wherein they diſagree; both which diſpoſitions of mind are uſeful: but to the prejudice of philoſophy men are apt to run into exceſs in each; while one ſort of genius dwells too much upon the groſs and ſum of things, and the other upon trifling minuteneſſes and ſhadowy diſtinctions [c].

10. UNDER the third head of prejudices and falſe notions this writer conſiders ſuch, as follow from the lax and indefinite uſe of words in ordinary diſcourſe; which occaſions great ambiguities and uncertainties in philoſophical debates (as another eminent philoſopher has ſince ſhewn more at large [d];) inſomuch that this our author thinks a ſtrict defining of terms to be ſcarce an infallible remedy againſt this inconvenience [e]. And perhaps he has no ſmall reaſon on his ſide: for the common inaccurate ſenſe of words, notwithſtanding the limitations given them by definitions, will offer it ſelf ſo conſtantly to

[a] Aph. 54.
[b] Aph. 56.
[c] Aph. 55.

[d] Locke. On human underſtanding, B. iii.
[e] Nov. Org. Aph. 59.

the mind, as to require great caution and circumſpection for us not to be deceived thereby. Of this we have a very eminent inſtance in the great diſputes, that have been raiſed about the uſe of the word attraction in philoſophy; of which we ſhall be obliged hereafter to make particular mention [a]. Words thus to be guarded againſt are of two kinds. Some are names of things, that are only imaginary [b]; ſuch words are wholly to be rejected. But there are other terms, that allude to what is real, though their ſignification is confuſed [c]. And theſe latter muſt of neceſſity be continued in uſe; but their ſenſe cleared up, and freed, as much as poſſible, from obſcurity.

11. THE laſt general head of theſe errors comprehends ſuch, as follow from the various ſects of falſe philoſophies; which this author divides into three ſorts, the ſophiſtical, empirical, and ſuperſtitious [d]. By the firſt of theſe he means a philoſophy built upon ſpeculations only without experiments [e]; by the ſecond, where experiments are blindly adhered to, without proper reaſoning upon them [f]; and by the third, wrong opinions of nature fixed in mens minds either through falſe religions, or from miſunderſtanding the declarations of the true [g].

12. THESE are the four principal canals, by which this judicious author thinks, that philoſophical errors have flowed in upon us. And he rightly obſerves, that the faulty method of

[a] In the concluſion.
[b] Nov. Org. L. i. Aph. 59.
[c] Ibid. Aph. 60.
[d] Ibid. Aph. 62.
[e] Aph. 63.
[f] Aph. 64.
[g] Aph. 65.

proceeding

proceeding in philoſophy, againſt which he writes [a], is ſo far from aſſiſting us towards overcoming theſe prejudices; that he apprehends it rather ſuited to rivet them more firmly to the mind [b]. How great reaſon then has his Lordſhip to call this way of philoſophizing the parent of error, and the bane of all knowledge [c]? For, indeed, what elſe but miſtakes can ſo bold and preſumptuous a treatment of nature produce? have we the wiſdom neceſſary to frame a world, that we ſhould think ſo eaſily, and with ſo ſlight a ſearch to enter into the moſt ſecret ſprings of nature, and diſcover the original cauſes of things? what chimeras, what monſters has not this prepoſterous method brought forth? what ſchemes, or what hypotheſis's of the ſubtileſt wits has not a ſtricter enquiry into nature not only overthrown, but manifeſted to be ridiculous and abſurd? Every new improvement, which we make in this ſcience, lets us ſee more and more the weakneſs of our gueſſes. Dr. HARVEY, by that one diſcovery of the circulation of the blood, has diſſipated all the ſpeculations and reaſonings of many ages upon the animal oeconomy. ASELLIUS, by detecting the lacteal veins, ſhewed how little ground all phyſicians and philoſophers had in conjecturing, that the nutritive part of the aliment was abſorbed by the mouths of the veins ſpread upon the bowels: and then PECQUET, by finding out the thoracic duct, as evidently proved the vanity of the opinion, which was perſiſted in after the lacteal veſſels were known, that the alimental juice was conveyed immediately to the liver, and there converted into blood.

[a] See above, § 4, 5.
[b] Nov. Org. L. i. Aph. 69.
[c] Ibid.

13. As theſe things ſet forth the great abſurdity of proceeding in philoſophy on conjectures, by informing us how far the operations of nature are above our low conceptions; ſo on the other hand, ſuch inſtances of ſucceſs from a more judicious method ſhew us, that our bountiful maker has not left us wholly without means of delighting our ſelves in the contemplation of his wiſdom. That by a juſt way of inquiry into nature, we could not fail of arriving at diſcoveries very remote from our apprehenſions; the Lord BACON himſelf argues from the experience of mankind. If, ſays he, the force of guns ſhould be deſcribed to any one ignorant of them, by their effects only; he might reaſonably ſuppoſe, that thoſe engines of deſtruction were only a more artificial compoſition, than he knew, of wheels and other mechanical powers: but it could never enter his thoughts, that their immenſe force ſhould be owing to a peculiar ſubſtance, which would enkindle into ſo violent an exploſion, as we experience in gunpowder: ſince he would no where ſee the leaſt example of any ſuch operation; except perhaps in earthquakes and thunder, which he would doubtleſs look upon as exalted powers of nature, greatly ſurpaſſing any art of man to imitate. In the ſame manner, if a ſtranger to the original of ſilk were ſhewn a garment made of it, he would be very far from imagining ſo ſtrong a ſubſtance to be ſpun out of the bowels of a ſmall worm; but muſt certainly believe it either a vegetable ſubſtance, like flax or cotton; or the natural covering of ſome animal, as wool is of ſheep. Or had we been told, before the invention of the magnetic needle among us, that another people was in poſſeſſion of a certain

contrivance

contrivance, by which they were inabled to diſcover the poſition of the heavens, with vaſtly more eaſe, than we could do; what could have been imagined more, than that they were provided with ſome fitter aſtronomical inſtrument for this purpoſe than we? That any ſtone ſhould have ſo amazing a property, as we find in the magnet, muſt have been the remoteſt from our thoughts [a].

14. BUT what ſurprizing advancements in the knowledge of nature may be made by purſuing the true courſe in philoſophical inquiries; when thoſe ſearches are conducted by a genius equal to ſo divine a work, will be beſt underſtood by conſidering Sir ISAAC NEWTON's diſcoveries. That my reader may apprehend as juſt a notion of theſe, as can be conveyed to him, by the brief account, which I intend to lay before him; I have ſet apart this introduction for explaining, in the fulleſt manner I am able, the principles, whereon Sir ISAAC NEWTON proceeds. For without a clear conception of theſe, it is impoſſible to form any true idea of the ſingular excellence of the inventions of this great philoſopher.

15. THE principles then of this philoſophy are; upon no conſideration to indulge conjectures concerning the powers and laws of nature, but to make it our endeavour with all diligence to ſearch out the real and true laws, by which the conſtitution of things is regulated. The philoſopher's firſt care muſt be to diſtinguiſh, what he ſees to be within his power, from what

[a] Ibid. Aph. 109.

is beyond his reach; to aſſume no greater degree of knowledge, than what he finds himſelf poſſeſſed of; but to advance by ſlow and cautious ſteps; to ſearch gradually into natural cauſes; to ſecure to himſelf the knowledge of the moſt immediate cauſe of each appearance, before he extends his views farther to cauſes more remote. This is the method, in which philoſophy ought to be cultivated; which does not pretend to ſo great things, as the more airy ſpeculations; but will perform abundantly more: we ſhall not perhaps ſeem to the unskilful to know ſo much, but our real knowledge will be greater. And certainly it is no objection againſt this method, that ſome others promiſe, what is nearer to the extent of our wiſhes: ſince this, if it will not teach us all we could deſire to be informed of, will however give us ſome true light into nature; which no other can do. Nor has the philoſopher any reaſon to think his labour loſt, when he finds himſelf ſtopt at the cauſe firſt diſcovered by him, or at any other more remote cauſe, ſhort of the original: for if he has but ſufficiently proved any one cauſe, he has entered ſo far into the real conſtitution of things, has laid a ſafe foundation for others to work upon, and has facilitated their endeavours in the ſearch after yet more diſtant cauſes; and beſides, in the mean time he may apply the knowledge of theſe intermediate cauſes to many uſeful purpoſes. Indeed the being able to make practical deductions from natural cauſes, conſtitutes the great diſtinction between the true philoſophy and the falſe. Cauſes aſſumed upon conjecture, muſt be ſo looſe and undefined, that nothing particular can be collected from them. But thoſe cauſes, which are brought to light by a ſtrict examination

of

of things, will be more diftinct. Hence it appears to have been no unufeful difcovery, that the afcent of water in pumps is owing to the preffure of the air by its weight or fpring; though the caufes, which make the air gravitate, and render it elaftic, be unknown: for notwithftanding we are ignorant of the original, whence thefe powers of the air are derived; yet we may receive much advantage from the bare knowledge of thefe powers. If we are but certain of the degree of force, wherewith they act, we fhall know the extent of what is to be expected from them; we fhall know the greateft height, to which it is poffible by pumps to raife water; and fhall thereby be prevented from making any ufelefs efforts towards improving thefe inftruments beyond the limits prefcribed to them by nature; whereas without fo much knowledge as this, we might probably have wafted in attempts of this kind much time and labour. How long did philofophers bufy themfelves to no purpofe in endeavouring to perfect telefcopes, by forming the glaffes into fome new figure; till Sir ISAAC NEWTON demonftrated, that the effects of telefcopes were limited from another caufe, than was fuppofed; which no alteration in the figure of the glaffes could remedy? What method Sir ISAAC NEWTON himfelf has found for the improvement of telefcopes fhall be explained hereafter [a]. But at prefent I fhall proceed to illuftrate, by fome farther inftances, this diftinguifhing character of the true philofophy, which we have now under confideration. It was no trifling difcovery, that the contraction of the mufcles of animals puts their limbs in motion, though the original caufe of that contraction

[a] Book III. Chap. iv.

remains

remains a ſecret, and perhaps may always do ſo; for the knowledge of thus much only has given riſe to many ſpeculations upon the force and artificial diſpoſition of the muſcles, and has opened no narrow proſpect into the animal fabrick. The finding out, that the nerves are great agents in this action, leads us yet nearer to the original cauſe, and yields us a wider view of the ſubject. And each of theſe ſteps affords us aſſiſtance towards reſtoring this animal motion, when impaired in our ſelves, by pointing out the ſeats of the injuries, to which it is obnoxious. To neglect all this, becauſe we can hitherto advance no farther, is plainly ridiculous. It is confeſſed by all, that GALILEO greatly improved philoſophy, by ſhewing, as we ſhall relate hereafter, that the power in bodies, which we call gravity, occaſions them to move downwards with a velocity equably accelerated[a]; and that when any body is thrown forwards, the ſame power obliges it to deſcribe in its motion that line, which is called by geometers a parabola[b]: yet we are ignorant of the cauſe, which makes bodies gravitate. But although we are unacquainted with the ſpring, whence this power in nature is derived, nevertheleſs we can eſtimate its effects. When a body falls perpendicularly, it is known, how long time it takes in deſcending from any height whatever: and if it be thrown forwards, we know the real path, which it deſcribes; we can determine in what direction, and with what degree of ſwiftneſs it muſt be projected, in order to its ſtriking againſt any object deſired; and we can alſo aſcertain the very force, wherewith it will ſtrike.

[a] Book I. Chap. 2. § 14. [b] Ibid. § 85, &c.

Sir

Sir ISAAC NEWTON has farther taught, that this power of gravitation extends up to the moon, and cauſes that planet to gravitate as much towards the earth, as any of the bodies, which are familiar to us, would, if placed at the ſame diſtance [a]: he has proved likewiſe, that all the planets gravitate towards the ſun, and towards one another; and that their reſpective motions follow from this gravitation. All this he has demonſtrated upon indiſputable geometrical principles, which cannot be rendered precarious for want of knowing what it is, which cauſes theſe bodies thus mutually to gravitate: any more than we can doubt of the propenſity in all the bodies about us, to deſcend towards the earth; or can call in queſtion the forementioned propoſitions of GALILEO, which are built upon that principle. And as GALILEO has ſhewn more fully, than was known before, what effects were produced in the motion of bodies by their gravitation towards the earth; ſo Sir ISAAC NEWTON, by this his invention, has much advanced our knowledge in the celeſtial motions. By diſcovering that the moon gravitates towards the ſun, as well as towards the earth; he has laid open thoſe intricacies in the moon's motion, which no aſtronomer, from obſervations only, could ever find out [b]: and one kind of heavenly bodies, the comets, have their motion now clearly aſcertained; whereof we had before no true knowledge at all [c].

16. DOUBTLESS it might be expected, that ſuch ſurprizing ſucceſs ſhould have ſilenced, at once, every cavil. But we

[a] See Book II. Ch. 3. § 3, 4. of this treatiſe.
[b] See Book II. Ch. 3. of this treatiſe.
[c] See Chap. 4.

D have

have ſeen the contrary. For becauſe this philoſophy profeſſes modeſtly to keep within the extent of our faculties, and is ready to confeſs its imperfections, rather than to make any fruitleſs attempts to conceal them, by ſeeking to cover the defects in our knowledge with the vain oſtentation of raſh and groundleſs conjectures; hence has been taken an occaſion to inſinuate that we are led to miraculous cauſes, and the occult qualities of the ſchools.

17. But the firſt of theſe accuſations is very extraordinary. If by calling theſe cauſes miraculous nothing more is meant than only, that they often appear to us wonderful and ſurprizing, it is not eaſy to ſee what difficulty can be raiſed from thence; for the works of nature diſcover every where ſuch proofs of the unbounded power, and the conſummate wiſdom of their author, that the more they are known, the more they will excite our admiration: and it is too manifeſt to be inſiſted on, that the common ſenſe of the word miraculous can have no place here, when it implies what is above the ordinary courſe of things. The other imputation, that theſe cauſes are occult upon the account of our not perceiving what produces them, contains in it great ambiguity. That ſomething relating to them lies hid, the followers of this philoſophy are ready to acknowledge, nay deſire it ſhould be carefully remarked, as pointing out proper ſubjects for future inquiry. But this is very different from the proceeding of the ſchoolmen in the cauſes called by them occult. For as their occult qualities were underſtood to operate in a manner occult, and not apprehended by us; ſo they were ob-

truded upon us for ſuch original and eſſential properties in bodies, as made it vain to ſeek any farther cauſe; and a greater power was attributed to them, than any natural appearances authorized. For inſtance, the riſe of water in pumps was aſcribed to a certain abhorrence of a vacuum, which they thought fit to aſſign to nature. And this was ſo far a true obſervation, that the water does move, contrary to its uſual courſe, into the ſpace, which otherwiſe would be left void of any ſenſible matter; and, that the procuring ſuch a vacuity was the apparent cauſe of the water's aſcent. But while we were not in the leaſt informed how this power, called an abhorrence of a vacuum, produced the viſible effects; inſtead of making any advancement in the knowledge of nature, we only gave an artificial name to one of her operations: and when the ſpeculation was puſhed ſo beyond what any appearances required, as to have it concluded, that this abhorrence of a vacuum was a power inherent in all matter, and ſo unlimited as to render it impoſſible for a vacuum to exiſt at all; it then became a much greater abſurdity, in being made the foundation of a moſt ridiculous manner of reaſoning; as at length evidently appeared, when it came to be diſcovered, that this riſe of the water followed only from the preſſure of the air, and extended it ſelf no farther, than the power of that cauſe. The ſcholaſtic ſtile in diſcourſing of theſe occult qualities, as if they were eſſential differences in the very ſubſtances, of which bodies conſiſted, was certainly very abſurd; by reaſon it tended to diſcourage all farther inquiry. But no ſuch ill conſequences can follow from the conſidering of any natural cauſes, which confeſſedly are not traced up to

their firſt original. How ſhall we ever come to the knowledge of the ſeveral original cauſes of things, otherwiſe than by ſtoring up all intermediate cauſes which we can diſcover? Are all the original and eſſential properties of matter ſo very obvious, that none of them can eſcape our firſt view? This is not probable. It is much more likely, that, if ſome of the eſſential properties are diſcovered by our firſt obſervations, a ſtricter examination ſhould bring more to light.

18. But in order to clear up this point concerning the eſſential properties of matter, let us conſider the ſubject a little diſtinctly. We are to conceive, that the matter, out of which the univerſe of things is formed, is furniſhed with certain qualities and powers, whereby it is rendered fit to anſwer the purpoſes, for which it was created. But every property, of which any particle of this matter is in it ſelf poſſeſſed, and which is not barely the conſequence of the union of this particle with other portions of matter, we may call an eſſential property: whereas all other qualities or attributes belonging to bodies, which depend on their particular frame and compoſition, are not eſſential to the matter, whereof ſuch bodies are made; becauſe the matter of theſe bodies will be deprived of thoſe qualities, only by the diſſolution of the body, without working any change in the original conſtitution of one ſingle particle of this maſs of matter. Extenſion we apprehend to be one of theſe eſſential properties, and impenetrability another. Theſe two belong univerſally to all matter; and are the principal ingredients in the idea, which this word matter uſually excites in the mind. Yet as the idea, marked by

by this name, is not purely the creature of our own understandings, but is taken for the repreſentation of a certain ſubſtance without us; if we ſhould diſcover, that every part of the ſubſtance, in which we find theſe two properties, ſhould likewiſe be endowed univerſally with any other eſſential qualities; all theſe, from the time they come to our notice, muſt be united under our general idea of matter. How many ſuch properties there are actually in all matter we know not; thoſe, of which we are at preſent apprized, have been found out only by our obſervations on things; how many more a farther ſearch may bring to light, no one can ſay; nor are we certain, that we are provided with ſufficient methods of perception to diſcern them all. Therefore, ſince we have no other way of making diſcoveries in nature, but by gradual inquiries into the properties of bodies; our firſt ſtep muſt be to admit without diſtinction all the properties, which we obſerve; and afterwards we muſt endeavour, as far as we are able, to diſtinguiſh between the qualities, wherewith the very ſubſtances themſelves are indued, and thoſe appearances, which reſult from the ſtructure only of compound bodies. Some of the properties, which we obſerve in things, are the attributes of particular bodies only; others univerſally belong to all, that fall under our notice. Whether ſome of the qualities and powers of particular bodies, be derived from different kinds of matter entring their compoſition, cannot, in the preſent imperfect ſtate of our knowledge, abſolutely be decided; though we have not yet any reaſon to conclude, but that all the bodies, with which we converſe, are framed out of the very ſame kind of matter, and that their diſtinct

quali-

qualities are occaſioned only by their ſtructure; through the variety whereof the general powers of matter are cauſed to produce different effects. On the other hand, we ſhould not haſtily conclude, that whatever is found to appertain to all matter, which falls under our examination, muſt for that reaſon only be an eſſential property thereof, and not be derived from ſome unſeen diſpoſition in the frame of nature. Sir ISAAC NEWTON has found reaſon to conclude, that gravity is a property univerſally belonging to all the perceptible bodies in the univerſe, and to every particle of matter, whereof they are compoſed. But yet he no where aſſerts this property to be eſſential to matter. And he was ſo far from having any deſign of eſtabliſhing it as ſuch, that, on the contrary, he has given ſome hints worthy of himſelf at a cauſe for it [a]; and expreſly ſays, that he propoſed thoſe hints to ſhew, that he had no ſuch intention [b].

19. IT appears from hence, that it is not eaſy to determine, what properties of bodies are eſſentially inherent in the matter, out of which they are made, and what depend upon their frame and compoſition. But certainly whatever properties are found to belong either to any particular ſyſtems of matter, or univerſally to all, muſt be conſidered in philoſophy; becauſe philoſophy will be otherwiſe imperfect. Whether thoſe properties can be deduced from ſome other appertaining to matter, either among thoſe, which are already known, or among ſuch as can be diſcovered by us, is afterwards to be ſought for the farther improvement of our knowledge. But this

[a] At the end of his Optics. in Qu. 21.

[b] See the ſame treatiſe, in Advertiſement 2.

inquiry

inquiry cannot properly have place in the deliberation about admitting any property of matter or bodies into philoſophy; for that purpoſe it is only to be conſidered, whether the exiſtence of ſuch a property has been juſtly proved or not. Therefore to decide what cauſes of things are rightly received into natural philoſophy, requires only a diſtinct and clear conception of what kind of reaſoning is to be allowed of as convincing, when we argue upon the works of nature.

20. THE proofs in natural philoſophy cannot be ſo abſolutely concluſive, as in the mathematics. For the ſubjects of that ſcience are purely the ideas of our own minds. They may be repreſented to our ſenſes by material objects, but they are themſelves the arbitrary productions of our own thoughts; ſo that as the mind can have a full and adequate knowledge of its own ideas, the reaſoning in geometry can be rendered perfect. But in natural knowledge the ſubject of our contemplation is without us, and not ſo compleatly to be known: therefore our method of arguing muſt fall a little ſhort of abſolute perfection. It is only here required to ſteer a juſt courſe between the conjectural method of proceeding, againſt which I have ſo largely ſpoke; and demanding ſo rigorous a proof, as will reduce all philoſophy to mere ſcepticiſm, and exclude all proſpect of making any progreſs in the knowledge of nature.

21. THE conceſſions, which are to be allowed in this ſcience, are by Sir ISAAC NEWTON included under a very few ſimple precepts.

22. The firſt is, that more cauſes are not to be received into philoſophy, than are ſufficient to explain the appearances of nature. That this rule is approved of unanimouſly, is evident from thoſe expreſſions ſo frequent among all philoſophers, that nature does nothing in vain; and that a variety of means, where fewer would ſuffice, is needleſs. And certainly there is the higheſt reaſon for complying with this rule. For ſhould we indulge the liberty of multiplying, without neceſſity, the cauſes of things, it would reduce all philoſophy to mere uncertainty; ſince the only proof, which we can have, of the exiſtence of a cauſe, is the neceſſity of it for producing known effects. Therefore where one cauſe is ſufficient, if there really ſhould in nature be two, which is in the laſt degree improbable, we can have no poſſible means of knowing it, and conſequently ought not to take the liberty of imagining, that there are more than one.

23. The ſecond precept is the direct conſequence of the firſt, that to like effects are to be aſcribed the ſame cauſes. For inſtance, that reſpiration in men and in brutes is brought about by the ſame means; that bodies deſcend to the earth here in Europe, and in America from the ſame principle; that the light of a culinary fire, and of the ſun have the ſame manner of production; that the reflection of light is effected in the earth, and in the planets by the ſame power; and the like.

24. The third of theſe precepts has equally evident reaſon for it. It is only, that thoſe qualities, which in the ſame body can neither be leſſened nor increaſed, and which belong

to

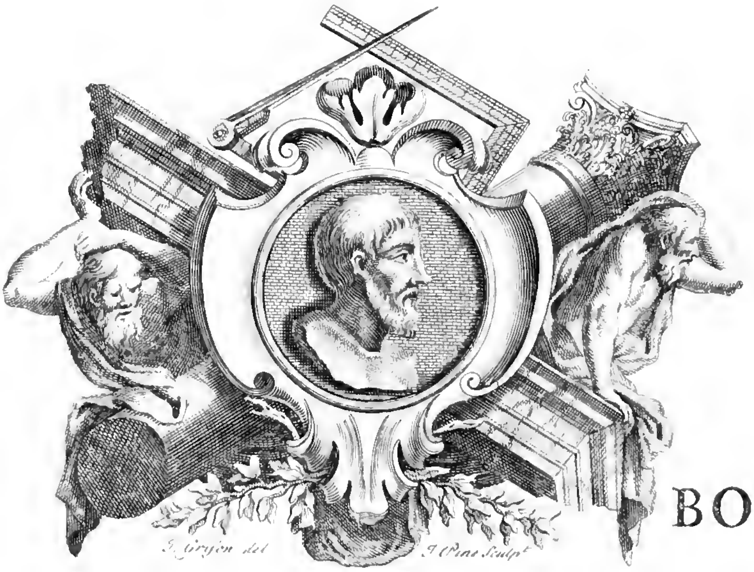
to all bodies that are in our power to make trial upon, ought to be accounted the univerſal properties of all bodies whatever.

25. In this precept is founded that method of arguing by induction, without which no progreſs could be made in natural philoſophy. For as the qualities of bodies become known to us by experiments only; we have no other way of finding the properties of ſuch bodies, as are out of our reach to experiment upon, but by drawing concluſions from thoſe which fall under our examination. The only caution here required is, that the obſervations and experiments, we argue upon, be numerous enough, and that due regard be paid to all objections, that occur, as the Lord Bacon very judiciouſly directs[a]. And this admonition is ſufficiently complied with, when by virtue of this rule we aſcribe impenetrability and extenſion to all bodies, though we have no ſenſible experiment, that affords a direct proof of any of the celeſtial bodies being impenetrable; nor that the fixed ſtars are ſo much as extended. For the more perfect our inſtruments are, whereby we attempt to find their viſible magnitude, the leſs they appear; inſomuch that all the ſenſible magnitude, which we obſerve in them, ſeems only to be an optical deception by the ſcattering of their light. However, I ſuppoſe no one will imagine they are without any magnitude, though their immenſe diſtance makes it undiſcernable by us. After the ſame manner, if it can be proved, that all

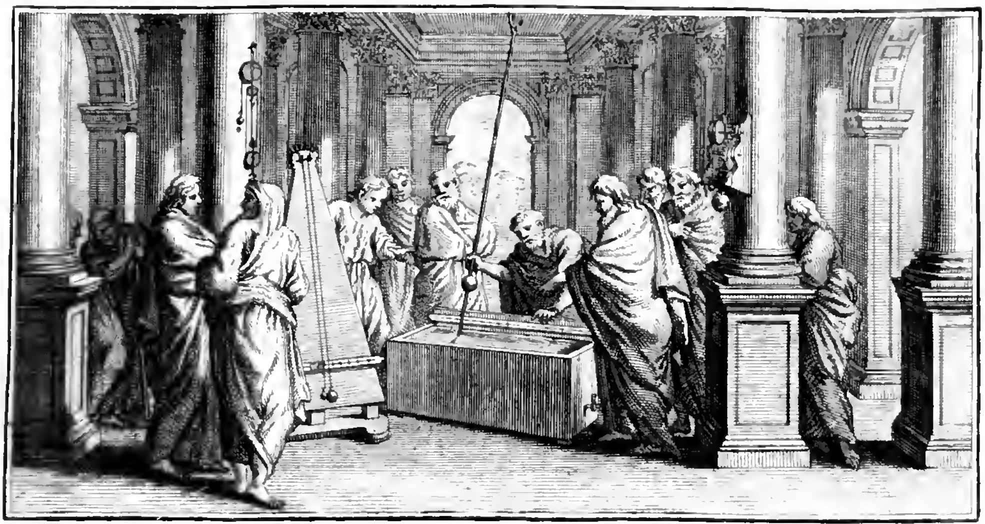
[a] Nov. Org. Lib. i. Ax. 105.

bodies here gravitate towards the earth, in proportion to the quantity of ſolid matter in each; and that the moon gravitates to the earth likewiſe, in proportion to the quantity of matter in it; and that the ſea gravitates towards the moon, and all the planets towards each other; and that the very comets have the ſame gravitating faculty; we ſhall have as great reaſon to conclude by this rule, that all bodies gravitate towards each other. For indeed this rule will more ſtrongly hold in this caſe, than in that of the impenetrability of bodies; becauſe there will more inſtances be had of bodies gravitating, than of their being impenetrable.

25. THIS is that method of induction, whereon all philoſophy is founded; which our author farther inforces by this additional precept, that whatever is collected from this induction, ought to be received, notwithſtanding any conjectural hypotheſis to the contrary, till ſuch times as it ſhall be contradicted or limited by farther obſervations on nature.



BOOK



J. Gryſon del. J. Pine Sculp.

BOOK I.

CONCERNING THE MOTION of BODIES IN GENERAL.

CHAP. I.
Of the LAWS of MOTION.

HAVING thus explained Sir ISAAC NEWTON's method of reaſoning in philoſophy, I ſhall now proceed to my intended account of his diſcoveries. Theſe are contained in two treatiſes. In one of them, the MATHEMATICAL PRINCIPLES OF NATURAL PHILOSOPHY, his chief deſign is to ſhew by what laws the heavenly

motions are regulated; in the other, his OPTICS, he difcourfes of the nature of light and colours, and of the action between light and bodies. This fecond treatife is wholly confined to the fubject of light: except fome conjectures propofed at the end concerning other parts of nature, which lie hitherto more concealed. In the other treatife our author was obliged to fmooth the way to his principal intention, by explaining many things of a more general nature: for even fome of the moft fimple properties of matter were fcarce well eftablifhed at that time. We may therefore reduce Sir ISAAC NEWTON's doctrine under three general heads; and I fhall accordingly divide my account into three books. In the firft I fhall fpeak of what he has delivered concerning the motion of bodies, without regard to any particular fyftem of matter; in the fecond I fhall treat of the heavenly motions; and the third fhall be employed upon light.

2. IN the firft part of my defign, we muft begin with an account of the general laws of motion.

3. THESE laws are fome univerfal affections and properties of matter drawn from experience, which are made ufe of as axioms and evident principles in all our arguings upon the motion of bodies. For as it is the cuftom of geometers to affume in their demonftrations fome propofitions, without exhibiting the proof of them; fo in philofophy, all our reafoning muft be built upon fome properties of matter, firft admitted as principles whereon to argue. In geometry thefe axioms are thus affumed, on account of their being fo evident

as

as to make any proof in form needlefs. But in philofophy no properties of bodies can be in this manner received for felf-evident; fince it has been obferved above, that we can conclude nothing concerning matter by any reafonings upon its nature and effence, but that we owe all the knowledge, we have thereof, to experience. Yet when our obfervations on matter have inform'd us of fome of its properties, we may fecurely reafon upon them in our farther inquiries into nature. And thefe laws of motion, of which I am here to fpeak, are found fo univerfally to belong to bodies, that there is no motion known, which is not regulated by them. Thefe are by Sir ISAAC NEWTON reduced to three [a].

4. THE firft law is, that all bodies have fuch an indifference to reft, or motion, that if once at reft they remain fo, till difturbed by fome power acting upon them: but if once put in motion, they perfift in it; continuing to move right forwards perpetually, after the power, which gave the motion, is removed; and alfo preferving the fame degree of velocity or quicknefs, as was firft communicated, not ftopping or remitting their courfe, till interrupted or otherwife difturbed by fome new power impreffed.

5. THE fecond law of motion is, that the alteration of the ftate of any body, whether from reft to motion, or from motion to reft, or from one degree of motion to another, is always proportional to the force impreffed. A body at reft, when

[a] Princip. philof. pag. 13, 14.

acted

acted upon by any power, yields to that power, moving in the ſame line, in which the power applied is directed; and moves with a leſs or greater degree of velocity, according to the degree of the power; ſo that twice the power ſhall communicate a double velocity, and three times the power a threefold velocity. If the body be moving, and the power impreſſed act upon the body in the direction of its motion, the body ſhall receive an addition to its motion, as great as the motion, into which that power would have put it from a ſtate of reſt; but if the power impreſſed upon a moving body act directly oppoſite to its former motion, that power ſhall then take away from the body's motion, as much as in the other caſe it would have added to it. Laſtly, if the power be impreſſed obliquely, there will ariſe an oblique motion differing more or leſs from the former direction, according as the new impreſſion is greater or leſs. For example, if the body A (in fig. 1.) be moving in the direction A B, and when it is at the point A, a power be impreſſed upon it in the direction A C, the body ſhall from henceforth neither move in its firſt direction A B, nor in the direction of the adventitious power, but ſhall take a courſe as A D between them: and if the power laſt impreſſed be juſt equal to that, which firſt gave to the body its motion; the line A D ſhall paſs in the middle between A B and A C, dividing the angle under B A C into two equal parts; but if the power laſt impreſſed be greater than the firſt, the line AD ſhall incline moſt to AC; whereas if the laſt impreſſion be leſs than the firſt, the line A D ſhall incline moſt to A B. To be more particular, the ſituation of the

the line A D is always to be determined after this manner. Let A E be the ſpace, which the body would have moved through in the line A B during any certain portion of time; provided that body, when at A, had received no ſecond impulſe. Suppoſe likewiſe, that A F is the part of the line A C, through which the body would have moved during an equal portion of time, if it had been at reſt in A, when it received the impulſe in the direction A C: then if from E be drawn a line parallel to, or equidiſtant from A C, and from F another line parallel to A B, thoſe two lines will meet in the line A D.

6. THE third and laſt of theſe laws of motion is, that when any body acts upon another, the action of that body upon the other is equalled by the contrary reaction of that other body upon the firſt.

7. THESE laws of motion are abundantly confirmed by this, that all the deductions made from them, in relation to the motion of bodies, how complicated ſoever, are found to agree perfectly with obſervation. This ſhall be ſhewn more at large in the next chapter. But before we proceed to ſo diffuſive a proof; I chuſe here to point out thoſe appearances of bodies, whereby the laws of motion are firſt ſuggeſted to us.

8. DAILY obſervation makes it appear to us, that any body, which we once ſee at reſt, never puts it ſelf into freſh motion;

motion; but continues always in the ſame place, till removed by ſome power applied to it.

9. Again, whenever a body is once in motion, it continues in that motion ſome time after the moving power has quitted it, and it is left to it ſelf. Now if the body continue to move but a ſingle moment, after the moving power has left it, there can no reaſon be aſſigned, why it ſhould ever ſtop without ſome external force. For it is plain, that this continuance of the motion is cauſed only by the body's having already moved, the ſole operation of the power upon the body being the putting it in motion; therefore that motion continued will equally be the cauſe of its farther motion, and ſo on without end. The only doubt that can remain, is, whether this motion communicated continues intire, after the power, that cauſed it, ceaſes to act; or whether it does not gradually languiſh and decreaſe. And this ſuſpicion cannot be removed by a tranſient and ſlight obſervation on bodies, but will be fully cleared up by thoſe more accurate proofs of the laws of motion, which are to be conſidered in the next chapter.

10. Lastly, bodies in motion appear to affect a ſtraight courſe without any deviation, unleſs when diſturbed by ſome adventitious power acting upon them. If a body be thrown perpendicularly upwards or downwards, it appears to continue in the ſame ſtraight line during the whole time of its motion. If a body be thrown in any other direction, it is found to deviate from the line, in which it began to move, more and more

more continually towards the earth, whither it is directed by its weight: but ſince, when the weight of a body does not alter the direction of its motion, it always moves in a ſtraight line, without doubt in this other caſe the body's declining from its firſt courſe is no more, than what is cauſed by its weight alone. As this appears at firſt ſight to be unqueſtionable, ſo we ſhall have a very diſtinct proof thereof in the next chapter, where the oblique motion of bodies will be particularly conſidered.

11. THUS we ſee how the firſt of the laws of motion agrees with what appears to us in moving bodies. But here occurs this farther conſideration, that the real and abſolute motion of any body is not viſible to us: for we are our ſelves alſo in conſtant motion along with the earth whereon we dwell; inſomuch that we perceive bodies to move ſo far only, as their motion is different from our own. When a body appears to us to lie at reſt, in reality it only continues the motion, it has received, without putting forth any power to change that motion. If we throw a body in the courſe or direction, wherein we are carried our ſelves; ſo much motion as we ſeem to have given to the body, ſo much we have truly added to the motion, it had, while it appeared to us to be at reſt. But if we impel a body the contrary way, although the body appears to us to have received by ſuch an impulſe as much motion, as when impelled the other way; yet in this caſe we have taken from the body ſo much real motion, as we ſeem to have given it. Thus the motion, which we ſee in bodies,

is not their real motion, but only relative with respect to us; and the forementioned observations only shew us, that this first law of motion has place in this relative or apparent motion. However, though we cannot make any observation immediately on the absolute motion of bodies, yet by reasoning upon what we observe in visible motion, we can discover the properties and effects of real motion.

12. With regard to this first law of motion, which is now under consideration, we may from the foregoing observations most truly collect, that bodies are disposed to continue in the absolute motion, which they have once received, without increasing or diminishing their velocity. When a body appears to us to lie at rest, it really preserves without change the motion, which it has in common with our selves: and when we put it into visible motion, and we see it continue that motion; this proves, that the body retains that degree of its absolute motion, into which it is put by our acting upon it: if we give it such an apparent motion, which adds to its real motion, it preserves that addition; and if our acting on the body takes off from its real motion, it continues afterwards to move with no more real motion, than we have left it.

13. Again, we do not observe in bodies any disposition or power within themselves to change the direction of their motion; and if they had any such power, it would easily be discovered. For suppose a body by the structure or disposition of its parts, or by any other circumstance in its make, was indued

duced with a power of moving it ſelf; this ſelf-moving principle, which ſhould be thus inherent in the body, and not depend on any thing external, muſt change the direction wherein it would act, as often as the poſition of the body was changed: ſo that for inſtance, if a body was lying before me in ſuch a poſition, that the direction, wherein this principle diſpoſes the body to move, was pointed directly from me; if I then gradually turned the body about, the direction of this ſelf-moving principle would no longer be pointed directly from me, but would turn about along with the body. Now if any body, which appears to us at reſt, were furniſhed with any ſuch ſelf-moving principle; from the body's appearing without motion we muſt conclude, that this ſelf-moving principle lies directed the ſame way as the earth is carrying the body; and ſuch a body might immediately be put into viſible motion only by turning it about in any degree, that this ſelf-moving principle might receive a different direction.

14. FROM theſe conſiderations it very plainly follows, that if a body were once abſolutely at reſt; not being furniſhed with any principle, whereby it could put it ſelf into motion, it muſt for ever continue in the ſame place, till acted upon by ſomething external: and alſo that when a body is put into motion, it has no power within it ſelf to make any change in the direction of that motion; and conſequently that the body muſt move on ſtraight forward without declining any way whatever. But it has before been ſhewn, that bodies do not appear to have in themſelves any power to

change the velocity of their motion: therefore this firſt law of motion has been illuſtrated and confirmed, as much as can be from the tranſient obſervations, which have here been diſcourſed upon; and in the next chapter all this will be farther eſtabliſhed by more correct obſervations.

15. But I ſhall now paſs to the ſecond law of motion; wherein, when it is aſſerted, that the velocity, with which any body is moved by the action of a power upon it, is proportional to that power; the degree of power is ſuppoſed to be meaſured by the greatneſs of the body, which it can move with a given celerity. So that the ſenſe of this law is, that if any body were put into motion with that degree of ſwiftneſs, as to paſs in one hour the length of a thouſand yards; the power, which would give the ſame degree of velocity to a body twice as great, would give this leſſer body twice the velocity, cauſing it to deſcribe in the ſame ſpace of an hour two thouſand yards. But by a body twice as great as another, I do not here mean ſimply of twice the bulk, but one that contains a double quantity of ſolid matter.

16. Why the power, which can move a body twice as great as another with the ſame degree of velocity, ſhould be called twice as great as the power, which can give the leſſer body the ſame velocity, is evident. For if we ſhould ſuppoſe the greater body to be divided into two equal parts, each equal to the leſſer body, each of theſe halves will require the ſame degree of power to move them with the velocity of the leſſer body, as the leſſer body it ſelf requires; and therefore both

thoſe halves, or the whole greater body, will require the moving power to be doubled.

17. THAT the moving power being in this ſenſe doubled, ſhould juſt double likewiſe the velocity of the ſame body, ſeems near as evident, if we conſider, that the effect of the power applied muſt needs be the ſame, whether that power be applied to the body at once, or in parts. Suppoſe then the double power not applied to the body at once, but half of it firſt, and afterwards the other half; it is not conceivable for what reaſon the half laſt applied ſhould come to have a different effect upon the body, from that which is applied firſt; as it muſt have, if the velocity of the body was not juſt doubled by the application of it. So far as experience can determine, we ſee nothing to favour ſuch a ſuppoſition. We cannot indeed (by reaſon of the conſtant motion of the earth) make trial upon any body perfectly at reſt, whereby to ſee whether a power applied in that caſe would have a different effect, from what it has, when the body is already moving; but we find no alteration in the effect of the ſame power on account of any difference there may be in the motion of the body, when the power is applied. The earth does not always carry bodies with the ſame degree of velocity; yet we find the viſible effects of any power applied to the ſame body to be at all times the very ſame: and a bale of goods, or other moveable body lying in a ſhip is as eaſily removed from place to place, while the ſhip is under ſail, if its motion be ſteady, as when it is fixed at anchor.

18. Now

18. Now this experience is alone ſufficient to ſhew to us the whole of this law of motion.

19. Since we find, that the ſame power will always produce the ſame change in the motion of any body, whether that body were before moving with a ſwifter or ſlower motion; the change wrought in the motion of a body depends only on the power applied to it, without any regard to the body's former motion: and therefore the degree of motion, which the body already poſſeſſes, having no influence on the power applied to diſturb its operation, the effects of the ſame power will not only be the ſame in all degrees of motion of the body; but we have likewiſe no reaſon to doubt, but that a body perfectly at reſt would receive from any power as much motion, as would be equivalent to the effect of the ſame power applied to that body already in motion.

20. Again, ſuppoſe a body being at reſt, any number of equal powers ſhould be ſucceſſively applied to it; puſhing it forward from time to time in the ſame courſe or direction. Upon the application of the firſt power the body would begin to move; when the ſecond power was applied, it appears from what has been ſaid, that the motion of the body would become double; the third power would treble the motion of the body; and ſo on, till after the operation of the laſt power the motion of the body would be as many times the motion, which the firſt power gave it, as there are powers in number. And the effect of this number of powers will be always the ſame,

ſame, without any regard to the ſpace of time taken up in applying them: ſo that greater or leſſer intervals between the application of each of theſe powers will produce no difference at all in their effects. Since therefore the diſtance of time between the action of each power is of no conſequence; without doubt the effect will ſtill be the ſame, though the powers ſhould all be applied at the very ſame inſtant; or although a ſingle power ſhould be applied equal in ſtrength to the collective force of all theſe powers. Hence it plainly follows, that the degree of motion, into which any body will be put out of a ſtate of reſt by any power, will be proportional to that power. A double power will give twice the velocity, a treble power three times the velocity, and ſo on. The foregoing reaſoning will equally take place, though the body were not ſuppoſed to be at reſt, when the powers began to be applied to it; provided the direction, in which the powers were applied, either conſpired with the action of the body, or was directly oppoſite to it. Therefore if any power be applied to a moving body, and act upon the body either in the direction wherewith the body moves, ſo as to accelerate the body; or if it act directly oppoſite to the motion of the body, ſo as to retard it: in both theſe caſes the change of motion will be proportional to the power applied; nay, the augmentation of the motion in one caſe, and the diminution thereof in the other, will be equal to that degree of motion, into which the ſame power would put the body, had it been at reſt, when the power was applied.

21. FARTHER

21. Farther, a power may be so applied to a moving body, as to act obliquely to the motion of the body. And the effects of such an oblique motion may be deduced from this observation; that as all bodies are continually moving along with the earth, we see that the visible effects of the same power are always the same, in whatever direction the power acts: and therefore the visible effects of any power upon a body, which seems only to be at rest, is always to appearance the same as the real effect would be upon a body truly at rest. Now suppose a body were moving along the line AB (in fig. 2.) and the eye accompanied it with an equal motion in the line CD equidistant from AB; so that when the body is at A, the eye shall be at C, and when the body is advanced to E in the line AB, the eye shall be advanced to F in the line CD, the distances AE and CF being equal. It is evident, that here the body will appear to the eye to be at rest; and the line FEG drawn from the eye through the body shall seem to the eye to be immoveable; though as the body and eye move forward together, this line shall really also move; so that when the body shall be advanced to H and the eye to K, the line FEG shall be transferred into the situation KHL, this line KHL being equidistant from FEG. Now if the body when at E were to receive an impulse in the direction of the line FEG; while the eye is moving on from F to I, and carrying along with it the line FEG, the body will appear to the eye to move along this line FEG: for this is what has just now been said; that while bodies are moving along with the earth, and the spectator's eye partakes of the same motion, the effect of any power upon the body will appear to be what it

it would really have been, had the body been truly at reſt, when the power was applied. From hence it follows, that when the eye is advanced to K, the body will appear ſomewhere in the line KHL. Suppoſe it appear in M; then it is manifeſt, from what has been premiſed at the beginning of this paragraph, that the diſtance HM is equal to what the body would have run upon the line EG, during the time, wherein the eye has paſſed from F to K, provided that the body had been at reſt, when acted upon in E. If it be farther asked, after what manner the body has moved from E to M? I anſwer, through a ſtraight line; for it has been ſhewn above in the explication of the firſt law of motion, that a moving body, from the time it is left to it ſelf, will proceed on in one continued ſtraight line.

22. IF EN be taken equal to HM and NM be drawn; ſince HM is equidiſtant from EN, NM will be equidiſtant from EH. Therefore the effect of any power upon a moving body, when that power acts obliquely to the motion of the body, is to be determined in this manner. Suppoſe the body is moving along the ſtraight line AEB, if when the body is come to E, a power gives it an impulſe in the direction of the line EG, to find what courſe the body will afterwards take we muſt proceed thus. Take in EB any length EH, and in EG take ſuch a length EN, that if the body had been at reſt in E, the power applied to it would have cauſed it to move over EN in the ſame ſpace of time, as it would have employed in paſſing over EH, if the power had not acted at all upon it. Then draw HL equidiſtant from EG, and NM equidiſtant

G from

from EB. After this, if a line be drawn from E to the point M, where theſe two lines meet, the line EM will be the courſe into which the body will be put by the action of the power upon it at E.

23. A MATHEMATICAL reader would here expect in ſome particulars more regular demonſtrations; but as I do not at preſent addreſs my ſelf to ſuch, ſo I hope, what I have now written will render my meaning evident enough to thoſe, who are unacquainted with that kind of reaſoning.

24. Now as we have been ſhewing, that ſome actual force is neceſſary either to put bodies out of a ſtate of reſt into motion, or to change the motion, which they have once received; it is proper here to obſerve, that this quality in bodies, whereby they preſerve their preſent ſtate, with regard to motion or reſt, till ſome active force diſturb them, is called the VIS INERTIAE of matter: and by this property, matter, ſluggiſh and unactive of it ſelf, retains all the power impreſſed upon it, and cannot be made to ceaſe from action, but by the oppoſition of as great a power, as that which firſt moved it. By the degree of this VIS INERTIAE, or power of inactivity, as we ſhall henceforth call it, we primarily judge of the quantity of ſolid matter in each body; for as this quality is inherent in all the bodies, upon which we can make any trial, we conclude it to be a property eſſential to all matter; and as we yet know no reaſon to ſuppoſe, that bodies are compoſed of different kinds of matter, we rather preſume, that the matter of all bodies is the ſame; and that the degree of this

this power of inactivity is in every body proportional to the quantity of the ſolid matter in it. But although we have no abſolute proof, that all the matter in the univerſe is uniform, and poſſeſſes this power of inactivity in the ſame degree; yet we can with certainty compare together the different degrees of this power of inactivity in different bodies. Particularly this power is proportional to the weight of bodies, as Sir ISAAC NEWTON has demonſtrated [a]. However, notwithſtanding that this power of inactivity in any body can be more certainly known, than the quantity of ſolid matter in it; yet ſince there is no reaſon to ſuſpect that one is not proportional to the other, we ſhall hereafter ſpeak without heſitation of the quantity of matter in bodies, as the meaſure of the degree of their power of inactivity.

25. THIS being eſtabliſhed, we may now compare the effects of the ſame power upon different bodies, as hitherto we have ſhewn the effects of different powers upon the ſame body. And here if we limit the word motion to the peculiar ſenſe given to it in philoſophy, we may comprehend all that is to be ſaid upon this head under one ſhort precept; that the ſame power, to whatever body it is applied, will always produce the ſame degree of motion. But here motion does not ſignify the degree of celerity or velocity with which a body moves, in which ſenſe only we have hitherto uſed it; but it is made uſe of particularly in philoſophy to ſignify the force with which a body moves: as if two bodies A and B be-

[a] Princ. Philoſ. L. II. prop. 24. corol. 7. See alſo B. II. Ch. 5. § 3. of this treatiſe.

ing in motion, twice the force would be required to ſtop A as to ſtop B, the motion of A would be eſteemed double the motion of B. In moving bodies, theſe two things are carefully to be diſtinguiſhed; their velocity or celerity, which is meaſured by the ſpace they paſs through during any determinate portion of time; and the quantity of their motion, or the force, with which they will preſs againſt any reſiſtance. Which force, when different bodies move with the ſame velocity, is proportional to the quantity of ſolid matter in the bodies; but if the bodies are equal, this force is proportional to their reſpective velocities, and in other caſes it is proportional both to the quantity of ſolid matter in the body, and alſo to its velocity. To inſtance in two bodies A and B: if A be twice as great as B, and they have both the ſame velocity, the motion of A ſhall be double the motion of B; and if the bodies be equal, and the velocity of A be twice that of B, the motion of A ſhall likewiſe be double that of B; but if A be twice as large as B, and move twice as ſwift, the motion of A will be four times the motion of B; and laſtly, if A be twice as large as B, and move but half as faſt, the degree of their motion ſhall be the ſame.

26. THIS is the particular ſenſe given to the word motion by philoſophers, and in this ſenſe of the word the ſame power always produces the ſame quantity or degree of motion. If the ſame power act upon two bodies A and B, the velocities, it ſhall give to each of them, ſhall be ſo adjuſted to the reſpective bodies, that the ſame degree of motion ſhall be produced in each. If A be twice as great as B, its velocity ſhall be half

that

that of B; if A has three times as much ſolid matter as B, the velocity of A ſhall be one third of the velocity of B; and generally the velocity given to A ſhall bear the ſame proportion to the velocity given to B, as the quantity of ſolid matter contained in the body B bears to the quantity of ſolid matter contained in A.

27. THE reaſon of all this is evident from what has gone before. If a power were applied to B, which ſhould bear the ſame proportion to the power applied to A, as the body B bears to A, the bodies B and A would both receive the ſame velocity; and the velocity, which B will receive from this power, will bear the ſame proportion to the velocity, which it would receive from the action of the power applied to A, as the former of theſe powers bears to the latter: that is, the velocity, which A receives from the power applied to it, will bear to the velocity, which B would receive from the ſame power, the ſame proportion as the body B bears to A.

28. FROM hence we may now paſs to the third law of motion, where this diſtinction between the velocity of a body and its whole motion is farther neceſſary to be regarded, as ſhall immediately be ſhewn; after having firſt illuſtrated the meaning of this law by a familiar inſtance. If a ſtone or other load be drawn by a horſe; the load re-acts upon the horſe, as much as the horſe acts upon the load; for the harneſs, which is ſtrained between them, preſſes againſt the horſe as much as againſt the load; and the progreſſive motion of the

horſe

horſe forward .. hindred as much by the load, as the motion of the load is p. omoted by the endeavour of the horſe: that is, if the horſe put forth the ſame ſtrength, when looſened from the load, he would move himſelf forwards with greater ſwiftneſs in proportion to the difference between the weight of his own body and the weight of himſelf and load together.

29. This inſtance will afford ſome general notion of the meaning of this law. But to proceed to a more philoſophical explication: if a body in motion ſtrike againſt another at reſt, let the body ſtriking be ever ſo ſmall, yet ſhall it communicate ſome degree of motion to the body it ſtrikes againſt, though the leſs that body be in compariſon of that it impinges upon, and the leſs the velocity is, with which it moves, the ſmaller will be the motion communicated. But whatever degree of motion it gives to the reſting body, the ſame it ſhall loſe it ſelf. This is the neceſſary conſequence of the forementioned power of inactivity in matter. For ſuppoſe the two bodies equal, it is evident from the time they meet, both the bodies are to be moved by the ſingle motion of the firſt; therefore the body in motion by means of its power of inactivity retaining the motion firſt given it, ſtrikes upon the other with the ſame force, wherewith it was acted upon it ſelf: but now both the bodies being to be moved by that force, which before moved one only, the enſuing velocity will be the ſame, as if the power, which was applied to one of the bodies, and put it into motion, had been applied to both; whence it appears, that they will proceed forwards,

with half the velocity, which the body firſt in [illegible] that is, the body firſt moved will have loſt hal[illegible] and the other will have gained exactly as much. [illegible] is juſt, provided the bodies keep contiguous after m[illegible] they would always do, if it were not for a certain cau[illegible] often intervenes, and which muſt now be explained. [illegible] upon ſtriking againſt each other, ſuffer an alteration in their figure, having their parts preſſed inwards by the ſtroke, which for the moſt part recoil again afterwards, the bodies endeavouring to recover their former ſhape. This power, whereby bodies are inabled to regain their firſt figure, is uſually called their elaſticity, and when it acts, it forces the bodies from each other, and cauſes them to ſeparate. Now the effect of this elaſticity in the preſent caſe is ſuch, that if the bodies are perfectly elaſtic, ſo as to recoil with as great a force as they are bent with, that they recover their figure in the ſame ſpace of time, as has been taken up in the alteration made in it by their compreſſion together; then this power will ſeparate the bodies as ſwiftly, as they before approached, and acting upon both equally, upon the body firſt in motion contrary to the direction in which it moves, and upon the other as much in the direction of its motion, it will take from the firſt, and add to the other equal degrees of velocity: ſo that the power being ſtrong enough to ſeparate them with as great a velocity, as they approached with, the firſt will be quite ſtopt, and that which was at reſt, will receive all the motion of the other. If the bodies are elaſtic in a leſs degree, the firſt will not loſe all its motion, nor will the other acquire the motion of the firſt, but fall as much ſhort of it, as the other retains.

For

For this rule is never deviated from, that though the degree of elaſticity determines how much more than half its velocity the body firſt in motion ſhall loſe; yet in every caſe the loſs in the motion of this body ſhall be transferred to the other, that other body always receiving by the ſtroke as much motion, as is taken from the firſt.

30. THIS is the caſe of a body ſtriking directly againſt an equal body at reſt, and the reaſoning here uſed is fully confirmed by experience. There are many other caſes of bodies impinging againſt one another: but the mention of theſe ſhall be reſerved to the next chapter, where we intend to be more particular and diffuſive in the proof of theſe laws of motion, than we have been here.

CHAP. II.

Farther proofs of the LAWS of MOTION.

HAVING in the preceding chapter deduced the three laws of motion, delivered by our great philoſopher, from the moſt obvious obſervations, that ſuggeſt them to us; I now intend to give more particular proofs of them, by recounting ſome of the diſcoveries which have been made in philoſophy before Sir ISAAC NEWTON. For as they were all collected by reaſoning upon thoſe laws; ſo the conformity of theſe diſcoveries to experience makes them ſo many proofs of the truth of the principles, from which they were derived.

2. LET

2. LET us begin with the ſubject, which concluded the laſt chapter. Although the body in motion be not equal to the body at reſt, on which it ſtrikes; yet the motion after the ſtroke is to be eſtimated in the ſame manner as above. Let A (in fig. 3.) be a body in motion towards another body B lying at reſt. When A is arrived at B, it cannot proceed farther without putting B into motion; and what motion it gives to B, it muſt loſe it ſelf, that the whole degree of motion of A and B together, if neither of the bodies be elaſtic, ſhall be equal, after the meeting of the bodies, to the ſingle motion of A before the ſtroke. Therefore, from what has been ſaid above, it is manifeſt, that as ſoon as the two bodies are met, they will move on together with a velocity, which will bear the ſame proportion to the original velocity of A, as the body A bears to the ſum of both the bodies.

3. IF the bodies are elaſtic, ſo that they ſhall ſeparate after the ſtroke, A muſt loſe a greater part of its motion, and the ſubſequent motion of B will be augmented by this elaſticity, as much as the motion of A is diminiſhed by it. The elaſticity acting equally between both the bodies, it will communicate to each the ſame degree of motion; that is, it will ſeparate the bodies by taking from the body A and adding to the body B different degrees of velocity, ſo proportioned to their reſpective quantities of matter, that the degree of motion, wherewith A ſeparates from B, ſhall be equal to the degree of motion, wherewith B ſeparates from A. It follows therefore, that the velocity taken from A by the elaſticity bears to the velocity, which the ſame elaſticity adds to B, the

ſame proportion, as B bears to A: conſequently the velocity, which the elaſticity takes from A, will bear the ſame proportion to the whole velocity, wherewith this elaſticity cauſes the two bodies to ſeparate from each other, as the body B bears to the ſum of the two bodies A and B; and the velocity, which is added to B by the elaſticity, bears to the velocity, wherewith the bodies ſeparate, the ſame proportion, as the body A bears to the ſum of the two bodies A and B. Thus is found, how much the elaſticity takes from the velocity of A, and adds to the velocity of B; provided the degree of elaſticity be known, whereby to determine the whole velocity wherewith the bodies ſeparate from each other after the ſtroke [a].

4. After this manner is determined in every caſe the reſult of a body in motion ſtriking againſt another at reſt. The ſame principles will alſo determine the effects, when both bodies are in motion.

5. Let two equal bodies move againſt each other with equal ſwiftneſs. Then the force, with which each of them preſſes forwards, being equal when they ſtrike; each preſſing in its own direction with the ſame energy, neither ſhall ſurmount the other, but both be ſtopt, if they be not elaſtic: for if they be elaſtic, they ſhall from thence recover new motion, and recede from each other, as ſwiftly as they met, if they be perfectly elaſtic; but more ſlowly, if leſs ſo. In the ſame manner, if two bodies of unequal bigneſs ſtrike againſt each other, and their velocities be ſo related, that the velocity

[a] How this degree of elaſticity is to be found by experiment, will be ſhewn below in § 74.

of

of the leſſer body ſhall exceed the velocity of the greater in the ſame proportion, as the greater body exceeds the leſſer (for inſtance, if one body contains twice the ſolid matter as the other, and moves but half as faſt) two ſuch bodies will entirely ſuppreſs each other's motion, and remain from the time of their meeting fixed; if, as before, they are not elaſtic: but, if they are ſo in the higheſt degree, they ſhall recede again, each with the ſame velocity, wherewith they met. For this elaſtic power, as in the preceding caſe, ſhall renew their motion, and preſſing equally upon both, ſhall give the ſame motion to both; that is, ſhall cauſe the velocity, which the leſſer body receives, to bear the ſame proportion to the velocity, which the greater receives, as the greater body bears to the leſſer: ſo that the velocities ſhall bear the ſame proportion to each other after the ſtroke, as before. Therefore if the bodies, by being perfectly elaſtic, have the ſum of their velocities after the ſtroke equal to the ſum of their velocities before the ſtroke, each body after the ſtroke will receive its firſt velocity. And the ſame proportion will hold likewiſe between the velocities, wherewith they go off, though they are elaſtic but in a leſs degree; only then the velocity of each will be leſs in proportion to the defect of elaſticity.

6. IF the velocities, wherewith the bodies meet, are not in the proportion here ſuppoſed; but if one of the bodies, as A, has a ſwifter velocity in compariſon to the velocity of the other; then the effect of this exceſs of velocity in the body A muſt be joined to the effect now mentioned, after the manner of this following example. Let A be twice as great as B, and

move with the ſame ſwiftneſs as B. Here A moves with twice that degree of ſwiftneſs, which would anſwer to the forementioned proportion. For A being double to B, if it moved but with half the ſwiftneſs, wherewith B advances, it has been juſt now ſhewn, that the two bodies upon meeting would ſtop, if they were not elaſtic; and if they were elaſtic, that they would each recoil, ſo as to cauſe A to return with half the velocity, wherewith B would return. But it is evident from hence, that B by encountring A will annul half its velocity, if the bodies be not elaſtic; and the future motion of the bodies will be the ſame, as if A had advanced againſt B at reſt with half the velocity here aſſigned to it. If the bodies be elaſtic, the velocity of A and B after the ſtroke may be thus diſcovered. As the two bodies advance againſt each other, the velocity, with which they meet, is made up of the velocities of both bodies added together. After the ſtroke their elaſticity will ſeparate them again. The degree of elaſticity will determine what proportion the velocity, wherewith they ſeparate, muſt bear to that, wherewith they meet. Divide this velocity, with which the bodies ſeparate into two parts, that one of the parts bear to the other the ſame proportion, as the body A bears to B; and aſcribe the leſſer part to the greater body A, and the greater part of the velocity to the leſſer body B. Then take the part aſcribed to A from the common velocity, which A and B would have had after the ſtroke, if they had not been elaſtic; and add the part aſcribed to B to the ſame common velocity. By this means the true velocities of A and B after the ſtroke will be made known.

7. If

7. IF the bodies are perfectly elaſtic, the great HUYGENS has laid down this rule for finding their motion after concourſe[a]. Any ſtraight line CD (in fig. 4, 5.) being drawn, let it be divided in E, that CE bear the ſame proportion to ED, as the ſwiftneſs of A bore to the ſwiftneſs of B before the ſtroke. Let the ſame line CD be alſo divided in F, that CF bear the ſame proportion to FD, as the body B bears to the body A. Then FG being taken equal to FE, if the point G falls within the line CD, both the bodies ſhall recoil after the ſtroke, and the velocity, wherewith the body A ſhall return, will bear the ſame proportion to the velocity, wherewith B ſhall return, as GC bears to GD; but if the point G falls without the line CD, then the bodies after their concourſe ſhall both proceed to move the ſame way, and the velocity of A ſhall bear to the velocity of B the ſame proportion, that GC bears to GD, as before.

8. IF the body B had ſtood ſtill, and received the impulſe of the other body A upon it; the effect has been already explained in the caſe, when the bodies are not elaſtic. And when they are elaſtic, the reſult of their colliſion is found by combining the effect of the elaſticity with the other effect, in the ſame manner as in the laſt caſe.

9. WHEN the bodies are perfectly elaſtic, the rule of HUYGENS[b] here is to divide the line CD (fig. 6.) in E as before, and to take EG equal to ED. And by theſe points

[a] In oper. poſthum. de Motu corpor. ex percuſſion. prop. 9.

[b] In the above-cited place.

thus;

thus found, the motion of each body after the ſtroke is determined, as before.

10. IN the next place, ſuppoſe the bodies A and B were both moving the ſame way, but A with a ſwifter motion, ſo as to overtake B, and ſtrike againſt it. The effect of the percuſſion or ſtroke, when the bodies are not elaſtic, is diſcovered by finding the common motion, which the two bodies would have after the ſtroke, if B were at reſt, and A were to advance againſt it with a velocity equal to the exceſs of the preſent velocity of A above the velocity of B; and by adding to this common velocity thus found the velocity of B.

11. IF the bodies are elaſtic, the effect of the elaſticity is to be united with this other, as in the former caſes.

12. WHEN the bodies are perfectly elaſtic, the rule of HUYGENS [a] in this caſe is to prolong CD (fig. 7.) and to take in it thus prolonged CE in the ſame proportion to ED, as the greater velocity of A bears to the leſſer velocity of B; after which FG being taken equal to FE, the velocities of the two bodies after the ſtroke will be determined, as in the two preceding caſes.

13. THUS I have given the ſum of what has been written concerning the effects of percuſſion, when two bodies freely in motion ſtrike directly againſt each other; and the reſults here ſet down, as the conſequence of our reaſoning

[a] In the place above-cited.

from

from the laws of motion, anſwer moſt exactly to experience. A particular ſet of experiments has been invented to make trial of theſe effects of percuſſion with the greateſt exactneſs. But I muſt defer theſe experiments, till I have explained the nature of pendulums [a]. I ſhall therefore now proceed to deſcribe ſome of the appearances, which are cauſed in bodies from the influence of the power of gravity united with the general laws of motion; among which the motion of the pendulum will be included.

14. THE moſt ſimple of theſe appearances is, when bodies fall down merely by their weight. In this caſe the body increaſes continually its velocity, during the whole time of its fall, and that in the very ſame proportion as the time increaſes. For the power of gravity acts conſtantly on the body with the ſame degree of ſtrength: and it has been obſerved above in the firſt law of motion, that a body being once in motion will perpetually preſerve that motion without the continuance of any external influence upon it: therefore, after a body has been once put in motion by the force of gravity, the body would continue that motion, though the power of gravity ſhould ceaſe to act any farther upon it; but, if the power of gravity continues ſtill to draw the body down, freſh degrees of motion muſt continually be added to the body; and the power of gravity acting at all times with the ſame ſtrength, equal degrees of motion will conſtantly be added in equal portions of time.

[a] Theſe experiments are deſcribed in § 73.

15. THIS

15. This conclusion is not indeed absolutely true: for we shall find hereafter [a], that the power of gravity is not of the same strength at all distances from the center of the earth. But nothing of this is in the least sensible in any distance, to which we can convey bodies. The weight of bodies is the very same to sense upon the highest towers or mountains, as upon the level ground; so that in all the observations we can make, the forementioned proportion between the velocity of a falling body and the time, in which it has been descending, obtains without any the least perceptible difference.

16. From hence it follows, that the space, through which a body falls, is not proportional to the time of the fall; for since the body increases its velocity, a greater space will be passed over in the same portion of time at the latter part of the fall, than at the beginning. Suppose a body let fall from the point A (in fig. 8.) were to descend from A to B in any portion of time; then if in an equal portion of time it were to proceed from B to C; I say, the space BC is greater than AB; so that the time of the fall from A to C being double the time of the fall from A to B, AC shall be more than double of AB.

17. The geometers have proved, that the spaces, through which bodies fall thus by their weight, are just in a duplicate or two-fold proportion of the times, in which the body has been falling. That is, if we were to take the line DE in the same proportion to AB, as the time, which the body has imployed in falling from A to C, bears to the time of the fall

[a] Book II. Chap. 5.

from

from A to B; then AC will be to DE in the ſame proportion. In particular, if the time of the fall through AC be twice the time of the fall through AB; then DE will be twice AB, and AC twice DE; or AC four times AB. But if the time of the fall through AC had been thrice the time of the fall through AB; DE would have been treble of AB, and AC treble of DE; that is, AC would have been equal to nine times AB.

18. IF a body fall obliquely, it will approach the ground by ſlower degrees, than when it falls perpendicularly. Suppoſe two lines AB, AC (in fig. 9.) were drawn, one perpendicular, and the other oblique to the ground DE: then if a body were to deſcend in the ſlanting line AC; becauſe the power of gravity draws the body directly downwards, if the line AC ſupports the body from falling in that manner, it muſt take off part of the effect of the power of gravity; ſo that in the time, which would have been ſufficient for the body to have fallen through the whole perpendicular line AB, the body ſhall not have paſſed in the line AC a length equal to AB; conſequently the line AC being longer than AB, the body ſhall moſt certainly take up more time in paſſing through AC, than it would have done in falling perpendicularly down through AB.

19. THE geometers demonſtrate, that the time, in which the body will deſcend through the oblique ſtraight line AC, bears the ſame proportion to the time of its deſcent through the perpendicular AB, as the line it ſelf AC bears to AB. And in reſpect to the velocity, which the body will have ac-

quired in the point C, they likewiſe prove, that the length of the time imployed in the deſcent through AC ſo compenſates the diminution of the influence of gravity from the obliquity of this line, that though the force of the power of gravity on the body is oppoſed by the obliquity of the line AC, yet the time of the body's deſcent ſhall be ſo much prolonged, that the body ſhall acquire the very ſame velocity in the point C, as it would have got at the point B by falling perpendicularly down.

20. IF a body were to deſcend in a crooked line, the time of its deſcent cannot be determined in ſo ſimple a manner; but the ſame property, in relation to the velocity, is demonſtrated to take place in all caſes: that is, in whatever line the body deſcends, the velocity will always be anſwerable to the perpendicular height, from which the body has fell. For inſtance, ſuppoſe the body A (in fig. 10.) were hung by a ſtring to the pin B. If this body were let fall, till it came to the point C perpendicularly under B, it will have moved from A to C in the arch of a circle. Then the horizontal line AD being drawn, the velocity of the body in C will be the ſame, as if it had fallen from the point D directly down to C.

21. IF a body be thrown perpendicularly upward with any force, the velocity, wherewith the body aſcends, ſhall continually diminiſh, till at length it be wholly taken away; and from that time the body will begin to fall down again, and paſs over a ſecond time in its deſcent the line, wherein it aſcended; falling through this line with an increaſing velocity in ſuch a manner, that in every point thereof, through which

which it falls, it ſhall have the very ſame velocity, as it had in the ſame place, when it aſcended; and conſequently ſhall come down into the place, whence it firſt aſcended, with the velocity which was at firſt given to it. Thus if a body were thrown perpendicularly up in the line AB (in fig. 11.) with ſuch a force, as that it ſhould ſtop at the point B, and there begin to fall again; when it ſhall have arrived in its deſcent to any point as C in this line, it ſhall there have the ſame velocity, as that wherewith it paſſed by this point C in its aſcent; and at the point A it ſhall have gained as great a velocity, as that wherewith it was firſt thrown upwards. As this is demonſtrated by the geometrical writers; ſo, I think, it will appear evident, by conſidering only, that while the body deſcends, the power of gravity muſt act over again, in an inverted order, all the influence it had on the body in its aſcent; ſo as to give again to the body the ſame degrees of velocity, which it had taken away before.

22. AFTER the ſame manner, if the body were thrown upwards in the oblique ſtraight line CA (in fig. 9.) from the point C, with ſuch a degree of velocity as juſt to reach the point A; it ſhall by its own weight return again through the line AC by the ſame degrees, as it aſcended.

23. AND laſtly, if a body were thrown with any velocity in a line continually incurvated upwards, the like effect will be produced upon its return to the point, whence it was thrown. Suppoſe for inſtance, the body A (in fig. 12.) were hung by a ſtring AB. Then if this body be impelled any

way, it muſt move in the arch of a circle. Let it receive ſuch an impulſe, as ſhall cauſe it to move in the arch A C; and let this impulſe be of ſuch ſtrength, that the body may be carried from A as far as D, before its motion is overcome by its weight: I ſay here, that the body forthwith returning from D, ſhall come again into the point A with the ſame velocity, as that wherewith it began to move.

24. It will be proper in this place to obſerve concerning the power of gravity, that its force upon any body does not at all depend upon the ſhape of the body; but that it continues conſtantly the ſame without any variation in the ſame body, whatever change be made in the figure of the body: and if the body be divided into any number of pieces, all thoſe pieces ſhall weigh juſt the ſame, as they did, when united together in one body: and if the body be of a uniform contexture, the weight of each piece will be proportional to its bulk. This has given reaſon to conclude, that the power of gravity acts upon bodies in proportion to the quantity of matter in them. Whence it ſhould follow, that all bodies muſt fall from equal heights in the ſame ſpace of time. And as we evidently ſee the contrary in feathers and ſuch like ſubſtances, which fall very ſlowly in compariſon of more ſolid bodies; it is reaſonable to ſuppoſe, that ſome other cauſe concurs to make ſo manifeſt a difference. This cauſe has been found by particular experiments to be the air. The experiments for this purpoſe are made thus. They ſet up a very tall hollow glaſs; within which near the top they lodge a feather and ſome very ponderous body, uſually a piece of gold, this

this metal being the moſt weighty of any body known to us. This glaſs they empty of the air contained within it, and by moving a wire, which paſſes through the top of the glaſs, they let the feather and the heavy body fall together; and it is always found, that as the two bodies begin to deſcend at the ſame time, ſo they accompany each other in the fall, and come to the bottom at the very ſame inſtant, as near as the eye can judge. Thus, as far as this experiment can be depended on, it is certain, that the effect of the power of gravity upon each body is proportional to the quantity of ſolid matter, or to the power of inactivity in each body. For in the limited ſenſe, which we have given above to the word motion, it has been ſhewn, that the ſame force gives to all bodies the ſame degree of motion, and different forces communicate different degrees of motion proportional to the reſpective powers [a]. In this caſe, if the power of gravity were to act equally upon the feather, and upon the more ſolid body, the ſolid body would deſcend ſo much ſlower than the feather, as to have no greater degree of motion than the feather: but as both bodies deſcend with equal ſwiftneſs, the degree of motion in the ſolid body is greater than in the feather, bearing the ſame proportion to it, as the quantity of matter in the ſolid body to the quantity of matter in the feather. Therefore the effect of gravity on the ſolid body is greater than on the feather, in proportion to the greater degree of motion communicated; that is, the effect of the power of gravity on the ſolid body bears the ſame proportion to its effect on the feather, as the quanti-

[a] Chap. I. § 25, 26, 27, compared with § 15, &c.

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ty of matter in the ſolid body bears to the quantity of matter in the feather. Thus it is the proper deduction from this experiment, that the power of gravity acts not on the ſurface of bodies only, but penetrates the bodies themſelves moſt intimately, and operates alike on every particle of matter in them. But as the great quickneſs, with which the bodies fall, leaves it ſomething uncertain, whether they do deſcend abſolutely in the ſame time, or only ſo nearly together, that the difference in their ſwift motion is not diſcernable to the eye; this property of the power of gravity, which has here been deduced from this experiment, is farther confirmed by pendulums, whoſe motion is ſuch, that a very minute difference would become ſufficiently ſenſible. This will be farther diſcourſed on in another place [a]; but here I ſhall make uſe of the principle now laid down to explain the nature of what is called the center of gravity in bodies.

25. THE center of gravity is that point, by which if a body be ſuſpended, it ſhall hang at reſt in any ſituation. In a globe of a uniform texture the center of gravity is the ſame with the center of the globe; for as the parts of the globe on every ſide of its center are ſimilarly diſpoſed, and the power of gravity acts alike on every part; it is evident, that the parts of the globe on each ſide of the center are drawn with equal force, and therefore neither ſide can yield to the other; but the globe, if ſupported at its center, muſt of neceſſity hang at reſt. In like manner, if two equal bodies A and B (in

[a] Book II. Chap. 5. § 3.

fig. 13.)

fig. 13.) be hung at the extremities of an inflexible rod C D: which ſhould have no weight; theſe bodies, if the rod be ſupported at its middle E, ſhall equiponderate; and the rod remain without motion. For the bodies being equal and at the ſame diſtance from the point of ſupport E, the power of gravity will act upon each with equal ſtrength, and in all reſpects under the ſame circumſtances; therefore the weight of one cannot overcome the weight of the other. The weight of A can no more ſurmount the weight of B, than the weight of B can ſurmount the weight of A. Again, ſuppoſe a body as A B (in fig. 14.) of a uniform texture in the form of a roller, or as it is more uſually called a cylinder, lying horizontally. If a ſtraight line be drawn between C and D, the centers of the extreme circles of this cylinder; and if this ſtraight line, commonly called the axis of the cylinder, be divided into two equal parts in E: this point E will be the center of gravity of the cylinder. The cylinder being a uniform figure, the parts on each ſide the point E are equal, and ſituated in a perfectly ſimilar manner; therefore this cylinder, if ſupported at the point E, muſt hang at reſt, for the ſame reaſon as the inflexible rod above-mentioned will remain without motion, when ſuſpended at its middle point. And it is evident, that the force applied to the point E, which would uphold the cylinder, muſt be equal to the cylinder's weight. Now ſuppoſe two cylinders of equal thickneſs A B and C D to be joined together at C B, ſo that the two axis's E F, and F G lie in one ſtraight line. Let the axis E F be divided into two equal parts at H, and the axis F G into two equal

equal parts at I. Then becauſe the cylinder AB would be upheld at reſt by a power applied in H equal to the weight of this cylinder, and the cylinder CD would likewiſe be upheld by a power applied in I equal to the weight of this cylinder; the whole cylinder AD will be ſupported by theſe two powers: but the whole cylinder may likewiſe be ſupported by a power applied to K, the middle point of the whole axis EG, provided that power be equal to the weight of the whole cylinder. It is evident therefore, that this power applied in K will produce the ſame effect, as the two other powers applied in H and I. It is farther to be obſerved, that HK is equal to half FG, and KI equal to half EF; for EK being equal to half EG, and EH equal to half EF, the remainder HK muſt be equal to half the remainder FG; ſo likewiſe GK being equal to half GE, and GI equal to half GF, the remainder IK muſt be equal to half the remainder EF. It follows therefore, that HK bears the ſame proportion to KI, as FG bears to EF. Beſides, I believe, my readers will perceive, and it is demonſtrated in form by the geometers, that the whole body of the cylinder CD bears the ſame proportion to the whole body of the cylinder AB, as the axis FG bears to the axis EF [a]. But hence it follows, that in the two powers applied at H and I, the power applied at H bears the ſame proportion to the power applied at I, as KI bears to KH. Now ſuppoſe two ſtrings HL and IM extended upwards, one from the point H and the other from I, and to be laid hold on by two powers, one ſtrong enough to hold up the cylinder AB, and the other of

[a] See Euclid's Elements, Book XII. prop. 13.

ſtrength

ſtrength ſufficient to ſupport the cylinder C D. Here as theſe two powers uphold the whole cylinder, and therefore produce an effect, equal to what would have been produced by a power applied to the point K of ſufficient force to ſuſtain the whole cylinder: it is manifeſt, that if the cylinder be taken away, the axis only being left, and from the point K a ſtring, as K N, be extended, which ſhall be drawn down by a power equivalent to the weight of the cylinder, this power ſhall act againſt the other two powers, as much as the cylinder acted againſt them; and conſequently theſe three powers ſhall be upon a balance, and hold the axis H I fixed between them. But if theſe three powers preſerve a mutual balance, the two powers applied to the ſtrings H L and I M are a balance to each other; the power applied to the ſtring H L bearing the ſame proportion to the power applied to the ſtring I M, as the diſtance I K bears to the diſtance K H. Hence it farther appears, that if an inflexible rod A B (in fig. 15.) be ſuſpended by any point C not in the middle thereof; and if at A the end of the ſhorter arm be hung a weight, and at B the end of the longer arm be alſo hung a weight leſs than the other, and that the greater of theſe weights bears to the leſſer the ſame proportion, as the longer arm of the rod bears to the ſhorter; then theſe two weights will equiponderate: for a power applied at C equal to both theſe weights will ſupport without motion the rod thus charged; ſince here nothing is changed from the preceding caſe but the ſituation of the powers, which are now placed on the contrary ſides of the line, to which they are fixed. Alſo for the

K ſame

ſame reaſon, if two weights A and B (in fig. 16.) were connected together by an inflexible rod CD, drawn from C the center of gravity of A to D the center of gravity of B; and if the rod CD were to be ſo divided in E, that the part DE bear the ſame proportion to the other part CE, as the weight A bears to the weight B: then this rod being ſupported at E will uphold the weights, and keep them at reſt without motion. This point E, by which the two bodies A and B will be ſupported, is called their common center of gravity. And if a greater number of bodies were joined together, the point, by which they could all be ſupported, is called the common center of gravity of them all. Suppoſe (in fig. 17.) there were three bodies A, B, C, whoſe reſpective centers of gravity were joined by the three lines DE, DF, EF: the line DE being ſo divided in G, that DG bear the ſame proportion to GE, as B bears to A; G is the center of gravity common to the two bodies A and B; that is, a power equal to the weight of both the bodies applied to G would ſupport them, and the point G is preſſed as much by the two weights A and B, as it would be, if they were both hung together at that point. Therefore, if a line be drawn from G to F, and divided in H, ſo that GH bear the ſame proportion to HF, as the weight C bears to both the weights A and B, the point H will be the common center of gravity of all the three weights; for H would be their common center of gravity, if both the weights A and B were hung together at G, and the point G is preſſed as much by them in their preſent ſituation, as it would be in that caſe. In the ſame manner from the common center of theſe three weights,

weights, you might proceed to find the common center, if a fourth weight were added, and by a gradual progreſs might find the common center of gravity belonging to any number of weights whatever.

26. As all this is the obvious conſequence of the propoſition laid down for aſſigning the common center of gravity of any two weights, by the ſame propoſition the center of gravity of all figures is found. In a triangle, as A B C (in fig. 18.) the center of gravity lies in the line drawn from the middle point of any one of the ſides to the oppoſite angle, as the line B D is drawn from D the middle of the line A C to the oppoſite angle B [a]; ſo that if from the middle of either of the other ſides, as from the point E in the ſide A B, a line be drawn, as E C, to the oppoſite angle; the point F, where this line croſſes the other line B D, will be the center of gravity of the triangle [b]. Likewiſe D F is equal to half F B, and E F equal to half F C [c]. In a hemiſphere, as A B C (fig. 19.) if from D the center of the baſe the line D B be erected perpendicular to that baſe, and this line be ſo divided in E, that D E be equal to three fifths of B E, the point E is the center of gravity of the hemiſphere [d].

27. It will be of uſe to obſerve concerning the center of gravity of bodies; that ſince a power applied to this center alone can ſupport a body againſt the power of gravity, and

[a] Archimed. de æquipond. prop. 11.
[b] Ibid. prop. 12.
[c] Lucas Valerius De centr. gravit. ſolid. L. I. prop. 2.
[d] Idem L. II. prop. 2.

 hold

hold it fixed at reſt; the effect of the power of gravity on a body is the ſame, as if that whole power were to exert itſelf on the center of gravity only. Whence it follows, that, when the power of gravity acts on a body ſuſpended by any point, if the body is ſo ſuſpended, that the center of gravity of the body can deſcend; the power of gravity will give motion to that body, otherwiſe not: or if a number of bodies are ſo connected together, that, when any one is put into motion, the reſt ſhall, by the manner of their being joined, receive ſuch motion, as ſhall keep their common center of gravity at reſt; then the power of gravity ſhall not be able to produce any motion in theſe bodies, but in all other caſes it will. Thus, if the body AB (in fig. 20, 21.) whoſe center of gravity is C, be hung on the point A, and the center C be perpendicularly under A (as in fig. 20.) the weight of the body will hold it ſtill without motion, becauſe the center C cannot deſcend any lower. But if the body be removed into any other ſituation, where the center C is not perpendicularly under A (as in fig. 21.) the body by its weight will be put into motion towards the perpendicular ſituation of its center of gravity. Alſo if two bodies A, B (in fig. 22.) be joined together by the rod CD lying in an horizontal ſituation, and be ſupported at the point E; if this point be the center of gravity common to the two bodies, their weight will not put them into motion; but if this point E is not their common center of gravity, the bodies will move; that part of the rod CD deſcending, in which the common center of gravity is found. So in like manner, if theſe two bodies were connected together by any more complex contrivance; yet if

if one of the bodies cannot move without ſo moving the other, that their common center of gravity ſhall reſt, the weight of the bodies will not put them in motion, otherwiſe it will.

28. I SHALL proceed in the next place to ſpeak of the mechanical powers. Theſe are certain inſtruments or machines, contrived for the moving great weights with ſmall force; and their effects are all deducible from the obſervation we have juſt been making. They are uſually reckoned in number five; the lever, the wheel and axis, the pulley, the wedge, and the ſcrew; to which ſome add the inclined plane. As theſe inſtruments have been of very ancient uſe, ſo the celebrated ARCHIMEDES ſeems to have been the firſt, who diſcovered the true reaſon of their effects. This, I think, may be collected from what is related of him, that ſome expreſſions, which he uſed to denote the unlimited force of theſe inſtruments, were received as very extraordinary paradoxes: whereas to thoſe, who had underſtood the cauſe of their great force, no expreſſions of that kind could have appeared ſurprizing.

29. ALL the effects of theſe powers may be judged of by this one rule, that, when two weights are applied to any of theſe inſtruments, the weights will equiponderate, if, when put into motion, their velocities will be reciprocally proportional to their reſpective weights. And what is ſaid of weights, muſt of neceſſity be equally underſtood of any other forces equi-

equivalent to weights, ſuch as the force of a man's arm, a ſtream of water, or the like.

30. But to comprehend the meaning of this rule, the reader muſt know, what is to be underſtood by reciprocal proportion; which I ſhall now endeavour to explain, as diſtinctly as I can; for I ſhall be obliged very frequently to make uſe of this term. When any two things are ſo related, that one increaſes in the ſame proportion as the other, they are directly proportional. So if any number of men can perform in a determined ſpace of time a certain quantity of any work, ſuppoſe drain a fiſh-pond, or the like; and twice the number of men can perform twice the quantity of the ſame work, in the ſame time; and three times the number of men can perform as ſoon thrice the work; here the number of men and the quantity of the work are directly proportional. On the other hand, when two things are ſo related, that one decreaſes in the ſame proportion, as the other increaſes, they are ſaid to be reciprocally proportional. Thus if twice the number of men can perform the ſame work in half the time, and three times the number of men can finiſh the ſame in a third part of the time; then the number of men and the time are reciprocally proportional. We ſhewed above [a] how to find the common center of gravity of two bodies, there the diſtances of that common center from the centers of gravity of the two bodies are reciprocally proportional to the reſpective bodies. For CE in fig. 16. being in the ſame pro-

[a] § 25.

portion

portion to E D, as B bears to A; CE is ſo much greater in proportion than E D, as A is leſs in proportion than B.

31. Now this being underſtood, the reaſon of the rule here ſtated will eaſily appear. For if theſe two bodies were put in motion, while the point E reſted, the velocity, wherewith A would move, would bear the ſame proportion to the velocity, wherewith B would move, as E C bears to E D. The velocity therefore of each body, when the common center of gravity reſts, is reciprocally proportional to the body. But we have ſhewn above[a], that if two bodies are ſo connected together, that the putting them in motion will not move their common center of gravity; the weight of thoſe bodies will not produce in them any motion. Therefore in any of theſe mechanical engines, if, when the bodies are put into motion, their velocities are reciprocally proportional to their reſpective weights, whereby the common center of gravity would remain at reſt; the bodies will not receive any motion from their weight, that is, they will equiponderate. But this perhaps will be yet more clearly conceived by the particular deſcription of each mechanical power.

32. The lever was firſt named above. This is a bar made uſe of to ſuſtain and move great weights. The bar is applied in one part to ſome ſtrong ſupport; as the bar A B (in fig. 23, 24.) is applied at the point C to the ſupport D. In ſome other part of the bar, as E, is applied the weight to be ſuſtained or moved; and in a third place, as F, is applied another weight or equivalent force, which is to ſuſtain or move the

[a] § 27.

the weight at E. Now here, if, when the lever ſhould be put in motion, and turned upon the point C, the velocity, wherewith the point F would move, bears the ſame proportion to the velocity, wherewith the point E would move, as the weight at E bears to the weight or force at F; then the lever thus charged will have no propenſity to move either way. If the weight or other force at F be not ſo great as to bear this proportion, the weight at E will not be ſuſtained; but if the force at F be greater than this, the weight at E will be ſurmounted. This is evident from what has been ſaid above[a], when the forces at E and F are placed (as in fig. 23.) on different ſides of the ſupport D. It will appear alſo equally manifeſt in the other caſe, by continuing the bar BC in fig. 24. on the other ſide of the ſupport D, till CG be equal to CF, and by hanging at G a weight equivalent to the power at F; for then, if the power at F were removed, the two weights at G and E would counterpoize each other, as in the former caſe: and it is evident, that the point F will be lifted up by the weight at G with the ſame degree of force, as by the other power applied to F; ſince, if the weight at E were removed, a weight hung at F equal to that at G would balance the lever, the diſtances CG and CF being equal.

33. If the two weights, or other powers, applied to the lever do not counterbalance each other; a third power may be applied in any place propoſed of the lever, which ſhall

[a] Pag. 65, 68.

hold

hold the whole in a juſt counterpoize. Suppoſe (in fig. 25.) the two powers at E and F did not equiponderate, and it were required to apply a third power to the point G, that might be ſufficient to balance the lever. Find what power in F would juſt counterbalance the power in E; then if the difference between this power and that, which is actually applied at F, bear the ſame proportion to the third power to be applied at G, as the diſtance CG bears to CF; the lever will be counterpoized by the help of this third power, if it be ſo applied as to act the ſame way with the power in F, when that power is too ſmall to counterbalance the power in E; but otherwiſe the power in G muſt be ſo applied, as to act againſt the power in F. In like manner, if a lever were charged with three, or any greater number of weights or other powers, which did not counterpoize each other, another power might be applied in any place propoſed, which ſhould bring the whole to a juſt balance. And what is here ſaid concerning a plurality of powers, may be equally applied to all the following caſes.

34. IF the lever ſhould conſiſt of two arms making an angle at the point C (as in fig. 26.) yet if the forces are applied perpendicularly to each arm, the ſame proportion will hold between the forces applied, and the diſtances of the center, whereon the lever reſts, from the points to which they are applied. That is, the weight at E will be to the force in F in the ſame proportion, as CF bears to CE.

35. BUT whenever the forces applied to the lever act obliquely to the arm, to which they are applied (as in fig. 27.)

 then

then the ſtrength of the forces is to be eſtimated by lines let fall from the center of the lever to the directions, wherein the forces act. To balance the levers in fig. 27, the weight or other force at F will bear the ſame proportion to the weight at E, as the diſtance C E bears to C G the perpendicular let fall from the point C upon the line, which denotes the direction wherein the force applied to F acts: for here, if the lever be put into motion, the power applied to F will begin to move in the direction of the line F G; and therefore its firſt motion will be the ſame, as the motion of the point G.

36. WHEN two weights hang upon a lever, and the point, by which the lever is ſupported, is placed in the middle between the two weights, that the arms of the lever are both of equal length; then this lever is particularly called a balance; and equal weights equiponderate as in common ſcales. When the point of ſupport is not equally diſtant from both weights, it conſtitutes that inſtrument for weighing, which is called a ſteelyard. Though both in common ſcales, and the ſteelyard, the point, on which the beam is hung, is not uſually placed juſt in the ſame ſtraight line with the points, that hold the weights, but rather a little above (as in fig. 28.) where the lines drawn from the point C, whereon the beam is ſuſpended, to the points E and F, on which the weights are hung, do not make abſolutely one continued line. If the three points E, C, and F were in one ſtraight line, thoſe weights, which equiponderated, when the beam hung horizontally, would alſo equiponderate in any other ſituation. But we ſee in theſe inſtruments, when they are charged with weights, which

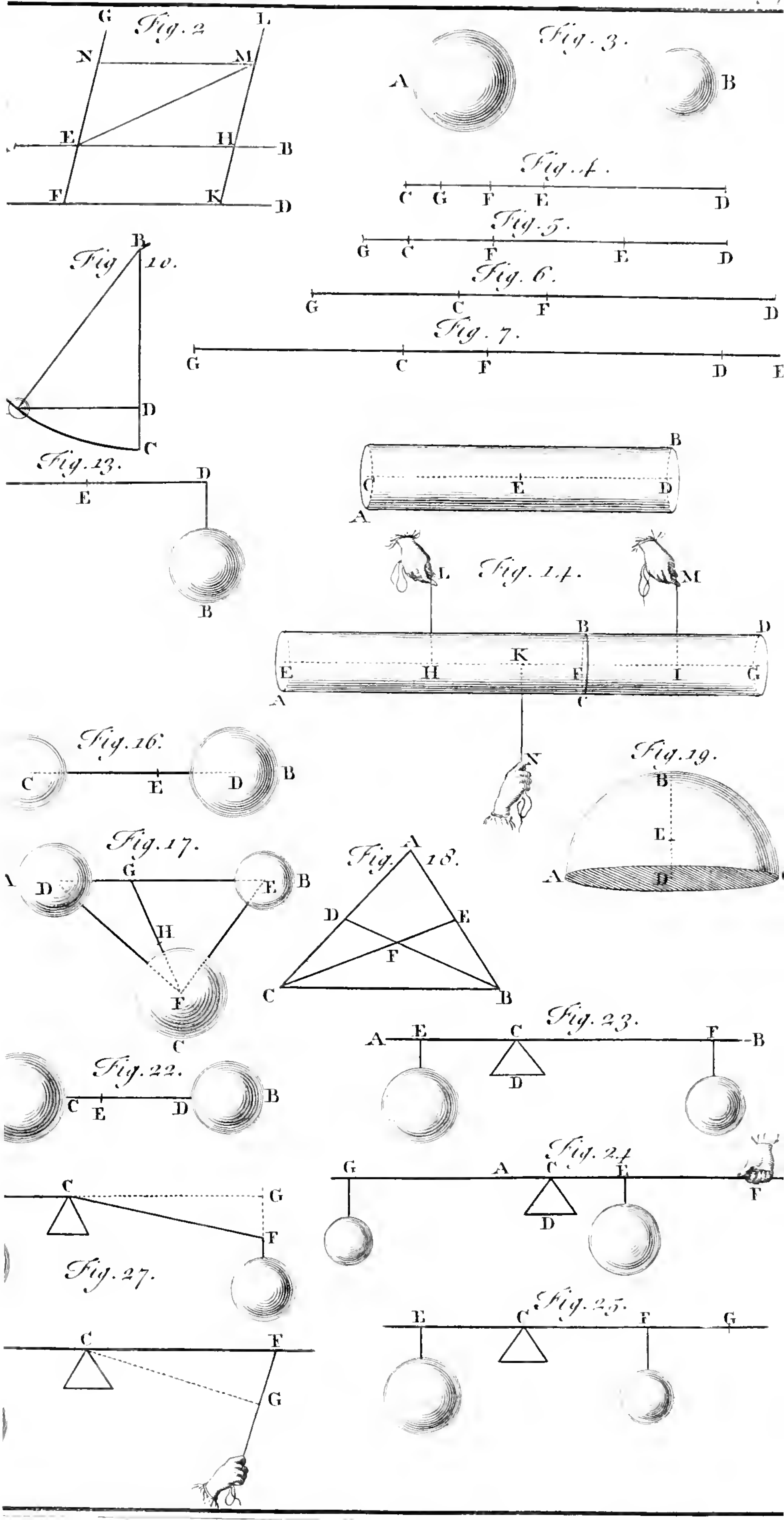
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 6
Fig. 7
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Fig. 25
Fig. 27

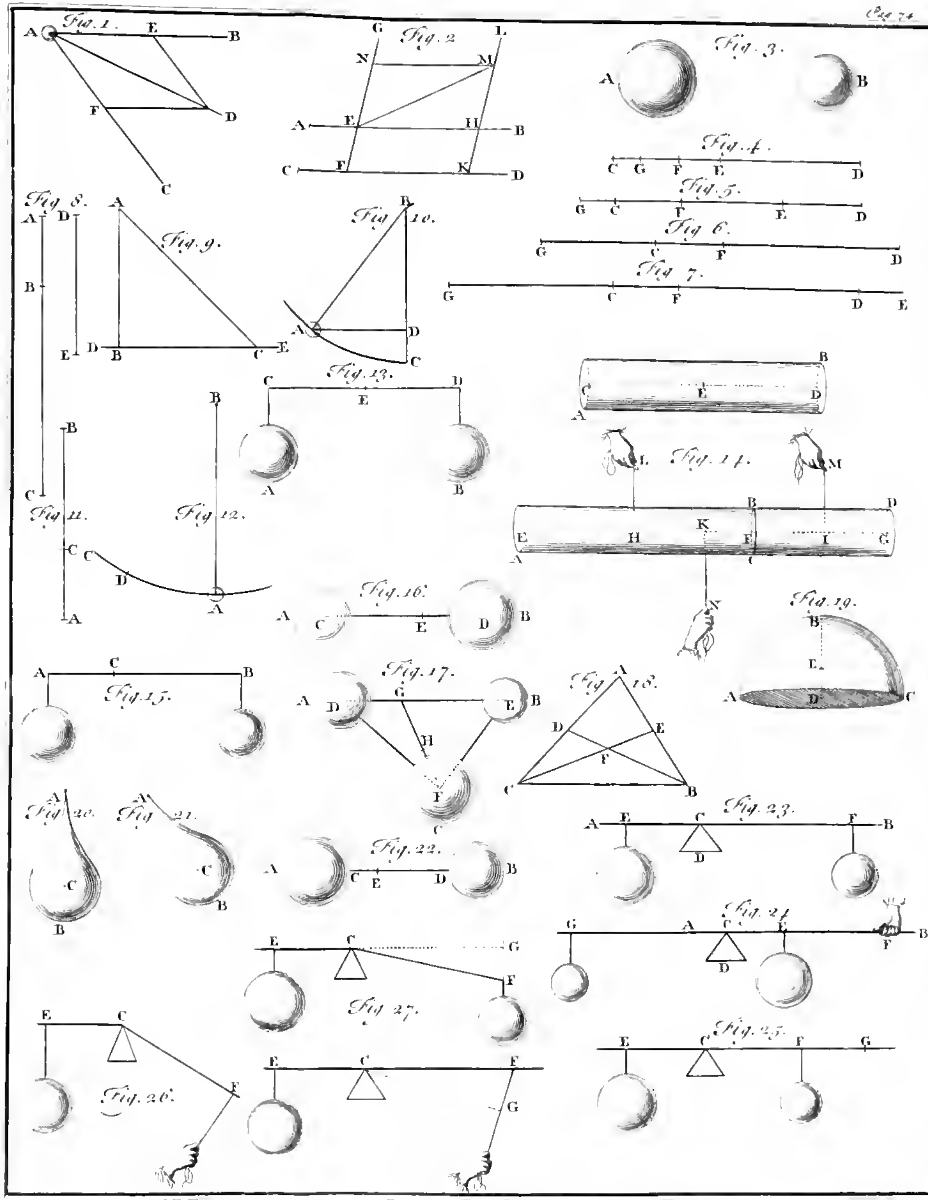

Fig. 1.
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Fig. 14.
Fig. 15.
Fig. 16.
Fig. 17.
Fig. 18.
Fig. 19.
Fig. 20.
Fig. 21.
Fig. 22.
Fig. 23.
Fig. 24.
Fig. 25.
Fig. 26.
Fig. 27.

which equiponderate with the beam hanging horizontally; that, if the beam be inclined either way, the weight moſt elevated ſurmounts the other, and deſcends, cauſing the beam to ſwing, till by degrees it recovers its horizontal poſition. This effect ariſes from the forementioned ſtructure: for by this ſtructure theſe inſtruments are levers compoſed of two arms, which make an angle at the point of ſupport (as in fig. 29, 30.) the firſt of which repreſents the caſe of the common balance, the ſecond the caſe of the ſteelyard. In the firſt, where CE and CF are equal, equal weights hung at E and F will equiponderate, when the points E and F are in an horizontal ſituation. Suppoſe the lines EG and FH to be perpendicular to the horizon, then they will denote the directions, wherein the forces applied to E and F act. Therefore the proportion between the weights at E and F, which ſhall equiponderate, are to be judged of by perpendiculars, as CI, CK, let fall from C upon EG and FH: ſo that the weights being equal, the lines CI, CK, muſt be equal alſo, when the weights equiponderate. But I believe my readers will eaſily ſee, that ſince CE and CF are equal, the lines CI and CK will be equal, when the points E and F are horizontally ſituated.

37. IF this lever be ſet into any other poſition (as in fig. 31.) then the weight, which is raiſed higheſt, will outweigh the other. Here, if the point F be raiſed higher than E, the perpendicular CK will be longer than CI: and therefore the weights would equiponderate, if the weight at F

were leſs than the weight at E. But the weight at F is equal to that at E; therefore is greater, than is neceſſary to counterbalance the weight at E, and conſequently will outweigh it, and draw the beam of the lever down.

38. In like manner in the caſe of the ſteelyard (fig. 32.) if the weights at E and F are ſo proportioned, as to equiponderate, when the points E and F are horizontally ſituated; then in any other ſituation of this lever the weight, which is raiſed higheſt, will preponderate. That is, if in the horizontal ſituation of the points E and F the weight at F bears the ſame proportion to the weight at E, as CI bears to CK; then, if the point F be raiſed higher than E (as in fig. 32.) the weight at F ſhall bear a greater proportion to the weight at E, than CI bears to CK.

39. Farther a lever may be hung upon an axis, and then the two arms of the lever need not be continuous, but fixed to different parts of this axis; as in fig. 33, where the axis AB is ſupported by its two extremities A and B. To this axis one arm of the lever is fixed at the point C, the other at the point D. Now here, if a weight be hung at E, the extremity of that arm, which is fixed to the axis at the point C; and another weight be hung at F, the extremity of the arm, which is fixed on the axis at D; then theſe weights will equiponderate, when the weight at E bears the ſame proportion to the weight at F, as the arm DF bears to CE.

40. This

40. THIS is the caſe, if both the arms are perpendicular to the axis, and lie (as the geometers expreſs themſelves) in the ſame plane; or, in other words, if the arms are ſo fixed perpendicularly upon the axis, that, when one of them lies horizontally, the other ſhall alſo be horizontal. If either arm ſtand not perpendicular to the axis; then, in determining the proportion between the weights, inſtead of the length of that arm, you muſt uſe the perpendicular let fall upon the axis from the extremity of that arm. If the arms are not ſo fixed as to become horizontal, at the ſame time; the method of aſſigning the proportion between the weights is analogous to that made uſe of above in levers, which make an angle at the point, whereon they are ſupported.

41. FROM this caſe of the lever hung on an axis, it is eaſy to make a tranſition to another mechanical power, the wheel and axis.

42. THIS inſtrument is a wheel fixed on a roller, the roller being ſupported at each extremity ſo as to turn round freely with the wheel, in the manner repreſented in fig. 34, where AB is the wheel, CD the roller, and EF its two ſupports. Now ſuppoſe a weight G hung by a cord wound round the roller, and another weight H hung by a cord wound about the wheel the contrary way: that theſe weights may ſupport each other, the weight H muſt bear the ſame proportion to the weight G, as the thickneſs of the roller bears to the diameter of the wheel.

43. SUPPOSE

43. Suppose the line kl to be drawn through the middle of the roller; and from the place of the roller, where the cord, on which the weight G hangs, begins to leave the roller, as at m, let the line mn be drawn perpendicularly to kl; and from the point, where the cord holding the weight H begins to leave the wheel, as at o, let the line op be drawn perpendicular to kl. This being done, the two lines op and mn repreſent two arms of a lever fixed on the axis kl; conſequently the weight H will bear to the weight G the ſame proportion, as mn bears to op. But mn bears the ſame proportion to op, as the thickneſs of the roller bears to the diameter of the wheel; for mn is half the thickneſs of the roller, and op half the diameter of the wheel.

44. If the wheel be put into motion, and turned once round, that the cord, on which the weight G hangs, be wound once more round the axis; then at the ſame time the cord, whereon the weight H hangs, will be wound off from the wheel one circuit. Therefore the velocity of the weight G will bear the ſame proportion to the velocity of the weight H, as the circumference of the roller to the circumference of the wheel. But the circumference of the roller bears the ſame proportion to the circumference of the wheel, as the thickneſs of the roller bears to the diameter of the wheel, conſequently the velocity of the weight G bears to the velocity of the weight H the ſame proportion, as the thickneſs of the roller bears to the diameter of the wheel, which is the proportion that the weight H bears to the weight G. Therefore as before in the lever, ſo here alſo the general rule laid down

down above is verified, that the weights equiponderate, when their velocities would be reciprocally proportional to their respective weights.

45. IN like manner, if on the same axis two wheels of different sizes are fixed (as in fig. 35.) and a weight hung on each; the weights will equiponderate, if the weight hung on the greater wheel bear the same proportion to the weight hung on the lesser, as the diameter of the lesser wheel bears to the diameter of the greater.

46. IT is usual to join many wheels together in the same frame, which by the means of certain teeth, formed in the circumference of each wheel, shall communicate motion to each other. A machine of this nature is represented in fig. 36. Here ABC is a winch, upon which is fixed a small wheel D indented with teeth, which move in the like teeth of a larger wheel EF fixed on the axis GH. Let this axis carry another wheel I, which shall move in like manner a greater wheel KL fixed on the axis MN. Let this axis carry another small wheel O, which after the same manner shall turn about a larger wheel PQ fixed on the roller RS, on which a cord shall be wound, that holds a weight, as T. Now the proportion required between the weight T and a power applied to the winch at A sufficient to support the weight, will most easily be estimated, by computing the proportion, which the velocity of the point A would bear to the velocity of the weight. If the winch be turned round, the point A will describe a circle as AV. Suppose the wheel EF to have ten times the number of teeth, as the

the wheel D; then the winch muſt turn round ten times to carry the wheel E F once round. If the wheel K L has alſo ten times the number of teeth, as I, the wheel I muſt turn round ten times to carry the wheel K L once round; and conſequently the winch A B C muſt turn round an hundred times to turn the wheel K L once round. Laſtly, if the wheel P Q has ten times the number of teeth, as the wheel O, the winch muſt turn about one thouſand times in order to turn the wheel P Q, or the roller R S once round. Therefore here the point A muſt have gone over the circle A V a thouſand times, in order to lift the weight T through a ſpace equal to the circumference of the roller R S: whence it follows, that the power applied at A will balance the weight T, if it bear the ſame proportion to it, as the circumference of the roller to one thouſand times the circle A V; or the ſame proportion as half the thickneſs of the roller bears to one thouſand times A B.

47. I shall now explain the effect of the pulley. Let a weight hang by a pulley, as in fig. 37. Here it is evident, that the power A, by which the weight B is ſupported, muſt be equal to the weight; for the cord C D is equally ſtrained between them; and if the weight B move, the power A muſt move with equal velocity. The pulley E has no other effect, than to permit the power A to act in another direction, than it muſt have done, if it had been directly applied to ſupport the weight without the intervention of any ſuch inſtrument.

48. Again, let a weight be ſupported, as in fig. 38; where the weight A is fixed to the pulley B, and the cord, by which

which the weight is upheld, is annexed by one extremity to a hook C, and at the other end is held by the power D. Here the weight is ſupported by a cord doubled; inſomuch that although the cord were not ſtrong enough to hold the weight ſingle, yet being thus doubled it might ſupport it. If the end of the cord held by the power D were hung on the hook C, as well as the other end; then, when both ends of the cord were tied to the hook, it is evident, that the hook would bear the whole weight; and each end of the ſtring would bear againſt the hook with the force of half the weight only, ſeeing both ends together bear with the force of the whole. Hence it is evident, that, when the power D holds one end of the weight, the force, which it muſt exert to ſupport the weight, muſt be equal to juſt half the weight. And the ſame proportion between the weight and power might be collected from comparing the reſpective velocities, with which they would move; for it is evident, that the power muſt move through a ſpace equal to twice the diſtance of the pulley from the hook, in order to lift the pulley up to the hook.

49. IT is equally eaſy to eſtimate the effect, when many pulleys are combined together, as in fig. 39, 40; in the firſt of which the under ſet of pulleys, and conſequently the weight is held by ſix ſtrings; and in the latter figure by five: therefore in the firſt of theſe figures the power to ſupport the weight, muſt be one ſixth part only of the weight, and in the latter figure the power muſt be one fifth part.

50. There are two other ways of ſupporting a weight by pulleys, which I ſhall particularly conſider.

51. One of theſe ways is repreſented in fig. 41. Here the weight being connected to the pulley B, a power equal to half the weight A would ſupport the pulley C, if applied immediately to it. Therefore the pulley C is drawn down with a force equal to half the weight A. But if the pulley D were to be immediately ſupported by half the force, with which the pulley C is drawn down, this pulley D will uphold the pulley C; ſo that if the pulley D be upheld with a force equal to one fourth part of the weight A, that force will ſupport the weight. But, for the ſame reaſon as before, if the power in E be equal to half the force neceſſary to uphold the pulley D; this pulley, and conſequently the weight A, will be upheld: therefore, if the power in E be one eighth part of the weight A, it will ſupport the weight.

52. Another way of applying pulleys to a weight is repreſented in fig. 42. To explain the effect of pulleys thus applied, it will be proper to conſider different weights hanging, as in fig. 43. Here, if the power and weights balance each other, the power A is equal to the weight B; the weight C is equal to twice the power A, or the weight B; and for the ſame reaſon the weight D is equal to twice the weight C, or equal to four times the power A. It is evident therefore, that all the three weights B, C, D together are equal to ſeven times the power A. But if theſe three weights were joined in one, they would produce the caſe of fig. 40: ſo that in that figure the weight

weight A, where there are three pulleys, is ſeven times the power B. If there had been but two pulleys, the weight would have been three times the power; and if there had ben four pulleys, the weight would have been fifteen times the power.

53. THE wedge is next to be conſidered. The form of this inſtrument is ſufficiently known. When it is put under any weight (as in fig. 44.) the force, with which the wedge will lift the weight, when drove under it by a blow upon the end A B, will bear the ſame proportion to the force, where-with the blow would act on the weight, if directly applied to it; as the velocity, which the wedge receives from the blow, bears to the velocity, wherewith the weight is lifted by the wedge.

54. THE ſcrew is the fifth mechanical power. There are two ways of applying this inſtrument. Sometimes it is ſcrewed into a hole, as in fig. 45, where the ſcrew A B is ſcrewed through the plank C D. Sometimes the ſcrew is applied to the teeth of a wheel, as in fig. 46, where the thred of the ſcrew A B turns in the teeth of a wheel C D. In both theſe caſes, if a bar, as A E, be fixed to the end A of the ſcrew; the force, wherewith the end B of the ſcrew in fig. 45 is forced down, and the force, wherewith the teeth of the wheel C D in fig. 44 are held, bears the ſame proportion to the power applied to the end E of the bar; as the velocity, wherewith the end E will move, when the ſcrew is turned, bears to the velocity, wherewith the end B of the ſcrew in fig. 43, or the teeth of the wheel C D in fig. 46, will be moved.

55. The inclined plane affords alſo a means of raiſing a weight with leſs force, than what is equal to the weight it ſelf. Suppoſe it were required to raiſe the globe A (in fig. 47.) from the ground BC up to the point, whoſe perpendicular height from the ground is ED. If this globe be drawn along the ſlant DF, leſs force will be required to raiſe it, than if it were lifted directly up. Here if the force applied to the globe bear the ſame proportion only to its weight, as ED bears to FD, it will be ſufficient to hold up the globe; and therefore any addition to that force will put it in motion, and draw it up; unleſs the globe, by preſſing againſt the plane, whereon it lies, adhere in ſome degree to the plane. This indeed it muſt always do more or leſs, ſince no plane can be made ſo abſolutely ſmooth as to have no inequalities at all; nor yet ſo infinitely hard, as not to yield in the leaſt to the preſſure of the weight. Therefore the globe cannot be laid on ſuch a plane, whereon it will ſlide with perfect freedom, but they muſt in ſome meaſure rub againſt each other; and this friction will make it neceſſary to imploy a certain degree of force more, than what is neceſſary to ſupport the globe, in order to give it any motion. But as all the mechanical powers are ſubject in ſome degree or other to the like impediment from friction; I ſhall here only ſhew what force would be neceſſary to ſuſtain the globe, if it could lie upon the plane without cauſing any friction at all. And I ſay, that if the globe were drawn by the cord GH, lying parallel to the plane DF; and the force, wherewith the cord is pulled, bear the ſame proportion to the weight of the globe, as ED bears to DF; this

this force will ſuſtain the globe. In order to the making proof of this, let the cord G H be continued on, and turned over the pulley I, and let the weight K be hung to it. Now I ſay, if this weight bears the ſame proportion to the globe A, as D E bears to D F, the weight will ſupport the globe. I think it is very manifeſt, that the center of the globe A will lie in one continued line with the cord H G. Let L be the center of the globe, and M the center of gravity of the weight K. In the firſt place let the weight hang ſo, that a line drawn from L to M ſhall lie horizontally; and I ſay, if the globe be moved either up or down the plane D F, the weight will ſo move along with it, that the center of gravity common to both the weights ſhall continue in this line L M, and therefore ſhall in no caſe deſcend. To prove this more fully, I ſhall depart a little from the method of this treatiſe, and make uſe of a mathematical propoſition or two: but they are ſuch, as any perſon, who has read EUCLID's ELEMENTS, will fully comprehend; and are in themſelves ſo evident, that, I believe, my readers, who are wholly ſtrangers to geometrical writings, will make no difficulty of admitting them. This being premiſed, let the globe be moved up, till its center be at G, then will M the center of gravity of the weight K be ſunk to N; ſo that M N ſhall be equal to G L. Draw N G croſſing the line M L in O; then I ſay, that O is the common center of gravity of the two weights in this their new ſituation. Let G P be drawn perpendicular to M L; then G L will bear the ſame proportion to G P, as D F bears to D E; and M N being equal to G L, M N will bear the ſame proportion

to

to GP, as DF bears to DE. But NO bears the ſame proportion to OG, as MN bears to GP; conſequently NO will bear the ſame proportion to OG, as DF bears to DE. In the laſt place, the weight of the globe A bears the ſame proportion to the other weight K, as DF bears to DE; therefore NO bears the ſame proportion to OG, as the weight of the globe A bears to the weight K. Whence it follows, that, when the center of the globe A is in G, and the center of gravity of the weight K is in N, O will be the center of gravity common to both the weights. After the ſame manner, if the globe had been cauſed to deſcend, the common center of gravity would have been found in this line ML. Since therefore no motion of the globe either way will make the common center of gravity deſcend, it is manifeſt, from what has been ſaid above, that the weights A and K counterpoize each other.

56. I shall now conſider the caſe of pendulums. A pendulum is made by hanging a weight to a line, ſo that it may ſwing backwards and forwards. This motion the geometers have very carefully conſidered, becauſe it is the moſt commodious inſtrument of any for the exact meaſurement of time.

57. I have obſerved already [a], that if a body hanging perpendicularly by a ſtring, as the body A (in fig. 48.) hangs by the ſtring AB, be put ſo into motion, as to be made to aſcend up the circular arch AC; then as ſoon as it has arrived

[a] § 23.

at

at the higheſt point, to which the motion, that the body has received, will carry it; it will immediately begin to deſcend, and at A will receive again as great a degree of motion, as it had at firſt. This motion therefore will carry the body up the arch A D, as high as it aſcended before in the arch A C. Conſequently in its return through the arch D A it will acquire again at A its original velocity, and advance a ſecond time up the arch A C as high as at firſt; by this means continuing without end its reciprocal motion. It is true indeed, that in fact every pendulum, which we can put in motion, will gradually leſſen its ſwing, and at length ſtop, unleſs there be ſome power conſtantly applied to it, whereby its motion ſhall be renewed; but this ariſes from the reſiſtance, which the body meets with both from the air, and the ſtring by which it is hung: for as the air will give ſome obſtruction to the progreſs of the body moving through it; ſo alſo the ſtring, whereon the body hangs, will be a farther impediment; for this ſtring muſt either ſlide on the pin, whereon it hangs, or it muſt bend to the motion of the weight; in the firſt there muſt be ſome degree of friction, and in the latter the ſtring will make ſome reſiſtance to its inflection. However, if all reſiſtance could be removed, the motion of a pendulum would be perpetual.

58. BUT to proceed, the firſt property, I ſhall take notice of in this motion, is, that the greater arch the pendulous body moves through, the greater time it takes up: though the length of time does not increaſe in ſo great a proportion as the arch. Thus if C D be a greater arch, and E F a leſſer, where C A is equal to A D, and E A equal to A F; the body, when

when it ſwings through the greater arch CD, ſhall take up in its ſwing from C to D a longer time than in ſwinging from E to F, when it moves only in that leſſer arch; or the time in which the body let fall from C will deſcend through the arch CA is greater than the time, in which it will deſcend through the arch EA, when let fall from E. But the firſt of theſe times will not hold the ſame proportion to the latter, as the firſt arch CA bears to the other arch EA; which will appear thus. Let CG and EH be two horizontal lines. It has been remarked above [a], that the body in falling through the arch CA will acquire as great a velocity at the point A, as it would have gained by falling directly down through GA; and in falling through the arch EA it will acquire in the point A only that velocity, which it would have got in falling through HA. Therefore, when the body deſcends through the greater arch CA, it ſhall gain a greater velocity, than when it paſſes only through the leſſer; ſo that this greater velocity will in ſome degree compenſate the greater length of the arch.

59. The increaſe of velocity, which the body acquires in falling from a greater height, has ſuch an effect, that, if ſtraight lines be drawn from A to C and E, the body would fall through the longer ſtraight line CA juſt in the ſame time, as through the ſhorter ſtraight line EA. This is demonſtrated by the geometers, who prove, that if any circle, as ABCD (fig. 49.) be placed in a perpendicular ſituation; a body ſhall fall obliquely through every line, as AB drawn from the loweſt point A in the circle to any other point in the circum-

[a] § 20.

ference

ference juſt in the ſame time, as would be imployed by the body in falling perpendicularly down through the diameter C A. But the time in which the body will deſcend through the arch, is different from the time, which it would take up in falling through the line A B.

60. IT has been thought by ſome, that becauſe in very ſmall arches this correſpondent ſtraight line differs but little from the arch itſelf; therefore the deſcent through this ſtraight line would be performed in ſuch ſmall arches nearly in the ſame time as through the arches themſelves: ſo that if a pendulum were to ſwing in ſmall arches, half the time of a ſingle ſwing would be nearly equal to the time, in which a body would fall perpendicularly through twice the length of the pendulum. That is, the whole time of the ſwing, according to this opinion, will be four fold the time required for the body to fall through half the length of the pendulum; becauſe the time of the body's falling down twice the length of the pendulum is half the time required for the fall through one quarter of this ſpace, that is through half the pendulum's length. However there is here a miſtake; for the whole time of the ſwing, when the pendulum moves through ſmall arches, bears to the time required for a body to fall down through half the length of the pendulum very nearly the ſame proportion, as the circumference of a circle bears to its diameter; that is very nearly the proportion of 355 to 113, or little more than the proportion of 3 to 1. If the pendulum takes ſo great a ſwing, as to paſs over an arch equal to one ſixth part of the whole circumference of the

circle, it will ſwing 115 times, while it ought according to this proportion to have ſwung 117 times; ſo that, when it ſwings in ſo large an arch, it loſes ſomething leſs than two ſwings in an hundred. If it ſwing through $\frac{1}{10}$ only of the circle, it ſhall not loſe above one vibration in 160. If it ſwing in $\frac{1}{20}$ of the circle, it ſhall loſe about one vibration in 690. If its ſwing be confined to $\frac{1}{40}$ of the whole circle, it ſhall loſe very little more than one ſwing in 2600. And if it take no greater a ſwing than through $\frac{1}{60}$ of the whole circle, it ſhall not loſe one ſwing in 5800.

61. Now it follows from hence, that, when pendulums ſwing in ſmall arches, there is very nearly a conſtant proportion obſerved between the time of their ſwing, and the time, in which a body would fall perpendicularly down through half their length. And we have declared above, that the ſpaces, through which bodies fall, are in a two fold proportion of the times, which they take up in falling [a]. Therefore in pendulums of different lengths, ſwinging throug hſmall arches, the lengths of the pendulums are in a two fold or duplicate proportion of the times, they take in ſwinging; ſo that a pendulum of four times the length of another ſhall take up twice the time in each ſwing, one of nine times the length will make one ſwing only for three ſwings of the ſhorter, and ſo on.

62. This proportion in the ſwings of different pendulums not only holds in ſmall arches; but in large ones alſo,

[a] § 17.

provided

provided they be ſuch, as the geometers call ſimilar; that is, if the arches bear the ſame proportion to the whole circumferences of their reſpective circles. Suppoſe (in fig. 48.) A B, C D to be two pendulums. Let the arch E F be deſcribed by the motion of the pendulum A B, and the arch G H be deſcribed by the pendulum C D; and let the arch E F bear the ſame proportion to the whole circumference, which would be formed by turning the pendulum A B quite round about the point A, as the arch G H bears to the whole circumference, that would be formed by turning the pendulum C D quite round the point C. Then I ſay, the proportion, which the length of the pendulum A B bears to the length of the pendulum C D, will be two fold of the proportion, which the time taken up in the deſcription of the arch E F bears to the time employed in the deſcription of the arch G H.

63. THUS pendulums, which ſwing in very ſmall arches, are nearly an equal meaſure of time. But as they are not ſuch an equal meaſure to geometrical exactneſs; the mathematicians have found out a method of cauſing a pendulum ſo to ſwing, that, if its motion were not obſtructed by any reſiſtance, it would always perform each ſwing in the ſame time, whether it moved through a greater, or a leſſer ſpace. This was firſt diſcovered by the great HUYGENS, and is as follows. Upon the ſtraight line A B (in fig. 49.) let the circle C D E be ſo placed, as to touch the ſtraight line in the point C. Then let this circle roll along upon the ſtraight line A B, as a coach-wheel rolls along upon the ground. It is evident, that, as

ſoon as ever the circle begins to move, the point C in the circle will be lifted off from the ſtraight line AB; and in the motion of the circle will deſcribe a crooked courſe, which is repreſented by the line CFGH. Here the part CH of the ſtraight line included between the two extremities C and H of the line CFGH will be equal to the whole circumference of the circle CDE; and if CH be divided into two equal parts at the point I, and the ſtraight line IK be drawn perpendicular to CH, this line IK will be equal to the diameter of the circle CDE. Now in this line if a body were to be let fall from the point H, and were to be carried by its weight down the line HGK, as far as the point K, which is the loweſt point of the line CFGH; and if from any other point G a body were to be let fall in the ſame manner; this body, which falls from G, will take juſt the ſame time in coming to K, as the body takes up, which falls from H. Therefore if a pendulum can be ſo hung, that the ball ſhall move in the line AGFE, all its ſwings, whether long or ſhort, will be performed in the ſame time; for the time, in which the ball will deſcend to the point K, is always half the time of the whole ſwing. But the ball of a pendulum will be made to ſwing in this line by the following means. Let KI (in fig. 52.) be prolonged upwards to L, till IL is equal to IK. Then let the line LMH equal and like to KH be applied, as in the figure between the points L and H, ſo that the point which in this line LMH anſwers to the point H in the line KH ſhall be applied to the point L, and the point anſwering to the point K ſhall be applied to the point H. Alſo let ſuch another line LNC be applied between L and C in the ſame manner.

manner. This preparation being made; if a pendulum be hung at the point L of ſuch a length, that the ball thereof ſhall reach to K; and if the ſtring ſhall continually bend againſt the lines HML and LNC, as the pendulum ſwings to and fro; by this means the ball ſhall conſtantly keep in the line CKH.

64. Now in this pendulum, as all the ſwings, whether long or ſhort, will be performed in the ſame time; ſo the time of each will exactly bear the ſame proportion to the time required for a body to fall perpendicularly down, through half the length of the pendulum, that is from I to K, as the circumference of a circle bears to its diameter.

65. It may from hence be underſtood in ſome meaſure, why, when pendulums ſwing in circular arches, the times of their ſwings are nearly equal, if the arches are ſmall, though thoſe arches be of very unequal lengths; for if with the ſemidiameter LK the circular arch OKP be deſcribed, this arch in the lower part of it will differ very little from the line CKH.

66. It may not be amiſs here to remark, that a body will fall in this line CKH (fig. 53.) from C to any other point, as Q or R in a ſhorter ſpace of time, than if it moved through the ſtraight line drawn from C to the other point; or through any other line whatever, that can be drawn between theſe two points.

67. But as I have obſerved, that the time, which a pendulum takes in ſwinging, depends upon its length; I ſhall now ſay ſomething concerning the way, in which this length of the pendulum is to be eſtimated. If the whole ball of the pendulum could be crouded into one point, this length, by which the motion of the pendulum is to be computed, would be the length of the ſtring or rod. But the ball of the pendulum muſt have a ſenſible magnitude, and the ſeveral parts of this ball will not move with the ſame degree of ſwiftneſs; for thoſe parts, which are fartheſt from the point, whereon the pendulum is ſuſpended, muſt move with the greateſt velocity. Therefore to know the time in which the pendulum ſwings, it is neceſſary to find that point of the ball, which moves with the ſame degree of velocity, as if the whole ball were to be contracted into that point.

68. This point is not the center of gravity, as I ſhall now endeavour to ſhew. Suppoſe the pendulum AB (in fig. 54.) compoſed of an inflexible rod AC and ball CB, to be fixed on the point A, and lifted up into an horizontal ſituation. Here if the rod were not fixed to the point A, the body CB would deſcend directly with the whole force of its weight; and each part of the body would move down with the ſame degree of ſwiftneſs. But when the rod is fixed at the point A, the body muſt fall after another manner; for the parts of the body muſt move with different degrees of velocity, the parts more remote from A deſcending with a ſwifter motion, than the parts nearer to A; ſo that the body will receive a kind of rolling motion while it deſcends. But it has been

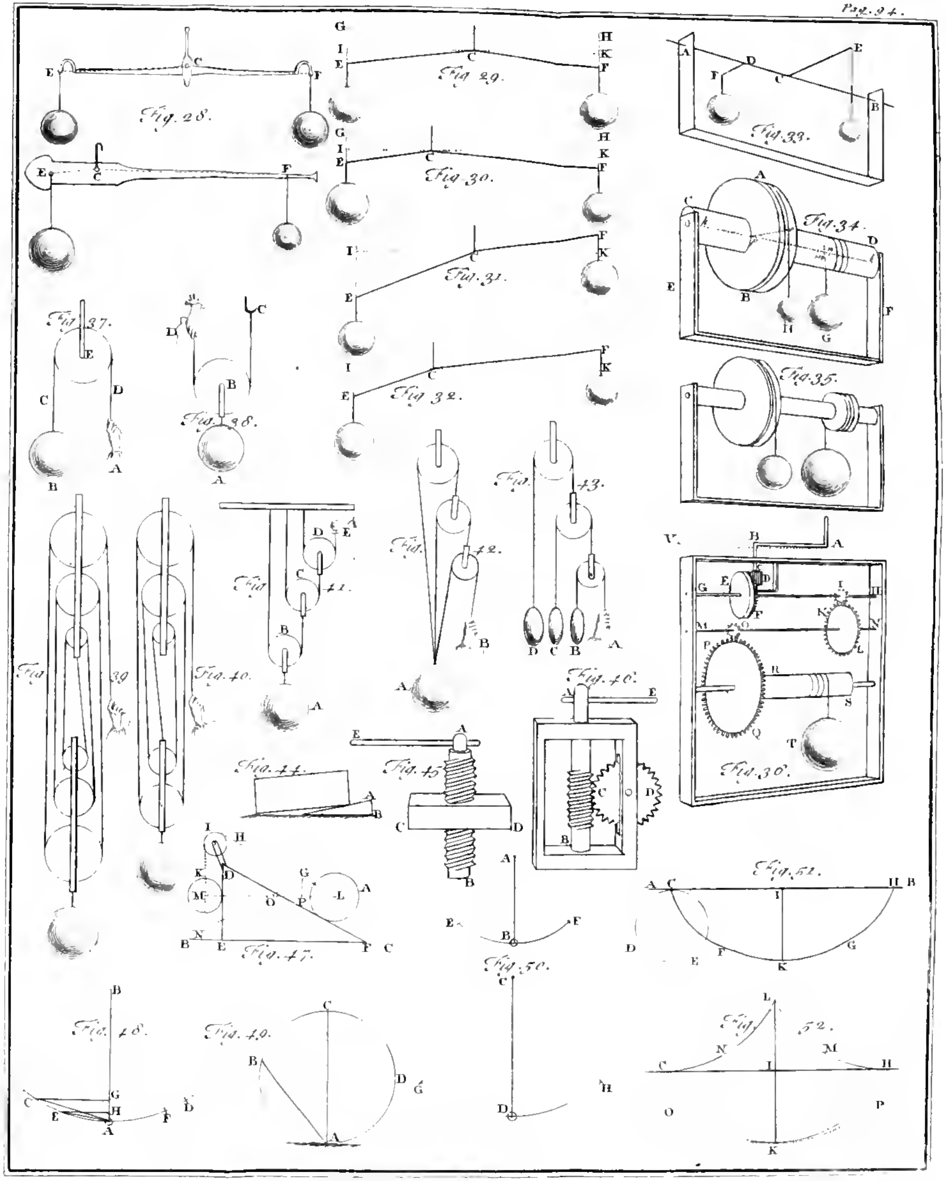
Fig. 28.
Fig. 29.
Fig. 30.
Fig. 31.
Fig. 32.
Fig. 33.
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Fig. 46.
Fig. 47.
Fig. 48.
Fig. 49.
Fig. 50.
Fig. 51.
Fig. 52.

been obſerved above, that the effect of gravity upon any body is the ſame, as if the whole force were exerted on the body's center of gravity [a]. Since therefore the power of gravity in drawing down the body muſt alſo communicate to it the rolling motion juſt deſcribed; it ſeems evident, that the center of gravity of the body cannot be drawn down as ſwiftly, as when the power of gravity has no other effect to produce on the body, than merely to draw it downward. If therefore the whole matter of the body CB could be crouded into its center of gravity, ſo that being united into one point, this rolling motion here mentioned might give no hindrance to its deſcent; this center would deſcend faſter, than it can now do. And the point, which now deſcends as faſt, as if the whole matter of the body CB were crouded into it, will be farther removed from the point A, than the center of gravity of the body CB.

69. AGAIN, ſuppoſe the pendulum AB (in fig. 55.) to hang obliquely. Here the power of gravity will operate leſs upon the ball of the pendulum, than before: but the line DE being drawn ſo, as to ſtand perpendicular to the rod AC of the pendulum; the force of gravity upon the body CB, now it is in this ſituation, will produce the ſame effect, as if the body were to glide down an inclined plane in the poſition of DE. But here the motion of the body, when the rod is fixed to the point A, will not be equal to the uninterrupted deſcent of the body down this plane; for the body

[a] § 27.

will

will here alſo receive the ſame kind of rotation in its motion, as before; ſo that the motion of the center of gravity will in like manner be retarded; and the point, which here deſcends with that degree of ſwiftneſs, which the body would have, if not hindered by being fixed to the point A; that is, the point, which deſcends as faſt, as if the whole body were crouded into it, will be as far removed from the point A, as before.

70. THIS point, by which the length of the pendulum is to be eſtimated, is called the center of oſcillation. And the mathematicians have laid down general directions, whereby to find this center in all bodies. If the globe AB (in fig. 56.) be hung by the ſtring CD, whoſe weight need not be regarded, the center of oſcillation is found thus. Let the ſtraight line drawn from C to D be continued through the globe to F. That it will paſs through the center of the globe is evident. Suppoſe E to be this center of the globe; and take the line G of ſuch a length, that it ſhall bear the ſame proportion to ED, as ED bears to EC. Then EH being made equal to $\frac{2}{5}$ of G, the point H ſhall be the center of oſcillation [a]. If the weight of the rod CD is too conſiderable to be neglected, divide CD (fig. 57) in I, that DI be equal to $\frac{1}{3}$, part of CD; and take K in the ſame proportion to CI, as the weight of the globe AB to the weight of the rod CD. Then having found H, the center of oſcillation of the globe, as before, divide IK in L, ſo that IL ſhall bear the ſame pro-

[a] Hugen. Horolog. oſcillat. pag. 141, 142.

portion

portion to LH, as the line CH bears to K; and L ſhall be the center of oſcillation of the whole pendulum.

71. THIS computation is made upon ſuppoſition, that the center of oſcillation of the rod CD, if that were to ſwing alone without any other weight annexed, would be the point I. And this point would be the true center of oſcillation, ſo far as the thickneſs of the rod is not to be regarded. If any one chuſes to take into conſideration the thickneſs of the rod, he muſt place the center of oſcillation thereof ſo much below the point I, that eight times the diſtance of the center from the point I ſhall bear the ſame proportion to the thickneſs of the rod, as the thickneſs of the rod bears to its length CD[a].

72. IT has been obſerved above, that when a pendulum ſwings in an arch of a circle, as here in fig. 58, the pendulum AB ſwings in the circular arch CD; if you draw an horizontal line, as EF, from the place whence the pendulum is let fall, to the line AG, which is perpendicular to the horizon: then the velocity, which the pendulum will acquire in coming to the point G, will be the ſame, as any body would acquire in falling directly down from F to G. Now this is to be underſtood of the circular arch, which is deſcribed by the center of oſcillation of the pendulum. I ſhall here farther obſerve, that if the ſtraight line EG be drawn from the point, whence the pendulum falls, to the loweſt point of the arch; in the ſame or in equal pendulums the velocity, which the

[a] See Hugen. Horolog. Oſcillat. p. 142.

pendulum acquires in G, is proportional to this line: that is, if the pendulum, after it has descended from E to G, be taken back to H, and let fall from thence, and the line HG be drawn; the velocity, which the pendulum shall acquire in G by its descent from H, shall bear the same proportion to the velocity, which it acquires in falling from E to G, as the straight line H G bears to the straight line E G.

73. We may now proceed to those experiments upon the percussion of bodies, which I observed above might be made with pendulums. This expedient for examining the effects of percussion was first proposed by our late great architect Sir CHRISTOPHER WREN. And it is as follows. Two balls, as A and B (in fig. 59.) either equal or unequal, are hung by two strings from two points C and D, so that, when the balls hang down without motion, they shall just touch each other, and the strings be parallel. Here if one of these balls be removed to any distance from its perpendicular situation, and then let fall to descend and strike against the other; by the last preceding paragraph it will be known, with what velocity this ball shall return into its first perpendicular situation, and consequently with what force it shall strike against the other ball; and by the height to which this other ball ascends after the stroke, the velocity communicated to this ball will be discovered. For instance, let the ball A be taken up to E, and from thence be let fall to strike against B, passing over in its descent the circular arch EF. By this impulse let B fly up to G, moving through the circular arch HG. Then EI and GK being drawn horizontally, the

the ball A will ſtrike againſt B with the velocity, which it would acquire in falling directly down from I; and the ball B has received a velocity, wherewith, if it had been thrown directly upward, it would have aſcended up to K. Likewiſe if ſtraight lines be drawn from E to F and from H to G, the velocity of A, wherewith it ſtrikes, will bear the ſame proportion to the velocity, which B has received by the blow, as the ſtraight line EF bears to the ſtraight line HG. In the ſame manner by noting the place to which A aſcends after the ſtroke, its remaining velocity may be compared with that, wherewith it ſtruck againſt B. Thus may be experimented the effects of the body A ſtriking againſt B at reſt. If both the bodies are lifted up, and ſo let fall as to meet and impinge againſt each other juſt upon the coming of both into their perpendicular ſituation; by obſerving the places into which they move after the ſtroke, the effects of their percuſſion in all theſe caſes may be found in the ſame manner as before.

74. Sir ISAAC NEWTON has deſcribed theſe experiments; and has ſhewn how to improve them to a greater exactneſs by making allowance for the reſiſtance, which the air gives to the motion of the balls [a]. But as this reſiſtance is exceeding ſmall, and the manner of allowing for it is delivered by himſelf in very plain terms, I need not enlarge upon it here. I ſhall rather ſpeak to a diſcovery, which he made by theſe experiments upon the elaſticity of bodies. It has been explained above [b], that when two bodies ſtrike, if they be not elaſtic,

[a] Princip. Philoſ. pag. 22. [b] Chap. 1 § 29.

they remain contiguous after the ſtroke; but that if they are elaſtic, they ſeparate, and that the degree of their elaſticity determines the proportion between the celerity wherewith they ſeparate, and the celerity wherewith they meet. Now our author found, that the degree of elaſticity appeared in the ſame bodies always the ſame, with whatever degree of force they ſtruck; that is, the celerity wherewith they ſeparated, always bore the ſame proportion to the celerity wherewith they met: ſo that the elaſtic power in all the bodies, he made trial upon, exerted it ſelf in one conſtant proportion to the compreſſing force. Our author made trial with balls of wool bound up very compact, and found the celerity with which they receded, to bear about the proportion of 5 to 9 to the celerity wherewith they met; and in ſteel he found nearly the ſame proportion; in cork the elaſticity was ſomething leſs; but in glaſs much greater; for the celerity, wherewith balls of that material ſeparated after percuſſion, he found to bear the proportion of 15 to 16 to the celerity wherewith they met [a].

75. I SHALL finiſh my diſcourſe on pendulums, with this farther obſervation only, that the center of oſcillation is alſo the center of another force. If a body be fixed to any point, and being put in motion turns round it; the body, if uninterrupted by the power of gravity or any other means, will continue perpetually to move about with the ſame equable motion. Now the force, with which ſuch a body

[a] Princip. Philoſ. pag. 25.

4 moves,

moves, is all united in the point, which in relation to the power of gravity is called the center of oſcillation. Let the cylinder ABCD (in fig. 60.) whoſe axis is EF, be fixed to the point E. And ſuppoſing the point E to be that on which the cylinder is ſuſpended, let the center of oſcillation be found in the axis EF, as has been explained above [a]. Let G be that center: then I ſay, that the force, wherewith this cylinder turns round the point E, is ſo united in the point G, that a ſufficient force applied in that point ſhall ſtop the motion of the cylinder, in ſuch a manner, that the cylinder ſhould immediately remain without motion, though it were to be looſened from the point E at the ſame inſtant, that the impediment was applied to G: whereas, if this impediment had been applied to any other point of the axis, the cylinder would turn upon the point, where the impediment was applied. If the impediment had been applied between E and G, the cylinder would ſo turn on the point, where the impediment was applied, that the end BC would continue to move on the ſame way it moved before along with the whole cylinder; but if the impediment were applied to the axis farther off from E than G, the end AD of the cylinder would ſtart out of its preſent place that way in which the cylinder moved. From this property of the center of oſcillation, it is alſo called the center of percuſſion. That excellent mathematician, Dr. BROOK TAYLOR, has farther improved this doctrine concerning the center of percuſſion, by ſhewing, that if through this point G a line, as GHI, be drawn perpendicular to EF, and lying

[a] § 71.

in

in the courſe of the body's motion; a ſufficient power applied to any point of this line will have the ſame effect, as the like power applied to G[a]: ſo that as we before ſhewed the center of percuſſion within the body on its axis; by this means we may find this center on the ſurface of the body alſo, for it will be where this line HI croſſes that ſurface.

76. I shall now proceed to the laſt kind of motion, to be treated on in this place, and ſhew what line the power of gravity will cauſe a body to deſcribe, when it is thrown forwards by any force. This was firſt diſcovered by the great Galileo, and is the principle, upon which engineers ſhould direct the ſhot of great guns. But as in this caſe bodies deſcribe in their motion one of thoſe lines, which in geometry are called conic ſections; it is neceſſary here to premiſe a deſcription of thoſe lines. In which I ſhall be the more particular, becauſe the knowledge of them is not only neceſſary for the preſent purpoſe, but will be alſo required hereafter in ſome of the principal parts of this treatiſe.

77. The firſt lines conſidered by the ancient geometers were the ſtraight line and the circle. Of theſe they compoſed various figures, of which they demonſtrated many properties, and reſolved divers problems concerning them. Theſe problems they attempted always to reſolve by the deſcribing ſtraight lines and circles. For inſtance, let a ſquare ABCD (fig. 61.) be propoſed, and let it be required to make ano-

[a] See Method. Increment. prop. 25.

ther

ther ſquare in any aſſigned proportion to this. Prolong one ſide, as D A, of this ſquare to E, till A E bear the ſame proportion to A D, as the new ſquare is to bear to the ſquare A C. If the oppoſite ſide B C of the ſquare A C be alſo prolonged to F, till B F be equal to A E, and E F be afterwards drawn, I ſuppoſe my readers will eaſily conceive, that the figure ABFE will bear to the ſquare A B C D the ſame proportion, as the line A E bears to the line A D. Therefore the figure A B F E will be equal to the new ſquare, which is to be found, but is not it ſelf a ſquare, becauſe the ſide A E is not of the ſame length with the ſide E F. But to find a ſquare equal to the figure A B F E you muſt proceed thus. Divide the line D E into two equal parts in the point G, and to the center G with the interval G D deſcribe the circle D H E I; then prolong the line A B, till it meets the circle in K; and make the ſquare AKLM, which ſquare will be equal to the figure ABFE, and bear to the ſquare A B C D the ſame proportion, as the line A E bears to A D.

78. I SHALL not proceed to the proof of this, having only here ſet it down as a ſpecimen of the method of reſolving geometrical problems by the deſcription of ſtraight lines and circles. But there are ſome problems, which cannot be reſolved by drawing ſtraight lines or circles upon a plane. For the management therefore of theſe they took into conſideration ſolid figures, and of the ſolid figures they found that, which is called a cone, to be the moſt uſeful.

79. A

79. A CONE is thus defined by EUCLIDE in his elements of geometry [a]. If to the ſtraight line A B (in fig. 62.) another ſtraight line, as A C, be drawn perpendicular, and the two extremities B and C be joined by a third ſtraight line compoſing the triangle A C B (for ſo every figure is called, which is included under three ſtraight lines:) then the two points A and B being held fixed, as two centers, and the triangle A C B being turned round upon the line A B, as on an axis; the line A C will deſcribe a circle, and the figure A C B will deſcribe a cone, of the form repreſented by the figure BCDEF (fig. 63.) in which the circle CDEF is uſually called the baſe of the cone, and B the vertex.

80. NOW by this figure may ſeveral problems be reſolved, which cannot by the ſimple deſcription of ſtraight lines and circles upon a plane. Suppoſe for inſtance, it were required to make a cube, which ſhould bear any aſſigned proportion to ſome other cube named. I need not here inform my readers, that a cube is the figure of a dye. This problem was much celebrated among the ancients, and was once inforced by the command of an oracle. This problem may be performed by a cone thus. Firſt make a cone from a triangle, whoſe ſide A C ſhall be half the length of the ſide B C. Then on the plane A B C D (fig. 64.) let the line E F be exhibited equal in length to the ſide of the cube propoſed; and let the line F G be drawn perpendicular to E F, and of ſuch a length, that it bear the ſame proportion to E F, as the

[a] Lib. XI. Def.

cube

cube to be ſought is required to bear to the cube propoſed. Through the points E, F, and G let the circle F H I be deſcribed. Then let the line E F be prolonged beyond F to K, that F K be equal to F E, and let the triangle F K L, having all its ſides F K, K L, L F equal to each other, be hung down perpendicularly from the plane A B C D. After this, let another plane M N O P be extended through the point L, ſo as to be equidiſtant from the former plane A B C D, and in this plane let the line Q L R be drawn ſo, as to be equidiſtant from the line E F K. All this being thus prepared, let ſuch a cone, as was above directed to be made, be ſo applied to the plane M N O P, that it touch this plane upon the line Q R, and that the vertex of the cone be applied to the point L. This cone, by cutting through the firſt plane A B C D, will croſs the circle F H I before deſcribed. And if from the point S, where the ſurface of this cone interſects the circle, the line S T be drawn ſo, as to be equidiſtant from the line E F; the line F T will be equal to the ſide of the cube ſought: that is, if there be two cubes or dyes formed, the ſide of one being equal to E F, and the ſide of the other equal to F T; the former of theſe cubes ſhall bear the ſame proportion to the latter, as the line E F bears to F G.

81. INDEED this placing a cone to cut through a plane is not a practicable method of reſolving problems. But when the geometers had diſcovered this uſe of the cone, they applied themſelves to conſider the nature of the lines, which will be produced by the interſection of the ſurface of a cone

and a plane; whereby they might be enabled both to reduce these kinds of solutions to practice, and also to render their demonstrations concise and elegant.

82. Whenever the plane, which cuts the cone, is equidistant from another plane, that touches the cone on the side; (which is the case of the present figure;) the line, wherein the plane cuts the surface of the cone, is called a parabola. But if the plane, which cuts the cone, be so inclined to this other, that it will pass quite through the cone (as in fig. 65.) such a plane by cutting the cone produces the figure called an ellipsis, in which we shall hereafter shew the earth and other planets to move round the sun. If the plane, which cuts the cone, recline the other way (as in fig. 66.) so as not to be parallel to any plane, whereon the cone can lie, nor yet to cut quite through the cone; such a plane shall produce in the cone a third kind of line, which is called an hyperbola. But it is the first of these lines named the parabola, wherein bodies, that are thrown obliquely, will be carried by the force of gravity; as I shall here proceed to shew, after having first directed my readers how to describe this sort of line upon a plane, by which the form of it may be seen.

83. To any straight line AB (fig. 67.) let a straight ruler CD be so applied, as to stand against it perpendicularly. Upon the edge of this ruler let another ruler EF be so placed, as to move along upon the edge of the first ruler CD, and keep always perpendicular to it. This being so disposed, let any point, as G, be taken in the line AB, and let a string equal

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in length to the ruler EF be faſtened by one end to the point G, and by the other to the extremity F of the ruler EF. Then if the ſtring be held down to the ruler EF by a pin H, as is repreſented in the figure; the point of this pin, while the ruler EF moves on the ruler CD, ſhall deſcribe the line IKL, which will be one part of the curve line, whoſe deſcription we were here to teach: and by applying the rulers in the like manner on the other ſide of the line AB, we may deſcribe the other part IM of this line. If the diſtance CG be equal to half the line EF in fig. 64, the line MIL will be that very line, wherein the plane ABCD in that figure cuts the cone.

84. THE line AI is called the axis of the parabola MIL, and the point G is called the focus.

85. NOW by comparing the effects of gravity upon falling bodies, with what is demonſtrated of this figure by the geometers, it is proved, that every body thrown obliquely is carried forward in one of theſe lines, the axis whereof is perpendicular to the horizon.

86. THE geometers demonſtrate, that if a line be drawn to touch a parabola in any point, as the line AB (in fig. 68.) touches the parabola CD, whoſe axis is YZ, in the point E; and ſeveral lines FG, HI, KL be drawn parallel to the axis of the parabola: then the line FG will be to HI in the duplicate proportion of EF to EH, and FG to KL in the duplicate proportion of EF to EK; likewiſe HI to KL in the duplicate proportion of EH to EK. What is to be underſtood by duplicate or two-fold

 pro-

proportion, has been already explained [a]. Accordingly I mean here, that if the line M be taken to bear the ſame proportion to E H, as E H bears to E F, H I will bear the ſame proportion to F G, as M bears to E F ; and if the line N bears the ſame proportion to E K, as E K bears to E F, K L will bear the ſame proportion to F G, as N bears to E F ; or if the line O bear the ſame proportion to E K, as E K bears to E H, K L will bear the ſame proportion to H I, as O bears to E H.

87. This property is eſſential to the parabola, being ſo connected with the nature of the figure, that every line poſſeſſing this property is to be called by this name.

88. Now ſuppoſe a body to be thrown from the point A (in fig. 69.) towards B in the direction of the line A B. This body, if left to it ſelf, would move on with a uniform motion through this line A B. Suppoſe the eye of a ſpectator to be placed at the point C juſt under the point A ; and let us imagine the earth to be ſo put into motion along with the body, as to carry the ſpectator's eye along the line C D parallel to A B ; and that the eye ſhould move on with the ſame velocity, wherewith the body would proceed in the line A B, if it were to be left to move without any diſturbance from its gravitation towards the earth. In this caſe if the body moved on without being drawn towards the earth, it would appear to the ſpectator to be at reſt. But if the power of gravity exerted it ſelf on the body, it would appear to the ſpe-

[a] Chap. 2. § 17.

ctator

ctator to fall directly down. Suppofe at the diftance of time, wherein the body by its own progreffive motion would have moved from A to E, it fhould appear to the fpectator to have fallen through a length equal to EF: then the body at the end of this time will actually have arrived at the point F. If in the fpace of time, wherein the body would have moved by its progreffive motion from A to G, it would have appeared to the fpectator to have fallen down the fpace GH: then the body at the end of this greater interval of time will be arrived at the point H. Now if the line AFHI be that, through which the body actually paffes; from what has here been faid, it will follow, that this line is one of thofe, which I have been defcribing under the name of the parabola. For the diftances EF, GH, through which the body is feen to fall, will increafe in the duplicate proportion of the times [a]; but the lines AE, AG will be proportional to the times wherein they would have been defcribed by the fingle progreffive motion of the body: therefore the lines EF, GH will be in the duplicate proportion of the lines AF, AG; and the line AFHI poffeffes the property of the parabola.

89. IF the earth be not fuppofed to move along with the body, the cafe will be a little different. For the body being conftantly drawn directly towards the center of the earth, the body in its motion will be drawn in a direction a little oblique to that, wherein it would be drawn by the earth in motion, as before fuppofed. But the diftance to the center of the

[a] See above Ch. 2. § 17.

earth

earth bears so vast a proportion to the greatest length, to which we can throw bodies, that this obliquity does not merit any regard. From the sequel of this discourse it may indeed be collected, what line the body being thrown thus would be found to describe, allowance being made for this obliquity of the earth's action[a]. This is the discovery of Sir Is. Newton; but has no use in this place. Here it is abundantly sufficient to consider the body as moving in a parabola.

90. The line, which a projected body describes, being thus known, practical methods have been deduced from hence for directing the shot of great guns to strike any object desired. This work was first attempted by Galileo, and soon after farther improved by his scholar Torricelli; but has lately been rendred more complete by the great Mr. Cotes, whose immature death is an unspeakable loss to mathematical learning. If it be required to throw a body from the point A (in fig. 70.) so as to strike the point B; through the points A, B draw the straight line CD, and erect the line AE perpendicular to the horizon, and of four times the height, from which a body must fall to acquire the velocity, wherewith the body is intended to be thrown. Through the points A and E describe a circle, that shall touch the line CD in the point A. Then from the point B draw the line BF perpendicular to the horizon, intersecting the circle in the points G and H. This being done, if the body be projected directly towards either of these points G or H, it shall fall upon the point B; but with this difference, that, if it be thrown

[a] From B. II. Ch. 3.

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in the direction AG, it ſhall ſooner arrive at B, than if it were projected in the direction AH. When the body is projected in the direction AG; the time, it will take up in arriving at B, will bear the ſame proportion to the time, wherein it would fall down through one fourth part of AE, as AG bears to half AE. But when the body is thrown in the direction of AH, the time of its paſſing to B will bear the ſame proportion to the time, wherein it would fall through one fourth part of AE, as AH bears to half AE.

91. If the line AI be drawn ſo as to divide the angle under EAD in the middle, and the line IK be drawn perpendicular to the horizon; this line will touch the circle in the point I, and if the body be thrown in the direction AI, it will fall upon the point K: and this point K is the fartheſt point in the line AD, which the body can be made to ſtrike, without increaſing its velocity.

92. The velocity, wherewith the body every where moves, may be found thus. Suppoſe the body to move in the parabola AB (fig. 71.) Erect AC perpendicular to the horizon, and equal to the height, from which a body muſt fall to acquire the velocity, wherewith the body ſets out from A. If you take any points as D and E in the parabola, and draw DF and EG parallel to the horizon; the velocity of the body in D will be equal to what a body will acquire in falling down by its own weight through CF, and in E the velocity will be the ſame, as would be acquired in falling through CG. Thus the body moves ſloweſt at the higheſt point H of the parabola; and at equal diſtances from this point will

move

move with equal ſwiftneſs, and deſcend from that higheſt point through the line HB altogether like to the line AH in which it aſcended; abating only the reſiſtance of the air, which is not here conſidered. If the line HI be drawn from the higheſt point H parallel to the horizon, AI will be equal to $\frac{1}{4}$ of BG in fig. 70, when the body is projected in the direction AG, and equal to $\frac{1}{4}$ of BH, when the body is thrown in the direction AH provided AD be drawn horizontally.

93. THUS I have recounted the principal diſcoveries, which had been made concerning the motion of bodies by Sir ISAAC NEWTON's predeceſſors; all theſe diſcoveries, by being found to agree with experience, contributing to eſtabliſh the laws of motion, from whence they were deduced. I ſhall therefore here finiſh what I had to ſay upon thoſe laws; and conclude this chapter with a few words concerning the diſtinction which ought to be made between abſolute and relative motion. For ſome have thought fit to confound them together; becauſe they obſerve the laws of motion to take place here on the earth, which is in motion, after the ſame manner as if it were at reſt. But Sir ISAAC NEWTON has been careful to diſtinguiſh between the relative and abſolute conſideration both of motion and time[a]. The aſtronomers anciently found it neceſſary to make this diſtinction in time. Time conſidered in it ſelf paſſes on equably without relation to any thing external, being the proper meaſure of the continuance and duration of all things. But it is moſt frequently conceived of by us under a relative view to ſome ſucceſſion in

[a] Prin. Philoſ. pag. 7, &c.

ſenſible

ſenſible things, of which we take cognizance. The ſucceſſion of the thoughts in our own minds is that, from whence we receive our firſt idea of time, but is a very uncertain meaſure thereof; for the thoughts of ſome men flow on much more ſwiftly, than the thoughts of others; nor does the ſame perſon think equally quick at all times. The motions of the heavenly bodies are more regular; and the eminent diviſion of time into night and day, made by the ſun, leads us to meaſure our time by the motion of that luminary: nor do we in the affairs of life concern our ſelves with any inequality, which there may be in that motion; but the ſpace of time which comprehends a day and night is rather ſuppoſed to be always the ſame. However aſtronomers anciently found theſe ſpaces of time not to be always of the ſame length, and have taught how to compute their differences. Now the time, when ſo equated as to be rendered perfectly equal, is the true meaſure of duration, the other not. And therefore this latter, which is abſolutely true time, differs from the other, which is only apparent. And as we ordinarily make no diſtinction between apparent time, as meaſured by the ſun, and the true; ſo we often do not diſtinguiſh in our uſual diſcourſe between the real, and the apparent or relative motion of bodies; but uſe the ſame words for one, as we ſhould for the other. Though all things about us are really in motion with the earth; as this motion is not viſible, we ſpeak of the motion of every thing we ſee, as if our ſelves and the earth ſtood ſtill. And even in other caſes, where we diſcern the motion of bodies, we often ſpeak of them not in relation to the whole motion we ſee, but with regard to other

bodies, to which they are contiguous. If any body were lying on a table; when that table ſhall be carried along, we ſay the body reſts upon the table, or perhaps abſolutely, that the body is at reſt. However philoſophers muſt not reject all diſtinction between true and apparent motions, any more than aſtronomers do the diſtinction between true and vulgar time; for there is as real a difference between them, as will appear by the following conſideration. Suppoſe all the bodies of the univerſe to have their courſes ſtopped, and reduced to perfect reſt. Then ſuppoſe their preſent motions to be again reſtored; this cannot be done without an actual impreſſion made upon ſome of them at leaſt. If any of them be left untouched, they will retain their former ſtate, that is, ſtill remain at reſt; but the other bodies, which are wrought upon, will have changed their former ſtate of reſt, for the contrary ſtate of motion. Let us now ſuppoſe the bodies left at reſt to be annihilated, this will make no alteration in the ſtate of the moving bodies; but the effect of the impreſſion, which was made upon them, will ſtill ſubſiſt. This ſhews the motion they received to be an abſolute thing, and to have no neceſſary dependence upon the relation which the body ſaid to be in motion has to any other body [a].

94. Besides abſolute and relative motion are diſtinguiſhable by their Effects. One effect of motion is, that bodies, when moved round any center or axis, acquire a certain

[a] See Newton. princip. philoſ. pag. 9. lin. 30.

power,

power, by which they forcibly preſs themſelves from that center or axis of motion. As when a body is whirled about in a ſling, the body preſſes againſt the ſling, and is ready to fly out as ſoon as liberty is given it. And this power is proportional to the true, not relative motion of the body round ſuch a center or axis. Of this Sir ISAAC NEWTON gives the following inſtance [a]. If a pail or ſuch like veſſel near full of water be ſuſpended by a ſtring of ſufficient length, and be turned about till the ſtring be hard twiſted. If then as ſoon as the veſſel and water in it are become ſtill and at reſt, the veſſel be nimbly turned about the contrary way the ſtring was twiſted, the veſſel by the ſtrings untwiſting it ſelf ſhall continue its motion a long time. And when the veſſel firſt begins to turn, the water in it ſhall receive little or nothing of the motion of the veſſel, but by degrees ſhall receive a communication of motion, till at laſt it ſhall move round as ſwiftly as the veſſel it ſelf. Now the definition of motion, which DES CARTES has given us upon this principle of making all motion meerly relative, is this: that motion, is a removal of any body from its vicinity to other bodies, which were in immediate contact with it, and are conſidered as at reſt [b]. And if this be compared with what he ſoon after ſays, that there is nothing real or poſitive in the body moved, for the ſake of which we aſcribe motion to it, which is not to be found as well in the contiguous bodies, which are conſidered as at reſt [c]; it will follow from thence, that we may conſider the veſſel as at reſt

[a] Princip. Philoſ. pag. 10.
[b] Renat. Des Cart. Princ. Philoſ. Part. II. § 25.
[c] Ibid. § 30.

and the water as moving in it: and the water in reſpect of the veſſel has the greateſt motion, when the veſſel firſt begins to turn, and loſes this relative motion more and more, till at length it quite ceaſes. But now, when the veſſel firſt begins to turn, the ſurface of the water remains ſmooth and flat, as before the veſſel began to move; but as the motion of the veſſel communicates by degrees motion to the water, the ſurface of the water will be obſerved to change, the water ſubſiding in the middle and riſing at the edges: which elevation of the water is cauſed by the parts of it preſſing from the axis, they move about; and therefore this force of receding from the axis of motion depends not upon the relative motion of the water within the veſſel, but on its abſolute motion; for it is leaſt, when that relative motion is greateſt, and greateſt, when that relative motion is leaſt, or none at all.

95. Thus the true cauſe of what appears in the ſurface of this water cannot be aſſigned, without conſidering the water's motion within the veſſel. So alſo in the ſyſtem of the world, in order to find out the cauſe of the planetary motions, we muſt know more of the real motions, which belong to each planet, than is abſolutely neceſſary for the uſes of aſtronomy. If the aſtronomer ſhould ſuppoſe the earth to ſtand ſtill, he could aſcribe ſuch motions to the celeſtial bodies, as ſhould anſwer all the appearances; though he would not account for them in ſo ſimple a manner, as by attributing motion to the earth. But the motion of the earth muſt of neceſſity be conſidered, before the real cauſes, which actuate the planetary ſyſtem, can be diſcovered.

CHAP. III.

Of CENTRIPETAL FORCES.

WE have juſt been deſcribing in the preceding chapter the effects produced on a body in motion, from its being continually acted upon by a power always equal in ſtrength, and operating in parallel directions [a]. But bodies may be acted upon by powers, which in different places ſhall have different degrees of force, and whoſe ſeveral directions ſhall be variouſly inclined to each other. The moſt ſimple of theſe in reſpect to direction is, when the power is pointed conſtantly to one center. This is truly the caſe of that power, whoſe effects we deſcribed in the foregoing chapter; though the center of that power is ſo far removed, that the ſubject then before us is moſt conveniently to be conſidered in the light, wherein we have placed it: But Sir ISAAC NEWTON has conſidered very particularly this other caſe of powers, which are conſtantly directed to the ſame center. It is upon this foundation, that all his diſcoveries in the ſyſtem of the world are raiſed. And therefore, as this ſubject bears ſo very great a ſhare in the philoſophy, of which I am diſcourſing, I think it proper in this place to take a ſhort view of ſome of the general effects of theſe powers, before we come to apply them particularly to the ſyſtem of the world.

[a] § 85, &c.

2. THESE

2. These powers or forces are by Sir Isaac Newton called centripetal; and their firſt effect is to cauſe the body, on which they act, to quit the ſtraight courſe, wherein it would proceed if undiſturbed, and to deſcribe an incurvated line, which ſhall always be bent towards the center of the force. It is not neceſſary, that ſuch a power ſhould cauſe the body to approach that center. The body may continue to recede from the center of the power, notwithſtanding its being drawn by the power; but this property muſt always belong to its motion, that the line, in which it moves, will continually be concave towards the center, to which the power is directed. Suppoſe A (in fig. 72.) to be the center of a force. Let a body in B be moving in the direction of the ſtraight line BC, in which line it would continue to move, if undiſturbed; but being attracted by the centripetal force towards A, the body muſt neceſſarily depart from this line BC, and being drawn into the curve line BD, muſt paſs between the lines AB and BC. It is evident therefore, that the body in B being gradually turned off from the ſtraight line BC, it will at firſt be convex toward the line BC, and conſequently concave towards the point A: for theſe centripetal powers are ſuppoſed to be in ſtrength proportional to the power of gravity, and, like that, not to be able after the manner of an impulſe to turn the body ſenſibly out of its courſe into a different one in an inſtant, but to take up ſome ſpace of time in producing a viſible effect. That the curve will always continue to have its concavity towards A may thus appear. In the line BC near to B take any point as E, from which the line EFG may be ſo drawn

drawn, as to touch the curve line B D in ſome point as F. Now when the body is come to F, if the centripetal power were immediately to be ſuſpended, the body would no longer continue to move in a curve line, but being left to it ſelf would forthwith reaſſume a ſtraight courſe; and that ſtraight courſe would be in the line F G: for that line is in the direction of the body's motion at the point F. But the centripetal force continuing its energy, the body will be gradually drawn from this line F G ſo as to keep in the line F D, and make that line near the point F to be convex toward F G, and concave toward A. After the ſame manner the body may be followed on in its courſe through the line B D, and every part of that line be ſhewn to be concave toward the point A.

3. THIS then is the conſtant character belonging to thoſe motions, which are carried on by centripetal forces; that the line, wherein the body moves, is throughout concave towards the center of the force. In reſpect to the ſucceſſive diſtances of the body from the center there is no general rule to be laid down; for the diſtance of the body from the center may either increaſe, or decreaſe, or even keep always the ſame. The point A (in fig. 73.) being the center of a centripetal force, let a body at B ſet out in the direction of the ſtraight line B C perpendicular to the line A B drawn from A to B. It will be eaſily conceived, that there is no other point in the line B C ſo near to A, as the point B; that A B is the ſhorteſt of all the lines, which can be drawn from A to any part of the line B C; all other lines, as A D, or A E, drawn from A to the line B C being longer than A B. Hence it follows, that the body ſetting

ting out from B, if it moved in the line BC, it would recede more and more from the point A. Now as the operation of a centripetal force is to draw a body towards the center of the force: if ſuch a force act upon a reſting body, it muſt neceſſarily put that body ſo into motion, as to cauſe it to move towards the center of the force: if the body were of it ſelf moving towards that center, the centripetal force would accelerate that motion, and cauſe it to move faſter down: but if the body were in ſuch a motion, as being left to itſelf it would recede from this center, it is not neceſſary, that the action of a centripetal power upon it ſhould immediately compel the body to approach the center, from which it would otherwiſe have receded; the centripetal power is not without effect, if it cauſe the body to recede more ſlowly from that center, than otherwiſe it would have done. Thus in the caſe before us, the ſmalleſt centripetal power, if it act on the body, will force it out of the line BC, and cauſe it to paſs in a bent line between BC and the point A, as has been before explained. When the body, for inſtance, has advanced to the line AD, the effect of the centripetal force diſcovers it ſelf by having removed the body out of the line BC, and brought it to croſs the line AD ſomewhere between A and D: ſuppoſe at F. Now AD being longer than AB, AF may alſo be longer than AB. The centripetal power may indeed be ſo ſtrong, that AF ſhall be ſhorter than AB; or it may be ſo evenly balanced with the progreſſive motion of the body, that AF and AB ſhall be juſt equal: and in this laſt caſe, when the centripetal force is of that ſtrength, as conſtantly to draw the body as much toward the

the center, as the progreſſive motion would carry it off, the body will deſcribe a circle about the center A, this center of the force being alſo the center of the circle.

4. IF the body, inſtead of ſetting out in the line BC perpendicular to AB, had ſet out in another line BG more inclined towards the line AB, moving in the curve line BH; then as the body, if it were to continue its motion in the line BG, would for ſome time approach the center A; the centripetal force would cauſe it to make greater advances toward that center. But if the body were to ſet out in the line BI reclined the other way from the perpendicular BC, and were to be drawn by the centripetal force into the curve line BK; the body, notwithſtanding any centripetal force, would for ſome time recede from the center; ſince ſome part at leaſt of the curve line BK lies between the line BI and the perpendicular BC.

5. THUS far we have explained ſuch effects, as attend every centripetal force. But as theſe forces may be very different in regard to the different degrees of ſtrength, wherewith they act upon bodies in different places; I ſhall now proceed to make mention in general of ſome of the differences attending theſe centripetal motions.

6. TO reaſſume the conſideration of the laſt mentioned caſe. Suppoſe a centripetal power directed toward the point A (in fig. 74.) to act on a body in B, which is moving in the direction of the ſtraight line BC, the line BC reclining off from AB. If from A the ſtraight lines AD, AE, AF are

drawn at pleaſure to the line C B; the line C B being prolonged beyond B to G, it appears that A D is inclined to the line G C more obliquely, than A B is inclined to it, A E is inclined more obliquely than A D, and A F more than A E. To ſpeak more correctly, the angle under A D G is leſs than that under A B G, the angle under A E G leſs than that under A D G, and the angle under A F G leſs than that under A E G. Now ſuppoſe the body to move in the curve line B H I K. Then it is here likewiſe evident, that the line B H I K being concave towards A, and convex towards the line B C, it is more and more turned off from the line B C; ſo that in the point H the line A H will be leſs obliquely inclined to the curve line B H I K, than the ſame line A H D is inclined to B C at the point D; at the point I the inclination of the line A I to the curve line will be more different from the inclination of the ſame line A I E to the line B C, at the point E; and in the points K and F the difference of inclination will be ſtill greater; and in both the inclination at the curve will be leſs oblique, than at the ſtraight line B C. But the ſtraight line A B is leſs obliquely inclined to B G, than A D is inclined towards D G: therefore although the line A H be leſs obliquely inclined towards the curve H B, than the ſame line A H D is inclined towards D G; yet it is poſſible, that the inclination at H may be more oblique, than the inclination at B. The inclination at H may indeed be leſs oblique than the other, or they may be both the ſame. This depends upon the degree of ſtrength, wherewith the centripetal force exerts it ſelf, during the paſſage of the body from B to H. After the ſame manner the inclinations at I and K depend entirely on the degree

gree of ſtrength, wherewith the centripetal force acts on the body in its paſſage from H to K: if the centripetal force be weak enough, the lines AH and AI drawn from the center A to the body at H and at I ſhall be more obliquely inclined to the curve, than the line AB is inclined towards BG. The centripetal force may be of that ſtrength as to render all theſe inclinations equal, or if ſtronger, the inclinations at I and K will be leſs oblique than at B. Sir ISAAC NEWTON has particularly ſhewn, that if the centripetal power decreaſes after a certain manner with the increaſe of diſtance, a body may deſcribe ſuch a curve line, that all the lines drawn from the center to the body ſhall be equally inclined to that curve line [a]. But I do not here enter into any particulars, my preſent intention being only to ſhew, that it is poſſible for a body to be acted upon by a force continually drawing it down towards a center, and yet that the body ſhall continue to recede from that center; for here as long as the lines AH, AI, &c drawn from the center A to the body do not become leſs oblique to the curve, in which the body moves; ſo long ſhall thoſe lines perpetually increaſe, and conſequently the body ſhall more and more recede from the center.

7. BUT we may obſerve farther, that if the centripetal power, while the body increaſes its diſtance from the center, retain ſufficient ſtrength to make the lines drawn from the center to the body to become at length leſs oblique to the curve; then if this diminution of the obliquity continue, till

[a] Princip. Philoſ. Lib. I. prop. 9.

at laſt the line drawn from the center to the body ſhall ceaſe to be obliquely inclined to the curve, and ſhall become perpendicular thereto; from this inſtant the body ſhall no longer recede from the center, but in its following motion it ſhall again deſcend, and ſhall deſcribe a curve line in all reſpects like to that, which it has deſcribed already; provided the centripetal power, every where at the ſame diſtance from the center, acts with the ſame ſtrength. So we obſerved in the preceding chapter, that, when the motion of a projectile became parallel to the horizon, the projectile no longer aſcended, but forthwith directed its courſe downwards, deſcending in a line altogether like that, wherein it had before aſcended[a].

8. This return of the body may be proved by the following propoſition: that if the body in any place, ſuppoſe at I, were to be ſtopt, and be thrown directly backward with the velocity, wherewith it was moving forward in that point I; then the body, by the action of the centripetal force upon it, would move back again over the path IHB, in which it had before advanced forward, and would arrive again at the point B in the ſame ſpace of time, as was taken up in its paſſage from B to I; the velocity of the body at its return to the point B being the ſame, as that wherewith it firſt ſet out from that point. To give a full demonſtration of this propoſition, would require that uſe of mathematics, which I here purpoſe to avoid; but, I believe, it will appear in great meaſure evident from the following conſiderations.

[a] § 92.

9. Sup-

9. SUPPOSE (in fig. 75.) that a body were carried after the following manner through the bent figure ABCDEF, composed of the straight lines AB, BC, CD, DE, EF. First let it be moving in the line AB, from A towards B, with any uniform velocity. At B let the body receive an impulse directed toward some point, as G, taken within the concavity of the figure. Now whereas this body, when once moving in the straight line AB, will continue to move on in this line, so long as it shall be left to it self; but being disturbed at the point B in its motion by the impulse, which there acts upon it, it will be turned out of this line AB into some other straight line, wherein it will afterwards continue to move, as long as it shall be left to itself. Therefore let this impulse have strength sufficient to turn the body into the line BC. Then let the body move on undisturbed from B to C, but at C let it receive another impulse pointed toward the same point G, and of sufficient strength to turn the body into the line CD. At D let a third impulse, directed like the rest to the point G, turn the body into the line DE. And at E let another impulse, directed likewise to the point G, turn the body into the line EF. Now, I say, if the body while moving in the line EF be stopt, and turned back again in this line with the same velocity, as that wherewith it was moving forward in this line; then by the repetition of the former impulse at E the body will be turned into the line ED, and move in it from E to D with the same velocity as before it moved with from D to E; by the repetition of the impulse at D, when the body shall have returned to that point, it will be turned into the line DC; and by the repetition of the other impulses at C and B

the body will be brought back again into the line B A, with the velocity, wherewith it firſt moved in that line.

10. This I prove as follows. Let D E and F E be continued beyond E. In D E thus continued take at pleaſure the length E H, and let H I be ſo drawn, as to be equidiſtant from the line G E. Then, by what has been written upon the ſecond law of motion [a], it follows, that after the impulſe on the body in E it will move through E I in the ſame time, as it would have imployed in moving from E to H, with the velocity which it had in the line D E. In F E prolonged take E K equal to E I, and draw K L equidiſtant from G E. Then, becauſe the body is thrown back in the line F E with the ſame velocity as that wherewith it went forward in that line; if, when the body was returned to E, it were permitted to go ſtraight on, it would paſs through E K in the ſame time, as it took up in paſſing through E I, when it went forward in the line E F. But, if at the body's return to the point E, ſuch an impulſe directed toward the point D were to be given it, whereby it ſhould be turned into the line D E; I ſay, that the impulſe neceſſary to produce this effect muſt be equal to that, which turned the body out of the line D E into E F; and that the velocity, with which the body will return into the line E D, is the ſame, as that wherewith it before moved through this line from D to E. Becauſe E K is equal to E I, and K L and H I, being each equidiſtant from G E, are by conſequence equidiſtant from each other; it follows, that the two

[a] Ch. II. § 22.

trian-

triangular figures I E H and K E L are altogether like and equal to each other. If I were writing to mathematicians, I might refer them to ſome propoſitions in the elements of EUCLID for the proof of this [a]: but as I do not here addreſs my ſelf to ſuch, ſo I think this aſſertion will be evident enough without a proof in form; at leaſt I muſt deſire my readers to receive it as a propoſition true in geometry. But theſe two triangular figures being altogether like each other and equal; as E K is equal to E I, ſo E L is equal to E H, and K L equal to H I. Now the body after its return to E being turned out of the line F E into E D by an impulſe acting upon it in E, after the manner above expreſſed; the body will receive ſuch a velocity by this impulſe, as will carry it through E L in the ſame time, as it would have imployed in paſſing through E K, if it had gone on in that line undiſturbed. And it has already been obſerved, that the time, in which the body would paſs over E K with the velocity wherewith it returns, is equal to the time it took up in going forward from E to I; that is, equal to the time, in which it would have gone through E H with the velocity, wherewith it moved from D to E. Therefore the time, in which the body will paſs through E L after its return into the line E D, is the ſame, as would have been taken up by the body in paſſing through E H with the velocity, wherewith the body firſt moved in the line D E. Since therefore E L and E H are equal, the body returns into the line D E with the velocity, which it had before in that line. Again I ſay, the ſecond impulſe in E is equal to the firſt. By what has

[a] Viz. L. I. prop. 30, 29, & 26.

been

been said on the ſecond law of motion concerning the effect of oblique impulſes[a], it will be underſtood, that the impulſe in E, whereby the body was turned out of the line DE into the line EF, is of ſuch ſtrength, that if the body had been at reſt, when this impulſe had acted upon it, this impulſe would have communicated ſo much motion to the body, as would have carried it through a length equal to HI, in the time wherein the body would have paſſed from E to H, or in the time wherein it paſſed from E to I. In the ſame manner, on the return of the body, the impulſe in E, whereby the body is turned out of the line FE into ED, is of ſuch ſtrength, that if it had acted on the body at reſt, it would have cauſed the body to move through a length equal to KL, in the ſame time, as the body would imploy in paſſing through EK with the velocity, wherewith it returns in the line FE. Therefore the ſecond impulſe, had it acted on the body at reſt, would have cauſed it to move through a length equal to KL in the ſame ſpace of time, as would be taken up by the body in paſſing through a length equal to HI, were the firſt impulſe to act on the body when at reſt. That is, the effects of the firſt and ſecond impulſe on the body when at reſt would be the ſame; for KL and HI are equal: conſequently the ſecond impulſe is equal to the firſt.

11. Thus if the body be returned through FE with the velocity, wherewith it moved forward; we have ſhewn how by the repetition of the impulſe, which acted on it at E, the

[a] Ch. II. § 21, 22.

body

body will return again into the line DE with the velocity, which it had before in that line. By the ſame proceſs of reaſoning it may be proved, that, when the body is returned back to D, the impulſe, which before acted on the body at that point, will throw the body into the line DC with the velocity, which it firſt had in that line; and the other impulſes being ſucceſſively repeated, the body will at length be brought back again into the line BA with the velocity, wherewith it ſet out in that line.

12. Thus theſe impulſes, by acting over again in an inverted order all their operation on the body, bring it back again through the path, in which it had proceeded forward. And this obtains equally, whatever be the number of the ſtraight lines, whereof this curve figure is compoſed. Now by a method of reaſoning, which Sir Isaac Newton makes great uſe of, and which he introduced into geometry, thereby greatly inriching that ſcience [a]; we might make a tranſition from this figure compoſed of a number of ſtraight lines to a figure of one continued curvature, and from a number of ſeparate impulſes repeated at diſtinct intervals to a continual centripetal force, and ſhew, that, becauſe what has been here advanced holds univerſally true, whatever be the number of ſtraight lines, whereof the curve figure ACF is compoſed, and howſoever frequently the impulſes at the angles of this figure are repeated; therefore the ſame will ſtill remain true, although this figure ſhould be converted into one of a continued curvature, and theſe diſtinct impulſes ſhould be

[a] viz. His doctrine of prime and ultimate ratios.

changed into a continual centripetal force. But as the explaining this method of reaſoning is foreign to my preſent deſign; ſo I hope my readers, after what has been ſaid, will find no difficulty in receiving the propoſition laid down above: that, if the body, which has moved through the curve line B H I (in fig. 74.) from B to I, when it is come to I, be thrown directly back with the ſame velocity as that, wherewith it proceeded forward, the centripetal force, by acting over again all its operation on the body, ſhall bring the body back again in the line I H B: and as the motion of the body in its courſe from B to I was every where in ſuch a manner oblique to the line drawn from the center to the body, that the centripetal power acted in ſome degree againſt the body's motion, and gradually diminiſhed it; ſo in the return of the body, the centripetal power will every where draw the body forward, and accelerate its motion by the ſame degrees, as before it retarded it.

13. THIS being agreed, ſuppoſe the body in K to have the line A K no longer obliquely inclined to its motion. In this caſe, if the body be turned back, in the manner we have been conſidering, it muſt be directed back perpendicularly to A K. But if it had proceeded forward, it would likewiſe have moved in a direction perpendicular to A K; conſequently, whether it move from this point K backward or forward, it muſt deſcribe the ſame kind of courſe. Therefore ſince by being turned back it will go over again the line K I H B; if it be permitted to go forward, the line K L, which it ſhall deſcribe, will be altogether ſimilar to the line K H B.

14. IN like manner we may determine the nature of the motion, if the line, wherein the body ſets out, be inclined (as in fig. 76.) down toward the line BA drawn between the body and the center. If the centripetal power ſo much increaſes in ſtrength, as the body approaches, that it can bend the path, in which the body moves, to that degree, as to cauſe all the lines as AH, AI, AK to remain no leſs oblique to the motion of the body, than AB is oblique to BC; the body ſhall continually more and more approach the center. But if the centripetal power increaſes in ſo much leſs a degree, as to permit the line drawn from the center to the body, as it accompanies the body in its motion, at length to become more and more erect to the curve wherein the body moves, and in the end, ſuppoſe at K, to become perpendicular thereto; from that time the body ſhall riſe again. This is evident from what has been ſaid above; becauſe for the very ſame reaſon here alſo the body ſhall proceed from the point K to deſcribe a line altogether ſimilar to the line, in which it has moved from B to K. Thus, as it was obſerved of the pendulum in the preceding chapter[a], that all the time it approaches towards being perpendicular to the horizon, it more and more deſcends; but, as ſoon as it is come into that perpendicular ſituation, it immediately riſes again by the ſame degrees, as it deſcended by before: ſo here the body more and more approaches the center all the time it is moving from B to K; but thence forward it riſes from the center again by the ſame degrees, as it approached by before.

[a] § 57.

15. If (in fig. 77.) the line BC be perpendicular to AB; then it has been obferved above [a], that the centripetal power may be fo balanced with the progreffive motion of the body, that the body may keep moving round the center A conftantly at the fame diftance; as a body does, when whirled about any point, to which it is tyed by a ftring. If the centripetal power be too weak to produce this effect, the motion of the body will prefently become oblique to the line drawn from itfelf to the center, after the manner of the firft of the two cafes, which we have been confidering. If the centripetal power be ftronger, than what is required to carry the body in a circle, the motion of the body will prefently fall in with the fecond of the cafes, we have been confidering.

16. If the centripetal power fo change with the change of diftance, that the body, after its motion has become oblique to the line drawn from itfelf to the center, fhall again become perpendicular thereto; which we have fhewn to be poffible in both the cafes treated of above; then the body fhall in its fubfequent motion return again to the diftance of AB, and from that diftance take a courfe fimilar to the former: and thus, if the body move in a fpace free from all refiftance, which has been here all along fuppofed; it fhall continue in a perpetual motion about the center, defcending and afcending alternately therefrom. If the body fetting out from B (in fig. 78.) in the line BC perpendicular to AB, defcribe the line BDE, which in D fhall be oblique to the line AD, but in E

[a] § 3

ſhall again become erect to AE drawn from the body in E to the center A; then from this point E the body ſhall deſcribe the line EFG altogether like to the line BDE, and at G ſhall be at the ſame diſtance from A, as it was at B. But likewiſe the line AG ſhall be erect to the body's motion. Therefore the body ſhall proceed to deſcribe from G the line GHI altogether ſimilar to the line GFE, and at I have the ſame diſtance from the center, as it had at E; and alſo have the line AI erect to its motion: ſo that its following motion muſt be in the line IKL ſimilar to IHG, and the diſtance AL equal to AG. Thus the body will go on in a perpetual round without ceaſing, alternately inlarging and contracting its diſtance from the center.

17. IF it ſo happen, that the point E fall upon the line BA continued beyond A; then the point G will fall on B, I on E, and L alſo on B; ſo that the body will deſcribe in this caſe a ſimple curve line round the center A, like the line BDEF in fig. 79, in which it will continually revolve from B to E and from E to B without end.

18. IF AE in fig. 78 ſhould happen to be perpendicular to AB, in this caſe alſo a ſimple line will be deſcribed; for the point G will fall on the line BA prolonged beyond A, the point I on the line AE prolonged beyond A, and the point L on B: ſo that the body will deſcribe a line like the curve line BEGI in fig. 80, in which the oppoſite points B and G are equally diſtant from A, and the oppoſite points E and I are alſo equally diſtant from the ſame point A.

19. In other cafes the line defcribed will have a more complex figure.

20. Thus we have endeavoured to fhew how a body, while it is conftantly attracted towards a center, may notwithftanding by its progreffive motion keep it felf from falling down to that center; but defcribe about it an endlefs circuit, fometimes approaching toward that center, and at other times as much receding from the fame.

21. But here we have fuppofed, that the centripetal power is of equal ftrength every where at the fame diftance from the center. And this is the cafe of that centripetal power, which will hereafter be fhewn to be the caufe, that keeps the planets in their courfes. But a body may be kept on in a perpetual circuit round a center, although the centripetal power have not this property. Indeed a body may by a centripetal force be kept moving in any curve line whatever, that fhall have its concavity turned every where towards the center of the force.

22. To make this evident I fhall firft propofe the cafe of a body moving through the incurvated figure ABCDE (in fig. 81.) which is compofed of the ftraight lines AB, BC, CD, DE, and EA; the motion being carried on in the following manner. Let the body firft move in the line AB with any uniform velocity. When it is arrived at the point B, let it receive an impulfe directed toward any point F taken within the figure; and let the impulfe be of that ftrength as to turn the body out

of

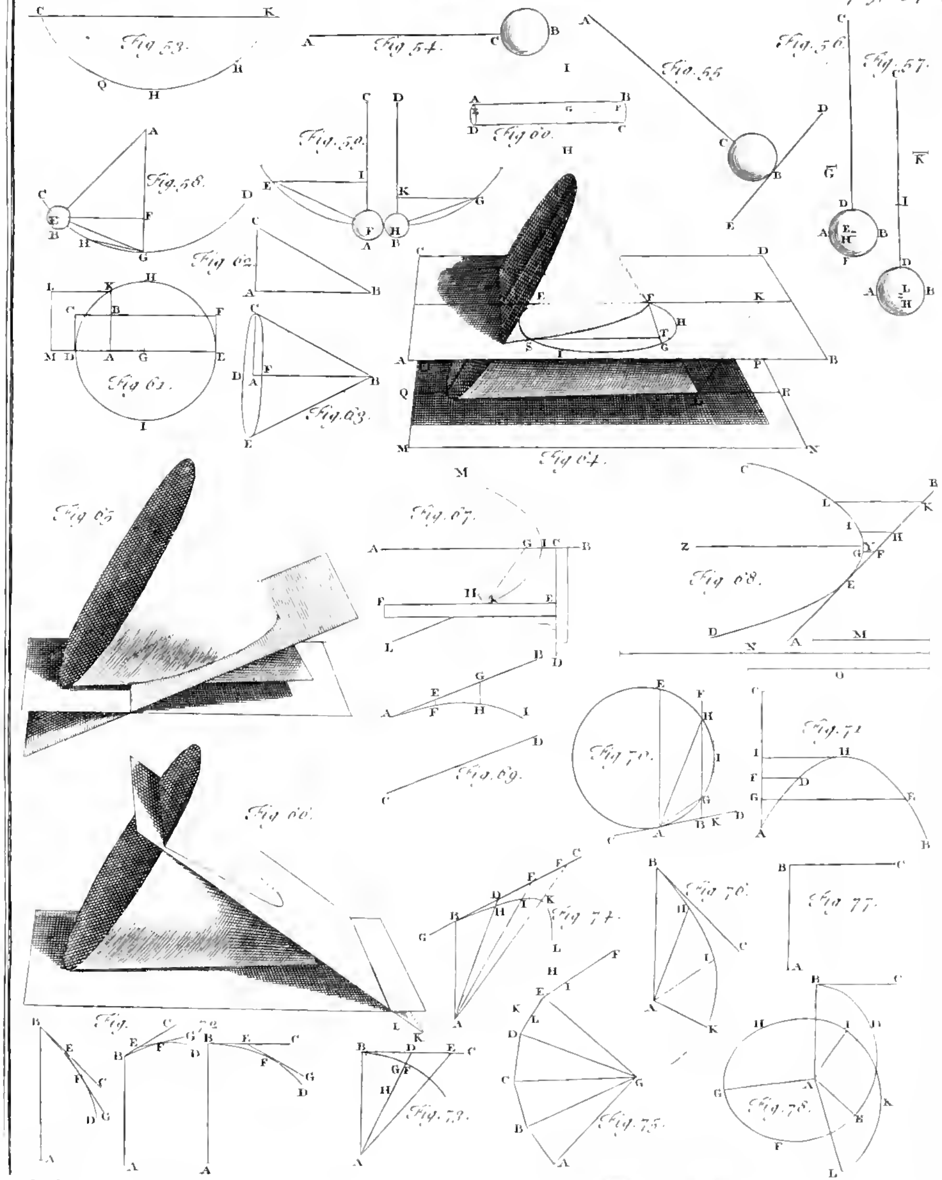


Fig. 53.
Fig. 54.
Fig. 55.
Fig. 56.
Fig. 57.
Fig. 58.
Fig. 59.
Fig. 60.
Fig. 61.
Fig. 62.
Fig. 63.
Fig. 64.
Fig. 65.
Fig. 66.
Fig. 67.
Fig. 68.
Fig. 69.
Fig. 70.
Fig. 71.
Fig. 72.
Fig. 73.
Fig. 74.
Fig. 75.
Fig. 76.
Fig. 77.
Fig. 78.

of the line A B into the line B C. The body after this impulſe, while left to itſelf, will continue moving in the line B C. At C let the body receive another impulſe directed towards the ſame point F, of ſuch ſtrength, as to turn the body from the line B C into the line C D. At D let the body by another impulſe, directed likewiſe to the point F, be turned out of the line C D into D E. And at E let another impulſe, directed toward the point F, turn the body from the line D E into E A. Thus we ſee how a body may be carried through the figure ABCDE by certain impulſes directed always toward the ſame center, only by their acting on the body at proper intervals, and with due degrees of ſtrength.

23. BUT farther, when the body is come to the point A, if it there receive another impulſe directed like the reſt toward the point F, and of ſuch a degree of ſtrength as to turn the body into the line A B, wherein it firſt moved; I ſay that the body ſhall return into this line with the ſame velocity, as it had at firſt.

24. Let A B be prolonged beyond B at pleaſure, ſuppoſe to G; and from G let G H be drawn, which if produced ſhould always continue equidiſtant from B F, or, according to the more uſual phraſe, let G H be drawn parallel to B F. Then it appears, from what has been ſaid upon the ſecond law of motion[a], that in the time, wherein the body would have moved from B to G, had it not received a new impulſe in B, by the means of that impulſe it will have acquired a velocity, which will carry it from B to H. After the ſame manner, if C I be

[a] Ch. 2. § 22.

taken

taken equal to BH, and IK be drawn equidiſtant from or parallel to CF; the body will have moved from C to K with the velocity, which it has in the line CD, in the ſame time, as it would have employed in moving from C to I with the velocity, it had in the line BC. Therefore ſince CI and BH are equal, the body will move through CK in the ſame time, as it would have taken up in moving from B to G with the original velocity, wherewith it moved through the line AB. Again, DL being taken equal to CK and LM drawn parallel to DF; for the ſame reaſon as before the body will move through DM with the velocity, which it has in the line DE, in the ſame time, as it would imploy in moving through BG with its original velocity. In the laſt place, if EN be taken equal to DM, and NO be drawn parallel to EF; likewiſe if AP be taken equal to EO, and PQ be drawn parallel to AF: then the body with the velocity, wherewith it returns into the line AB, will paſs through AQ in the ſame time, as it would have imployed in paſſing through BG with its original velocity. Now as all this follows directly from what has above been delivered, concerning the effect of oblique impulſes impreſſed upon bodies in motion; ſo we muſt here obſerve farther, that it can be proved by geometry, that AQ will always be equal to BG. The proof of this I am obliged, from the nature of my preſent deſign, to omit; but this geometrical propoſition being granted, it follows, that the body has returned into the line AB with the velocity, which it had, when it firſt moved in that line; for the velocity, with which it returns into the line AB, will carry it over the line AQ in the ſame time, as would

 have

have been taken up in its paſſing over an equal line B G with the original velocity.

25. THUS we have found, how a body may be carried round the figure A B C D E by the action of certain impulſes upon it, which ſhould all be pointed toward one center. And we likewiſe ſee, that when the body is brought back again to the point, whence it firſt ſet out; if it there meet with an impulſe ſufficient to turn it again into the line, wherein it moved at firſt, its original velocity will be again reſtored; and by the repetition of the ſame impulſes, the body will be carried again in the ſame round. Therefore if theſe impulſes, which act on the body at the points B, C, D, E, and A, continue always the ſame, the body will make round this figure innumerable revolutions.

26. THE proof, which we have here made uſe of, holds the ſame in any number of ſtraight lines, whereof the figure A B D ſhould be compoſed; and therefore by the method of reaſoning referred to above [a] we are to conclude, that what has here been ſaid upon this rectilinear figure, will remain true, if this figure were changed into one of a continued curvature, and inſtead of diſtinct impulſes acting by intervals at the angles of this figure, we had a continual centripetal force. We have therefore ſhewn, that a body may be carried round in any curve figure A B C (fig. 82.) which ſhall every where be concave towards any one point as D, by the continual action

[a] § 12.

of a centripetal power directed to that point, and when it is returned to the point, from whence it ſet out, it ſhall recover again the velocity, with which it departed from that point. It is not indeed always neceſſary, that it ſhould return again into its firſt courſe; for the curve line may have ſome ſuch figure as the line ABCDBE in fig. 83. In this curve line, if the body ſet out from B in the direction BF, and moved through the line BCD, till it returned to B; here the body would not enter again into the line BCD, becauſe the two parts BD and BC of the curve line make an angle at the point B: ſo that the centripetal power, which at the point B could turn the body from the line BF into the curve, will not be able to turn the body into the line BC from the direction, in which it returns to the point B; a forceable impulſe muſt be given the body in the point B to produce that effect.

27. IF at the point B, whence the body ſets out, the curve line return into it ſelf (as in fig. 82;) then the body, upon its arrival again at B, may return into its former courſe, and thus make an endleſs circuit about the center of the centripetal power.

28. WHAT has here been ſaid, I hope, will in ſome meaſure enable my readers to form a juſt idea of the nature of theſe centripetal motions.

29. I HAVE not attempted to ſhew, how to find particularly, what kind of centripetal force is neceſſary to carry a body in any curve line propoſed. This is to be deduced from the de-

gree

gree of curvature, which the figure has in each point of it, and requires a long and complex mathematical reaſoning. However I ſhall ſpeak a little to the firſt propoſition, which Sir ISAAC NEWTON lays down for this purpoſe. By this propoſition, when a body is found moving in a curve line, it may be known, whether the body be kept in its courſe by a power always pointed toward the ſame center; and if it be ſo, where that center is placed. The propoſition is this: that if a line be drawn from ſome fixed point to the body, and remaining by one extream united to that point, it be carried round along with the body; then, if the power, whereby the body is kept in its courſe, be always pointed to this fixed point as a center, this line will move over equal ſpaces in equal portions of time. Suppoſe a body were moving through the curve line ABCD (in fig. 84.) and paſſed over the arches AB, BC, CD in equal portions of time; then if a point, as E, can be found, from whence the line EA being drawn to the body in A, and accompanying the body in its motion, it ſhall make the ſpaces EAB, EBC, and ECD equal, over which it paſſes, while the body deſcribes the arches AB, BC, and CD: and if this hold the ſame in all other arches, both great and ſmall, of the curve line ABCD, that theſe ſpaces are always equal, where the times are equal; then is the body kept in this line by a power always pointed to E as a center.

30. THE principle, upon which Sir ISAAC NEWTON has demonſtrated this, requires but ſmall skill in geometry to comprehend. I ſhall therefore take the liberty to cloſe the pre-

ſent chapter with an explication of it; becauſe ſuch an example will give the cleareſt notion of our author's method of applying mathematical reaſoning to theſe philoſophical ſubjects.

31. HE reaſons thus. Suppoſe a body ſet out from the point A (in fig. 85.) to move in the ſtraight line AB; and after it had moved for ſome time in that line, it were to receive an impulſe directed to ſome point as C. Let it receive that impulſe at D; and thereby be turned into the line DE; and let the body after this impulſe take the ſame length of time in paſſing from D to E, as it imployed in the paſſing from A to D. Then the ſtraight lines CA, CD, and CE being drawn, Sir ISAAC NEWTON proves, that the and triangular ſpaces CAD and CDE are equal. This he does in the following manner.

32. LET EF be drawn parallel to CD. Then, from what has been ſaid upon the ſecond law of motion [a], it is evident, that ſince the body was moving in the line AB, when it received the impulſe in the direction DC; it will have moved after that impulſe through the line DE in the ſame time, as it would have taken up in moving through DF, provided it had received no diſturbance in D. But the time of the body's moving from D to E is ſuppoſed to be equal to the time of its moving through AD; therefore the time, which the body would have imployed in moving through DF, had it not been diſturbed in D, is equal to the time, wherein it moved through AD: conſequently DF is equal in length to AD; for if the

[a] Ch. 1. ſect. 21, 22.

body

body had gone on to move through the line A B without interruption, it would have moved through all parts thereof with the ſame velocity, and have paſſed over equal parts of that line in equal portions of time. Now C F being drawn, ſince A D and D F are equal, the triangular ſpace C D F is equal to the triangular ſpace C A D. Farther, the line E F being parallel to C D, it is proved by EUCLID, that the triangle C E D is equal to the triangle C F D [a]: therefore the triangle C E D is equal to the triangle C A D.

33. AFTER the ſame manner, if the body receive at E another impulſe directed toward the point C, and be turned by that impulſe into the line E G; if it move afterwards from E to G in the ſame ſpace of time, as was taken up by its motion from D to E, or from A to D; then C G being drawn, the triangle C E G is equal to C D E. A third impulſe at G directed as the two former to C, whereby the body ſhall be turned into the line G H, will have alſo the like effect with the reſt. If the body move over G H in the ſame time, as it took up in moving over E G, the triangle C G H will be equal to the triangle C E G. Laſtly, if the body at H be turned by a freſh impulſe directed toward C into the line H I, and at I by another impulſe directed alſo to C be turned into the line I K; and if the body move over each of the lines H I, and I K in the ſame time, as it imployed in moving over each of the preceding lines A D, D E, E G, and G H: then each of the triangles C H I, and C I K will be equal to each of the preceding. Like-

[a] Elem. Book I. p. 37.

wiſe

wise as the time, in which the body moves over ADE, is equal to the time of its moving over EGH, and to the time of its moving over HIK; the space CADE will be equal to the space CEGH, and to the space CHIK. In the same manner as the time, in which the body moved over ADEG is equal to the time of its moving over GHIK, so the space CADEG will be equal to the space CGHIK.

34. FROM this principle Sir ISAAC NEWTON demonstrates the proposition mentioned above, by that method of arguing introduced by him into geometry, whereof we have before taken notice [a], by making according to the principles of that method a transition from this incurvated figure composed of straight lines, to a figure of continued curvature; and by shewing, that since equal spaces are described in equal times in this present figure composed of straight lines, the same relation between the spaces described and the times of their description will also have place in a figure of one continued curvature. He also deduces from this proposition the reverse of it; and proves, that whenever equal spaces are continually described; the body is acted upon by a centripetal force directed to the center, at which the spaces terminate.

[a] § 12.

CHAP.

CHAP. IV.

Of the RESISTANCE of FLUIDS.

BEFORE the cauſe can be diſcovered, which keeps the planets in motion, it is neceſſary firſt to know, whether the ſpace, wherein they move, is empty and void, or filled with any quantity of matter. It has been a prevailing opinion, that all ſpace contains in it matter of ſome kind or other; ſo that where no ſenſible matter is found, there was yet a ſubtle fluid ſubſtance by which the ſpace was filled up; even ſo as to make an abſolute plenitude. In order to examine this opinion, Sir ISAAC NEWTON has largely conſidered the effects of fluids upon bodies moving in them.

2. THESE effects he has reduced under theſe three heads. In the firſt place he ſhews how to determine in what manner the reſiſtance, which bodies ſuffer, when moving in a fluid, gradually increaſes in proportion to the ſpace, they deſcribe in any fluid; to the velocity, with which they deſcribe it; and to the time they have been in motion. Under the ſecond head he conſiders what degree of reſiſtance different bodies moving in the ſame fluid undergo, according to the different proportion between the denſity of the fluid and the denſity of the body. The denſities of bodies, whether fluid or ſolid, are meaſured by the quantity of matter, which is comprehended under the ſame magnitude; that body being the

the moſt denſe or compact, which under the ſame bulk contains the greateſt quantity of ſolid matter, or which weighs moſt, the weight of every body being obſerved above to be proportional to the quantity of matter in it [a]. Thus water is more denſe than cork or wood, iron more denſe than water, and gold than iron. The third particular Sir Is. Newton conſiders concerning the reſiſtance of fluids is the influence, which the diverſity of figure in the reſiſted body has upon its reſiſtance.

3. For the more perfect illuſtration of the firſt of theſe heads, he diſtinctly ſhews the relation between all the particulars ſpecified upon three different ſuppoſitions. The firſt is, that the ſame body be reſiſted more or leſs in the ſimple proportion to its velocity; ſo that if its velocity be doubled, its reſiſtance ſhall become threefold. The ſecond is of the reſiſtance increaſing in the duplicate proportion of the velocity; ſo that, if the velocity of a body be doubled, its reſiſtance ſhall be rendered four times; and if the velocity be trebled, nine times as great as at firſt. But what is to be underſtood by duplicate proportion has been already explained [b]. The third ſuppoſition is, that the reſiſtance increaſes partly in the ſingle proportion of the velocity, and partly in the duplicate proportion thereof.

4. In all theſe ſuppoſitions, bodies are conſidered under two reſpects, either as moving, and oppoſing themſelves

[a] Ch. 1 § 24. [b] Ch. 2 ſelect. 17.

against

againſt the fluid by that power alone, which is eſſential to them, of reſiſting to the change of their ſtate from reſt to motion, or from motion to reſt, which we have above called their power of inactivity; or elſe, as deſcending or aſcending, and ſo having the power of gravity combined with that other power. Thus our author has ſhewn in all thoſe three ſuppoſitions, in what manner bodies are reſiſted in an uniform fluid, when they move with the aforeſaid progreſſive motion [a]; and what the reſiſtance is, when they aſcend or deſcend perpendicularly [b]. And if a body aſcend or deſcend obliquely, and the reſiſtance be ſingly proportional to the velocity, it is ſhewn how the body is reſiſted in a fluid of an uniform denſity, and what line it will deſcribe [c], which is determined by the meaſurement of the hyperbola, and appears to be no other than that line, firſt conſidered in particular by Dr. BARROW [d], which is now commonly known by the name of the logarithmical curve. In the ſuppoſition that the reſiſtance increaſes in the duplicate proportion of the velocity, our author has not given us the line which would be deſcribed in an uniform fluid; but has inſtead thereof diſcuſſed a problem, which is in ſome ſort the reverſe; to find the denſity of the fluid at all altitudes, by which any given curve line may be deſcribed; which problem is ſo treated by him, as to be applicable to any kind of reſiſtance whatever [e]. But here not unmindful of practice, he ſhews that a body in a fluid of uniform denſity, like the

[a] Newt. Princ. L. II. prop. 2; 5, 6, 7; 11, 12.
[b] Prop. 3; 8, 9; 13, 14.
[c] Prop. 4.
[d] Prælect. Geometr. pag. 123.
[e] Newton. Princ. Lib. II. prop. 10.

air, will deſcribe a line, which approaches towards an hyperbola; that is, its motion will be nearer to that curve line than to the parabola. And conſequent upon this remark, he ſhews how to determine this hyperbola by experiment, and briefly reſolves the chief of thoſe problems relating to projectiles, which are in uſe in the art of gunnery, in this curve[a]; as Torricelli and others have done in the parabola[b], whoſe inventions have been explained at large above[c].

5. Our author has alſo handled diſtinctly that particular ſort of motion, which is deſcribed by pendulums[d]; and has likewiſe conſidered ſome few caſes of bodies moving in reſiſting fluids round a center, to which they are impelled by a centripetal force, in order to give an idea of thoſe kinds of motions[e].

6. The treating of the reſiſtance of pendulums has given him an opportunity of inſerting into another part of his work ſome ſpeculations upon the motions of them without reſiſtance, which have a very peculiar elegance; wherein he treats of them as moved by a gravitation acting in the law, which he ſhews to belong to the earth below its ſurface[f]; performing in this kind of gravitation, where the force is proportional to the diſtance from the center, all that Huygens had before done in the common ſuppoſition of its being uniform, and acting in parallel lines[g].

[a] Newton. Princ. Lib II prop. 10. in ſchol.
[b] Torricelli de motu gravium.
[c] Ch 2 § 85, &c.
[d] Newt. Princ. L. II. ſect. 6.
[e] L. II. ſect 4.
[f] See B. II. Ch 6. § 7. of this treatiſe.
[g] Lib. I. ſect. 10.

7. Huy-

7. HUYGENS at the end of his treatise of the cause of gravity[a] informs us, that he likewise had carried his speculations on the first of these suppositions, of the resistance in fluids being proportional to the velocity of the body, as far as our author. But finding by experiment that the second was more conformable to nature, he afterwards made some progress in that, till he was stopt, by not being able to execute to his wish what related to the perpendicular descent of bodies; not observing that the measurement of the curve line, he made use of to explain it by, depended on the hyperbola. Which oversight may well be pardoned in that great man, considering that our author had not been pleased at that time to communicate to the publick his admirable discourse of the QUADRATURE OF MEASUREMENT OF CURVE LINES, with which he has since obliged the world: for without the use of that treatise, it is I think no injury even to our author's unparalleled abilities to believe, it would not have been easy for himself to have succeeded so happily in this and many other parts of his writings.

8. WHAT HUYGENS found by experiment, that bodies were in reality resisted in the duplicate proportion of their velocity, agrees with the reasoning of our author [b], who distinguishes the resistance, which fluids give to bodies by the tenacity of their parts, and the friction between them and the body, from that, which arises from the power of inactivity, with which the constituent particles of fluids are endued like all

[a] De la Pesanteur, pag. 169, and the following.

[b] Newton. Princ. L. II. prop. 4. schol.

 other

other portions of matter, by which power the particles of fluids like other bodies make refiftance againft being put into motion.

9. The refiftance, which arifes from the friction of the body againft the parts of the fluid, muft be very inconfiderable; and the refiftance, which follows from the tenacity of the parts of fluids, is not ufually very great, and does not depend much upon the velocity of the body in the fluid; for as the parts of the fluid adhere together with a certain degree of force, the refiftance, which the body receives from thence, cannot much depend upon the velocity, with which the body moves; but like the power of gravity, its effect muft be proportional to the time of its acting. This the reader may find farther explained by Sir Isaac Newton himfelf in the poftfcript to a difcourfe publifhed by me in the philosophical transactions, N° 371. The principal refiftance, which moft fluids give to bodies, arifes from the power of inactivity in the parts of the fluids, and this depends upon the velocity, with which the body moves, on a double account. In the firft place, the quantity of the fluid moved out of place by the moving body in any determinate fpace of time is proportional to the velocity, wherewith the body moves; and in the next place, the velocity with which each particle of the fluid is moved, will alfo be proportional to the velocity of the body: therefore fince the refiftance, which any body makes againft being put into motion, is proportional both to the quantity of matter moved and the velocity it is moved with; the refiftance, which a fluid gives on this account, will be doubly increafed with the increafe of the velocity in the moving body; that

that is, the reſiſtance will be in a two-fold or duplicate proportion of the velocity, wherewith the body moves through the fluid.

10. FARTHER it is moſt manifeſt, that this latter kind of reſiſtance increaſing with the increaſe of velocity, even in a greater degree than the velocity it ſelf increaſes, the ſwifter the body moves, the leſs proportion the other ſpecies of reſiſtance will bear to this: nay that this part of the reſiſtance may be ſo much augmented by a due increaſe of velocity, till the former reſiſtances ſhall bear a leſs proportion to this, than any that might be aſſigned. And indeed experience ſhews, that no other reſiſtance, than what ariſes from the power of inactivity in the parts of the fluid, is of moment, when the body moves with any conſiderable ſwiftneſs.

11. THERE is beſides theſe yet another ſpecies of reſiſtance, found only in ſuch fluids, as, like our air, are elaſtic. Elaſticity belongs to no fluid known to us beſide the air. By this property any quantity of air may be contracted into a leſs ſpace by a forcible preſſure, and as ſoon as the compreſſing power is removed, it will ſpring out again to its former dimenſions. The air we breath is held to its preſent denſity by the weight of the air above us. And as this incumbent weight, by the motion of the winds, or other cauſes, is frequently varied (which appears by the barometer;) ſo when this weight is greateſt, we breath a more denſe air than at other times. To what degree the air would expand it ſelf by its ſpring, if all preſſure were removed, is not known,

known, nor yet into how narrow a compaſs it is capable of being compreſſed. Mr. BOYLE found it by experiment capable both of expanſion and compreſſion to ſuch a degree, that he could cauſe a quantity of air to expand it ſelf over a ſpace ſome hundred thouſand times greater, than the ſpace to which he could confine the ſame quantity [a]. But I ſhall treat more fully of this ſpring in the air hereafter [b]. I am now only to conſider what reſiſtance to the motion of bodies ariſes from it.

12. BUT before our author ſhews in what manner this cauſe of reſiſtance operates, he propoſes a method, by which fluids may be rendered elaſtic, demonſtrating that if their particles be provided with a power of repelling each other, which ſhall exert it ſelf with degrees of ſtrength reciprocally proportional to the diſtances between the centers of the particles; that then ſuch fluids will obſerve the ſame rule in being compreſſed, as our air does, which is this, that the ſpace, into which it yields upon compreſſion, is reciprocally proportional to the compreſſing weight [c]. The term reciprocally proportional has been explained above [d]. And if the centrifugal force of the particles acted by other laws, ſuch fluids would yield in a different manner to compreſſion [e].

13. WHETHER the particles of the air be endued with ſuch a power, by which they can act upon each other out of contact, our author does not determine; but leaves that

[a] ... Tract on the admirable rarifaction of ...
[b] ... Ch. 6.

[c] Princ. philoſ. Lib. II. prop. 23.
[d] Book I. Ch. 2. § 30
[e] Princ. philoſ. Lib. II. prop. 23. in ſchol.

to future examination, and to be diſcuſſed by philoſophers. Only he takes occaſion from hence to conſider the reſiſtance in elaſtic fluids, under this notion; making remarks, as he paſſes along, upon the differences, which will ariſe, if their elaſticity be derived from any other fountain [a]. And this, I think, muſt be confeſſed to be done by him with great judgment; for this is far the moſt reaſonable account, which has been given of this ſurprizing power, as muſt without doubt be freely acknowledged by any one, who in the leaſt conſiders the inſufficiency of all the other conjectures, which have been framed; and alſo how little reaſon there is to deny to bodies other powers, by which they may act upon each other at a diſtance, as well as that of gravity; which we ſhall hereafter ſhew to be a property univerſally belonging to all the bodies of the univerſe, and to all their parts [b]. Nay we actually find in the loadſtone a very apparent repelling, as well as an attractive power. But of this more in the concluſion of this diſcourſe.

14. BY theſe ſteps our author leads the way to explain the reſiſtance, which the air and ſuch like fluids will give to bodies by their elaſticity; which reſiſtance he explains thus. If the elaſtic power of the fluid were to be varied ſo, as to be always in the duplicate proportion of the velocity of the reſiſted body, it is ſhewn that then the reſiſtance derived from the elaſticity, would increaſe in the duplicate proportion of the velocity; in ſo much that the

[a] Princ. philoſ. Lib. II. prop. 33. coroll. | [b] Lib. II. Ch. 5.

whole

whole resistance would be in that proportion, excepting only that small part, which arises from the friction between the body and the parts of the fluid. From whence it follows, that because the elastic power of the same fluid does in truth continue the same, if the velocity of the moving body be diminished, the resistance from the elasticity, and therefore the whole resistance, will decrease in a less proportion, than the duplicate of the velocity; and if the velocity be increased, the resistance from the elasticity will increase in a less proportion, than the duplicate of the velocity, that is in a less proportion, than the resistance made by the power of inactivity of the parts of the fluid. And from this foundation is raised the proof of a property of this resistance, given by the elasticity in common with the others from the tenacity and friction of the parts of the fluid; that the velocity may be increased, till this resistance from the fluid's elasticity shall bear no considerable proportion to that, which is produced by the power of inactivity thereof [a]. From whence our author draws this conclusion; that the resistance of a body, which moves very swiftly in an elastic fluid, is near the same, as if the fluid were not elastic; provided the elasticity arises from the centrifugal power of the parts of the medium, as before explained, especially if the velocity be so great, that this centrifugal power shall want time to exert it self [b]. But it is to be observed, that in the proof of all this our author proceeds upon the supposition of this centrifugal power in the parts of the fluid; but if the elasticity be caused by the expansion of the parts in the

[a] Ibid. Prop. 33. coroll. 2. | [b] Ibid. coroll. 3.

manner

manner of wool compreſſed, and ſuch like bodies, by which the parts of the fluid will be in ſome meaſure entangled together, and their motion be obſtructed, the fluid will be in a manner tenacious, and give a reſiſtance upon that account over and above what depends upon its elaſticity only [a]; and the reſiſtance derived from that cauſe is to be judged of in the manner before ſet down.

15. It is now time to paſs to the ſecond part of this theory; which is to aſſign the meaſure of reſiſtance, according to the proportion between the denſity of the body and the denſity of the fluid. What is here to be underſtood by the word denſity has been explained above [b]. For this purpoſe as our author before conſidered two diſtinct caſes of bodies moving in mediums; one when they oppoſed themſelves to the fluid by their power of inactivity only, and another when by aſcending or deſcending their weight was combined with that other power: ſo likewiſe, the fluids themſelves are to be regarded under a double capacity; either as having their parts at reſt, and diſpoſed freely without reſtraint, or as being compreſſed together by their own weight, or any other cauſe.

16. In the firſt caſe, if the parts of the fluid be wholly diſingaged from one another, ſo that each particle is at liberty to move all ways without any impediment, it is ſhewn, that if a globe move in ſuch a fluid, and the globe and par-

[a] Vid. ibid. coroll. 6. | [b] In § 2.

ticles of the fluid are endued with perfect elasticity; so that as the globe impinges upon the particles of it, they shall bound off and separate themselves from the globe, with the same velocity, with which the globe strikes upon them; then the resistance, which the globe moving with any known velocity suffers, is to be thus determined. From the velocity of the globe, the time, wherein it would move over two third parts of its own diameter with that velocity, will be known. And such proportion as the density of the fluid bears to the density of the globe, the same the resistance given to the globe will bear to the force, which acting, like the power of gravity, on the globe without intermission during the space of time now mentioned, would generate in the globe the same degree of motion, as that wherewith it moves in the fluid [a]. But if neither the globe nor the particles of the fluid be elastic, so that the particles, when the globe strikes against them, do not rebound from it, then the resistance will be but half so much [b]. Again, if the particles of the fluid and the globe are imperfectly elastic, so that the particles will spring from the globe with part only of that velocity wherewith the globe impinges upon them; then the resistance will be a mean between the two preceding cases, approaching nearer to the first or second, according as the elasticity is more or less [c].

17. The elasticity, which is here ascribed to the particles of the fluid, is not that power of repelling one another,

[a] Princ. philos. Lib. II. Prop. 35.
[b] Ibid.
[c] Id.

when out of contact, by which, as has before been mentioned, the whole fluid may be rendred elastic; but such an elasticity only, as many solid bodies have of recovering their figure, whenever any forcible change is made in it, by the impulse of another body or otherwise. Which elasticity has been explained above at large [a].

18. THIS is the case of discontinued fluids, where the body, by pressing against their particles, drives them before itself, while the space behind the body is left empty. But in fluids which are compressed, so that the parts of them removed out of place by the body resisted immediately retire behind the body, and fill that space, which in the other case is left vacant, the resistance is still less; for a globe in such a fluid which shall be free from all elasticity, will be resisted but half as much as the least resistance in the former case [b]. But by elasticity I now mean that power, which renders the whole fluid so; of which if the compressed fluid be possessed, in the manner of the air, then the resistance will be greater than by the foregoing rule; for the fluid being capable in some degree of condensation, it will resemble so far the case of uncompressed fluids [c]. But, as has been before related, this difference is most considerable in slow motions.

19. IN the next place our author is particular in determining the degrees of resistance accompanying bodies of different figures; which is the last of the three heads, we

[a] h. 1. § 29.
[b] Princ. philos. Lib. II. Prop. 38, compared with coroll. 1. of prop. 35.
[c] L. II. Lem. 7. schol. pag. 341.

divided the whole discourse of resistance into. And in this disquisition he finds a very surprizing and unthought of difference, between free and compressed fluids. He proves, that in the former kind, a globe suffers but half the resistance, which the cylinder, that circumscribes the globe, will do, if it move in the direction of its axis [a]. But in the latter he proves, that the globe and cylinder are resisted alike [b]. And in general, that let the shape of bodies be ever so different, yet if the greatest sections of the bodies perpendicular to the axis of their motion be equal, the bodies will be resisted equally [c].

20. Pursuant to the difference found between the resistance of the globe and cylinder in rare and uncompressed fluids, our author gives us the result of some other inquiries of the same nature. Thus of all the frustums of a cone, that can be described upon the same base and with the same altitude, he shews how to find that, which of all others will be the least resisted, when moving in the direction of its axis [d]. And from hence he draws an easy method of altering the figure of any spheroidical solid, so that its capacity may be enlarged, and yet the resistance of it diminished [e]: a note which he thinks may not be useless to shipwrights. He concludes with determining the solid, which will be resisted the least that is possible, in these discontinued fluids [f].

[a] Lib. II. Prop. 34.
[b] Lib. II. Lem. 7. p. 341.
[c] Schol. to Lem. 7.
[d] Prop. 34. schol.
[e] Ibid.
[f] Ibid.

21. That

21. THAT I may here be underſtood by readers unacquainted with mathematical terms, I ſhall explain what I mean by a fruſtum of a cone, and a ſpheroidical ſolid. A cone has been defined above. A fruſtum is what remains, when part of the cone next the vertex is cut away by a ſection parallel to the baſe of the cone, as in fig. 86. A ſpheroid is produced from an ellipſis, as a ſphere or globe is made from a circle. If a circle turn round on its diameter, it deſcribes by its motion a ſphere; ſo if an ellipſis (which figure has been defined above, and will be more fully explained hereafter[a]) be turned round either upon the longeſt or ſhorteſt line, that can be drawn through the middle of it, there will be deſcribed a kind of oblong or flat ſphere, as in fig. 87. Both theſe figures are called ſpheroids, and any ſolid reſembling theſe I here call ſpheroidical.

22. IF it ſhould be asked, how the method of altering ſpheroidical bodies, here mentioned, can contribute to the facilitating a ſhip's motion, when I juſt above affirmed, that the figure of bodies, which move in a compreſſed fluid not elaſtic, has no relation to the augmentation or diminution of the reſiſtance; the reply is, that what was there ſpoken relates to bodies deep immerged into ſuch fluids, but not of thoſe, which ſwim upon the ſurface of them; for in this latter caſe the fluid, by the appulſe of the anterior parts of the body, is raiſed above the level of the ſurface, and behind the body is ſunk ſomewhat below; ſo

[a] Book II. Ch. 1. § 6.

that

that by this inequality in the superficies of the fluid, that part of it, which at the head of the body is higher than the fluid behind, will resist in some measure after the manner of discontinued fluids [a], analogous to what was before observed to happen in the air through its elasticity, though the body be surrounded on every side by it [b]. And as far as the power of these causes extends, the figure of the moving body affects its resistance; for it is evident, that the figure, which presses least directly against the parts of the fluid, and so raises least the surface of a fluid not elastic, and least compresses one that is elastic, will be least resisted.

23. The way of collecting the difference of the resistance in rare fluids, which arises from the diversity of figure, is by considering the different effect of the particles of the fluid upon the body moving against them, according to the different obliquity of the several parts of the body upon which they respectively strike; as it is known, that any body impinging against a plane obliquely, strikes with a less force, than if it fell upon it perpendicularly; and the greater the obliquity is, the weaker is the force. And it is the same thing, if the body be at rest, and the plane move against it [c].

24. That there is no connexion between the figure of a body and its resistance in compressed fluids, is proved thus. Suppose A B C D (in fig.88.) to be a canal, having such a fluid, water for instance, running through it with an equable

[a] Vid. Newt. princ. in schol. to Lem. 7. of Lib. II. pag. 341.

[b] Sect. 17. of this chapter.

[c] See Princ. philos. Lib. II. prop. 34.

velocity;

J. Grison del. J. Pine Sculp.

BOOK II.

CONCERNING THE

SYSTEM of the WORLD.

CHAP. I.

That the Planets move in a ſpace empty of all ſenſible matter.

I HAVE now gone through the firſt part of my deſign, and have explained, as far as the nature of my undertaking would permit, what Sir ISAAC NEWTON has delivered in general concerning the motion of bodies. It follows now to ſpeak of the diſcoveries, he has made in the ſyſtem of the world;

and to ſhew from him what cauſe keeps the heavenly bodies in their courſes. But it will be neceſſary for the uſe of ſuch, as are not ſkilled in aſtronomy, to premiſe a brief deſcription of the planetary ſyſtem.

2. This ſyſtem is diſpoſed in the following manner. In the middle is placed the ſun. About him ſix globes continually roll. Theſe are the primary planets; that which is neareſt to the ſun is called Mercury, the next Venus, next to this is our earth, the next beyond is Mars, after him Jupiter, and the outermoſt of all Saturn. Beſides theſe there are diſcovered in this ſyſtem ten other bodies, which move about ſome of theſe primary planets in the ſame manner, as they move round the ſun. Theſe are called ſecondary planets. The moſt conſpicuous of them is the moon, which moves round our earth; four bodies move in like manner round Jupiter; and five round Saturn. Thoſe which move about Jupiter and Saturn, are uſually called ſatellites; and cannot any of them be ſeen without a teleſcope. It is not impoſſible, but there may be more ſecondary planets, beſide theſe; though our inſtruments have not yet diſcovered any other. This diſpoſition of the planetary or ſolar ſyſtem is repreſented in fig. 89.

3. The ſame planet is not always equally diſtant from the ſun. But the middle diſtance of Mercury is between $\frac{1}{3}$ and $\frac{2}{5}$ of the diſtance of the earth from the ſun; Venus is diſtant from the ſun almoſt $\frac{3}{4}$ of the diſtance of the earth; the middle diſtance of Mars is ſomething more than half

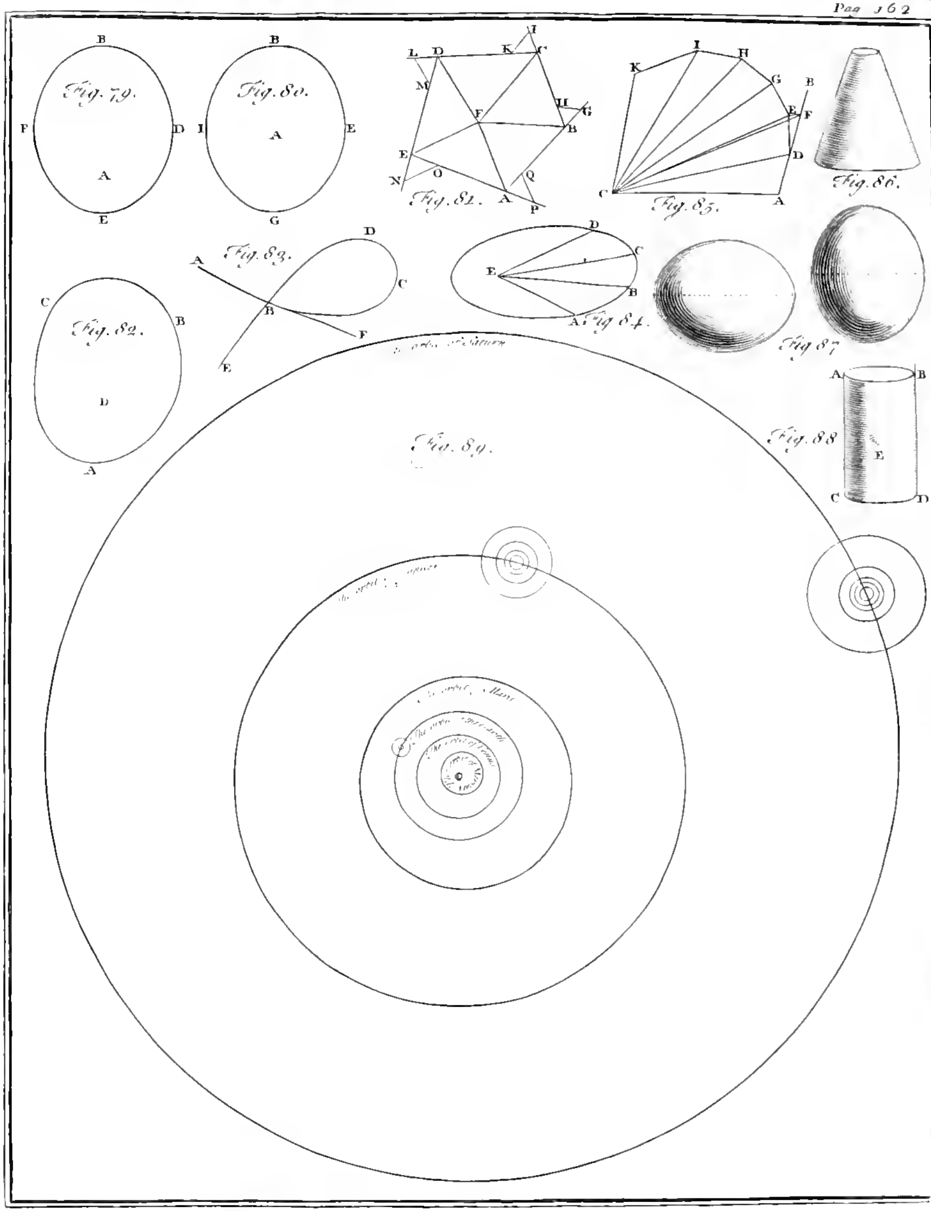


Fig. 79.
Fig. 80.
Fig. 81.
Fig. 85.
Fig. 86.
Fig. 82.
Fig. 83.
Fig. 84.
Fig. 87
Fig. 88
Fig. 89.

half as much again, as the diſtance of the earth; Jupiter's middle diſtance exceeds five times the diſtance of the earth, by between $\frac{1}{7}$ and $\frac{1}{6}$ part of this diſtance; Saturn's middle diſtance is ſcarce more than $9\frac{1}{2}$ times the diſtance between the earth and ſun; but the middle diſtance between the earth and ſun is about $217\frac{1}{8}$ times the ſun's ſemidiameter.

4. ALL theſe planets move one way, from weſt to eaſt; and of the primary planets the moſt remote is longeſt in finiſhing its courſe round the ſun. The period of Saturn falls ſhort only ſixteen days of 29 years and a half. The period of Jupiter is twelve years wanting about 50 days. The period of Mars falls ſhort of two years by about 43 days. The revolution of the earth conſtitutes the year. Venus performs her period in about $224\frac{1}{2}$ days, and mercury in about 88 days.

5. THE courſe of each planet lies throughout in one plane or flat ſurface, in which the ſun is placed; but they do not all move in the ſame plane, though the different planes, in which they move, croſs each other in very ſmall angles. They all croſs each other in lines, which paſs through the ſun; becauſe the ſun lies in the plane of each orbit. This inclination of the ſeveral orbits to each other is repreſented in fig. 90. The line, in which the plane of any orbit croſſes the plane of the earth's motion, is called the line of the nodes of that orbit.

6. Each planet moves round the ſun in the line, which we have mentioned above[a] under the name of ellipſis; which I ſhall here ſhew more particularly how to deſcribe. I have there ſaid how it is produced in the cone. I ſhall now ſhew how to form it upon a plane. Fix upon any plane two pins, as at A and B in fig. 91. To theſe tye a ſtring ACB of any length. Then apply a third pin D ſo to the ſtring, as to hold it ſtrained; and in that manner carrying this pin about, the point of it will deſcribe an ellipſis. If through the points A, B the ſtraight line EABF be drawn, to be terminated at the ellipſis in the points E and F, this is the longeſt line of any, that can be drawn within the figure, and is called the greater axis of the ellipſis. The line GH, drawn perpendicular to this axis EF, ſo as to paſs through the middle of it, is called the leſſer axis. The two points A and B are called focus's. Now each planet moves round the ſun in a line of this kind, ſo that the ſun is found in one focus. Suppoſe A to be the place of the ſun. Then E is the point, wherein the planet will be neareſt of all to the ſun, and at F it will be moſt remote. The point E is called the perihelion of the planet, and F the aphelion. In G and H the planet is ſaid to be in its middle or mean diſtance; becauſe the diſtance AG or AH is truly the middle between AE the leaſt, and AF the greateſt diſtance. In fig. 92. is repreſented how the greater axis of each orbit is ſituated in reſpect of the reſt. The proportion between the greateſt and leaſt diſtances of the planet from the ſun is very different in the different planets. In Saturn the proportion of the greateſt

[a] Book I. ch. 2. § 82.

Fig. 90

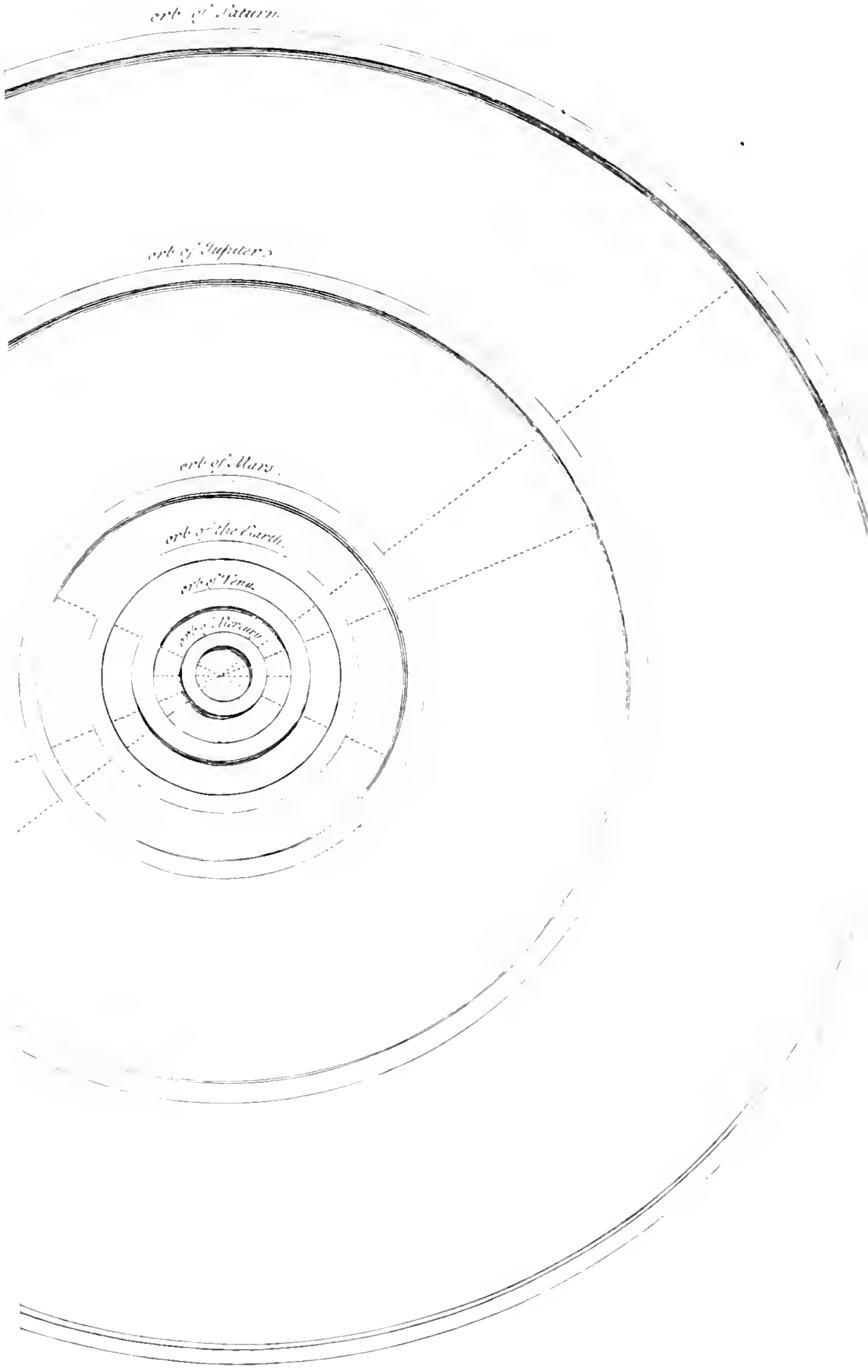
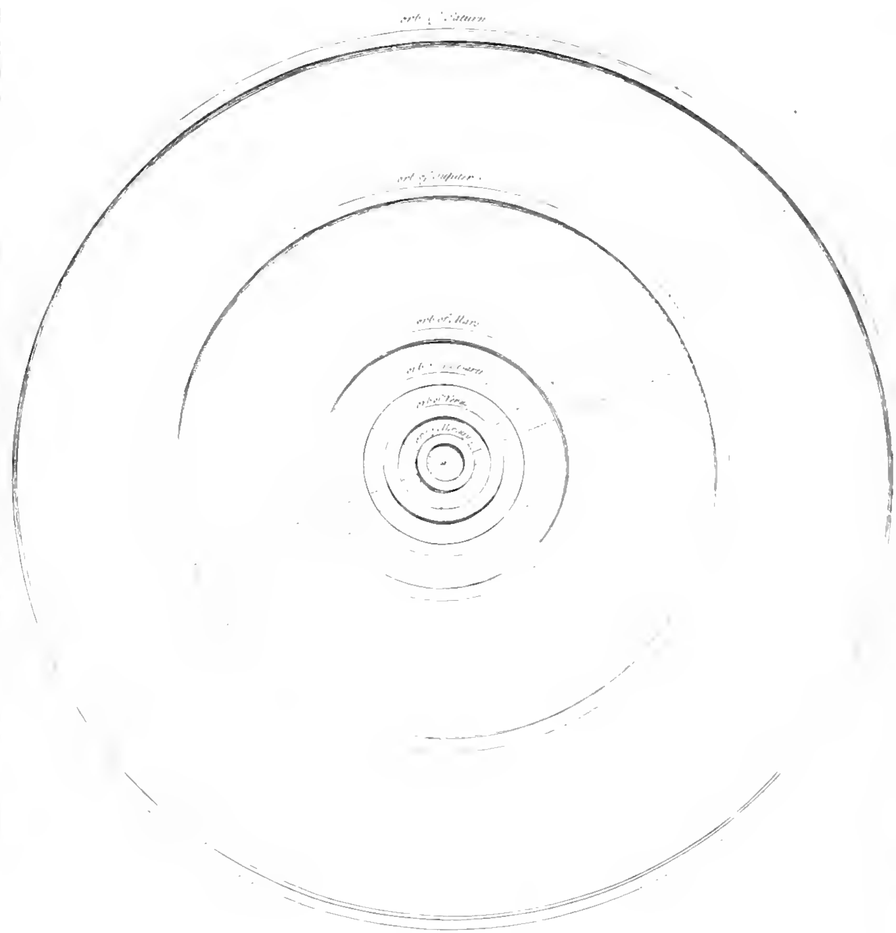

orb of Saturn.
orb of Jupiter.
orb of Mars.
orb of the Earth.
orb of Venu.
orb of Mercury.

Fig. 90



greateſt diſtance to the leaſt is ſomething leſs, than the proportion of 9 to 8; but much nearer to this, than to the proportion of 10 to 9. In Jupiter this proportion is a little greater, than that of 11 to 10. In Mars it exceeds the proportion of 6 to 5. In the earth it is about the proportion of 30 to 29. In Venus it is near to that of 70 to 69. And in Mercury it comes not a great deal ſhort of the proportion of 3 to 2.

7. EACH of theſe planets ſo moves through its ellipſis, that the line drawn from the ſun to the planet, by accompanying the planet in its motion, will deſcribe about the ſun equal ſpaces in equal times, after the manner ſpoke of in the chapter of centripetal forces[a]. There is alſo a certain relation between the greater axis's of theſe ellipſis's, and the times, in which the planets perform their revolutions through them. Which relation may be expreſſed thus. Let the period of one planet be denoted by the letter A, the greater axis of its orbit by D; let the period of another planet be denoted by B, and the greater axis of this planet's orbit by E. Then if C be taken to bear the ſame proportion to B, as B bears to A; likewiſe if F be taken to bear the ſame proportion to E, as E bears to D; and G taken to bear the ſame proportion likewiſe to F, as E bears to D; then A ſhall bear the ſame proportion to C, as D bears to G.

A	D
B	E
C	F
	G

8. THE ſecondary planets move round their reſpective primary, much in the ſame manner as the primary do round

[a] Book I. Ch. 3. § 29.

the

the ſun. But the motions of theſe ſhall be more fully explained hereafter [a]. And there is, beſides the planets, another ſort of bodies, which in all probability move round the ſun; I mean the comets. The farther deſcription of which bodies I alſo leave to the place, where they are to be particularly treated on [b].

9. FAR without this ſyſtem the fixed ſtars are placed. Theſe are all ſo remote from us, that we ſeem almoſt incapable of contriving any means to eſtimate their diſtance. Their number is exceeding great. Beſides two or three thouſand, which we ſee with the naked eye, teleſcopes open to our view vaſt numbers; and the farther improved theſe inſtruments are, we ſtill diſcover more and more. Without doubt theſe are luminous globes, like our ſun, and ranged through the wide extent of ſpace; each of which, it is to be ſuppoſed, perform the ſame office, as our ſun, affording light and heat to certain planets moving about them. But theſe conjectures are not to be purſued in this place.

10. I SHALL therefore now proceed to the particular deſign of this chapter, and ſhew, that there is no ſenſible matter lodged in the ſpace where the planets move.

11. THAT they ſuffer no ſenſible reſiſtance from any ſuch matter, is evident from the agreement between the obſervations of aſtronomers in different ages, with regard to the time, in which the planets have been found to perform their

[a] Ch. 3. of this preſent book. [b] Ch. 4.

periods.

periods. But it was the opinion of DES CARTES [a], that the planets might be kept in their courſes by the means of a fluid matter, which continually circulating round ſhould carry the planets along with it. There is one appearance that may ſeem to favour this opinion; which is, that the ſun turns round its own axis the ſame way, as the planets move. The earth alſo turns round its axis the ſame way, as the moon moves round the earth. And the planet Jupiter turns upon its axis the ſame way, as his ſatellites revolve round him. It might therefore be ſuppoſed, that if the whole planetary region were filled with a fluid matter, the ſun, by turning round on its own axis, might communicate motion firſt to that part of the fluid, which was contiguous, and by degrees propagate the like motion to the parts more remote. After the ſame manner the earth might communicate motion to this fluid, to a diſtance ſufficient to carry round the moon, and Jupiter communicate the like to the diſtance of its ſatellites. Sir ISAAC NEWTON has particularly examined what might be the reſult of ſuch a motion as this [b]; and he finds, that the velocities, with which the parts of this fluid will move in different diſtances from the center of the motion, will not agree with the motion obſerved in different planets: for inſtance, that the time of one intire circulation of the fluid, wherein Jupiter ſhould ſwim, would bear a greater proportion to the time of one intire circulation of the fluid, where the earth is; than the period of Jupiter bears to the period of the earth. But he alſo proves [c], that the planet cannot circulate in ſuch a fluid,

[a] In Princ. philoſ. part. 3.
[b] Philoſ. princ. mathem. Lib. II. prop. 2. & ſchol.
[c] Ibid. prop. 53.

ſo

ſo as to keep long in the ſame courſe, unleſs the planet and the contiguous fluid are of the ſame denſity, and the planet be carried along with the ſame degree of motion, as the fluid. There is alſo another remark made upon this motion by our author; which is, that ſome vivifying force will be continually neceſſary at the center of the motion [a]. The ſun in particular, by communicating motion to the ambient fluid, will loſe from it ſelf as much motion, as it imparts to the fluid; unleſs ſome acting principle reſide in the ſun to renew its motion continually. If the fluid be infinite, this gradual loſs of motion would continue till the whole ſhould ſtop [b]; and if the fluid were limited, this loſs of motion would continue, till there would remain no ſwifter a revolution in the ſun, than in the utmoſt part of the fluid; ſo that the whole would turn together about the axis of the ſun, like one ſolid globe [c].

12. It is farther to be obſerved, that as the planets do not move in perfect circles round the ſun; there is a greater diſtance between their orbits in ſome places, than in others. For inſtance, the diſtance between the orbit of Mars and Venus is near half as great again in one part of their orbits, as in the oppoſite place. Now here the fluid, in which the earth ſhould ſwim, muſt move with a leſs rapid motion, where there is this greater interval between the contiguous orbits; but on the contrary, where the ſpace is ſtraiteſt, the earth moves more ſlowly, than where it is wideſt [d].

[a] Philoſ. princ. prop. 52. coroll. 4.
[b] Ibid.
[c] Coroll. 11.
[d] See ibid. ſchol. poſt prop. 53.

13. Farther

13. FARTHER, if this our globe of earth ſwam in a fluid of equal denſity with the earth it ſelf, that is, in a fluid more denſe than water; all bodies put in motion here upon the earth's ſurface muſt ſuffer a great reſiſtance from it; where as, by Sir ISAAC NEWTON's experiments mentioned in the preceding chapter, bodies, that fell perpendicularly down through the air, felt about $\frac{1}{860}$ part only of the reſiſtance, which bodies ſuffered that fell in like manner through water.

14. Sir ISAAC NEWTON applies theſe experiments yet farther, and examines by them the general queſtion concerning the abſolute plenitude of ſpace. According to the Ariſtotelians, all ſpace was full without any the leaſt vacuities whatever. DES CARTES embraced the ſame opinion, and therefore ſuppoſed a ſubtile fluid matter, which ſhould pervade all bodies, and adequately fill up their pores. The Atomical philoſophers, who ſuppoſe all bodies both fluid and ſolid to be compoſed of very minute but ſolid atoms, aſſert that no fluid, how ſubtile ſoever the particles or atoms whereof it is compoſed ſhould be, can ever cauſe an abſolute plenitude; becauſe it is impoſſible that any body can paſs through the fluid without putting the particles of it into ſuch a motion, as to ſeparate them, at leaſt in part, from one another, and ſo perpetually to cauſe ſmall vacuities; by which theſe Atomiſts endeavour to prove, that a vacuum, or ſome ſpace empty of all matter, is abſolutely neceſſary to be in nature. Sir ISAAC NEWTON objects againſt the filling of ſpace with ſuch a ſubtile fluid, that all bodies in motion muſt be unmeaſurably re-

ſiſted by a fluid ſo denſe, as abſolutely to fill up all the ſpace, through which it is ſpread. And leſt it ſhould be thought, that this objection might be evaded by aſcribing to this fluid ſuch very minute and ſmooth parts, as might remove all adheſion or friction between them, whereby all reſiſtance would be loſt, which this fluid might otherwiſe give to bodies moving in it; Sir ISAAC NEWTON proves, in the manner above related, that fluids reſiſt from the power of inactivity of their particles; and that water and the air reſiſt almoſt entirely on this account: ſo that in this ſubtile fluid, however minute and lubricated the particles, which compoſe it, might be; yet if the whole fluid was as denſe as water, it would reſiſt very near as much as water does; and whereas ſuch a fluid, whoſe parts are abſolutely cloſe together without any intervening ſpaces, muſt be a great deal more denſe than water, it muſt reſiſt more than water in proportion to its greater denſity; unleſs we will ſuppoſe the matter, of which this fluid is compoſed, not to be endued with the ſame degree of inactivity as other matter. But if you deprive any ſubſtance of the property ſo univerſally belonging to all other matter, without impropriety of ſpeech it can ſcarce be called by this name.

15. Sir ISAAC NEWTON made alſo an experiment to try in particular, whether the internal parts of bodies ſuffered any reſiſtance. And the reſult did indeed appear to favour ſome ſmall degree of reſiſtance; but ſo very little, as to leave it doubtful, whether the effect did not ariſe from ſome other latent cauſe[a].

[a] Princ. philoſ. pag. 316, 317.

CHAP.

CHAP. II.

Concerning the cauſe, which keeps in motion the primary planets.

SINCE the planets move in a void ſpace and are free from reſiſtance; they, like all other bodies, when once in motion, would move on in a ſtraight line without end, if left to themſelves. And it is now to be explained what kind of action upon them carries them round the ſun. Here I ſhall treat of the primary planets only, and diſcourſe of the ſecondary apart in the next chapter. It has been juſt now declared, that theſe primary planets move ſo about the ſun, that a line extended from the ſun to the planet, will, by accompanying the planet in its motion, paſs over equal ſpaces in equal portions of time [a]. And this one property in the motion of the planets proves, that they are continually acted on by a power directed perpetually to the ſun as a center. This therefore is one property of the cauſe, which keeps the planets in their courſes, that it is a centripetal power, whoſe center is the ſun.

2. AGAIN, in the chapter upon centripetal forces [b] it was obſerv'd, that if the ſtrength of the centripetal power was ſuitably accommodated every where to the motion of any body round a center, the body might be carried in

[a] Ch. 1. § 7. [b] Book I. Ch. 3.

any bent line whatever, whoſe concavity ſhould be every where turned towards the center of the force. It was farther remarked, that the ſtrength of the centripetal force, in each place, was to be collected from the nature of the line, wherein the body moved [a]. Now ſince each planet moves in an ellipſis, and the ſun is placed in one focus; Sir Isaac Newton deduces from hence, that the ſtrength of this power is reciprocally in the duplicate proportion of the diſtance from the ſun. This is deduced from the properties, which the geometers have diſcovered in the ellipſis. The proceſs of the reaſoning is not proper to be enlarged upon here; but I ſhall endeavour to explain what is meant by the reciprocal duplicate proportion. Each of the terms reciprocal proportion, and duplicate proportion, has been already defined [b]. Their ſenſe when thus united is as follows. Suppoſe the planet moved in the orbit ABC (in fig. 93.) about the ſun in S. Then, when it is ſaid, that the centripetal power, which acts on the planet in A, bears to the power acting on it in B a proportion, which is the reciprocal of the duplicate proportion of the diſtance SA to the diſtance SB; it is meant that the power in A bears to the power in B the duplicate of the proportion of the diſtance SB to the diſtance SA. The reciprocal duplicate proportion may be explained alſo by numbers as follows. Suppoſe ſeveral diſtances to bear to each other proportions expreſſed by the numbers 1, 2, 3, 4, 5; that is, let the ſecond diſtance be double the firſt, the third be three times, the fourth four times, and the fifth five times as great as the

[a] Book I. Ch. 3. § 29. [b] Ibid. Ch. 2. § 30, 17.

firſt,

first. Multiply each of these numbers by it self, and 1 multiplied by 1 produces still 1, 2 multiplied by 2 produces 4, 3 by 3 makes 9, 4 by 4 makes 16, and 5 by 5 gives 25. This being done, the fractions $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, will respectively express the proportion, which the centripetal power in each of the following distances bears to the power at the first distance: for in the second distance, which is double the first, the centripetal power will be one fourth part only of the power at the first distance; at the third distance the power will be one ninth part only of the first power; at the fourth distance, the power will be but one sixteenth part of the first; and at the fifth distance, one twenty fifth part of the first power.

3. THUS is found the proportion, in which this centripetal power decreases, as the distance from the sun increases, within the compass of one planet's motion. How it comes to pass, that the planet can be carried about the sun by this centripetal power in a continual round, sometimes rising from the sun, then descending again as low, and from thence be carried up again as far remote as before, alternately rising and falling without end; appears from what has been written above concerning centripetal forces: for the orbits of the planets resemble in shape the curve line proposed in § 17 of the chapter on these forces [a].

4. BUT farther, in order to know whether this centripetal force extends in the same proportion throughout, and consequently whether all the planets are influenced by the very same

[a] Book I. Ch. 3.

power,

power, our author proceeds thus. He inquires what relation there ought to be between the periods of the different planets, provided they were acted upon by the ſame power decreaſing throughout in the forementioned proportion; and he finds, that the period of each in this caſe would have that very relation to the greater axis of its orbit, as I have declared above [a] to be found in the planets by the obſervations of aſtronomers. And this puts it beyond queſtion, that the different planets are preſſed towards the ſun, in the ſame proportion to their diſtances, as one planet is in its ſeveral diſtances. And thence in the laſt place it is juſtly concluded, that there is ſuch a power acting towards the ſun in the foreſaid proportion at all diſtances from it.

5. This power, when referred to the planets, our author calls centripetal, when to the ſun attractive; he gives it likewiſe the name of gravity, becauſe he finds it to be of the ſame nature with that power of gravity, which is obſerved in our earth, as will appear hereafter [b]. By all theſe names he deſigns only to ſignify a power endued with the properties before mentioned; but by no means would he have it underſtood, as if theſe names referred any way to the cauſe of it. In particular in one place where he uſes the name of attraction, he cautions us expreſly againſt implying any thing but a power directing a body to a center without any reference to the cauſe of it, whether reſiding in that center, or ariſing from any external impulſe [c].

[a] Ch. 1. § 7. [b] Chap. 5. § 8. [c] Princ. pag. 60.

6. But

6. BUT now, in theſe demonſtrations ſome very minute inequalities in the motion of the planets are neglected; which is done with a great deal of judgment; for whatever be their cauſe, the effects are very inconſiderable, they being ſo exceeding ſmall, that ſome aſtronomers have thought fit wholly to paſs them by [a]. However the excellency of this philoſophy, when in the hands of ſo great a geometer as our author, is ſuch, that it is able to trace the leaſt variations of things up to their cauſes. The only inequalities, which have been obſerved common to all the planets, are the motion of the aphelion and the nodes The tranſverſe axis of each orbit does not always remain fixed, but moves about the ſun with a very ſlow progreſſive motion: nor do the planets keep conſtantly the ſame plane, but change them, and the lines in which thoſe planes interſect each other by inſenſible degrees. The firſt of theſe inequalities, which is the motion of the aphelion, may be accounted for, by ſuppoſing the gravitation of the planets towards the ſun to differ a little from the forementioned reciprocal duplicate proportion of the diſtances; but the ſecond, which is the motion of the nodes, cannot be accounted for by any power directed towards the ſun; for no ſuch can give the planet any lateral impulſe to divert it from the plane of its motion into any new plane, but of neceſſity muſt be derived from ſome other center. Where that power is lodged, remains to be diſcovered. Now it is proved, as ſhall be explained in the following chapter, that the three primary planets Saturn, Jupiter, and the earth, which have ſatellites revolving about them, are endued with a power of

[a] Street, in Aſtron. Carolin.

cauſing,

causing bodies, in particular those satellites, to gravitate towards them with a force, which is reciprocally in the duplicate proportion of their distances; and the planets are in all respects, in which they come under our examination, so similar and alike, that there is no reason to question, but they have all the same property. Though it be sufficient for the present purpose to have it proved of Jupiter and Saturn only; for these planets contain much greater quantities of matter than the rest, and proportionally exceed the others in power [a]. But the influence of these two planets being allowed, it is evident how the planets come to shift continually their planes: for each of the planets moving in a different plane, the action of Jupiter and Saturn upon the rest will be oblique to the planes of their motion; and therefore will gradually draw them into new ones. The same action of these two planets upon the rest will cause likewise a progressive motion of the aphelion; so that there will be no necessity of having recourse to the other cause for this motion, which was before hinted at [b]; viz, the gravitation of the planets towards the sun differing from the exact reciprocal duplicate proportion of the distances. And in the last place, the action of Jupiter and Saturn upon each other will produce in their motions the same inequalities, as their joint action produces in the rest. All this is effected in the same manner, as the sun produces the same kind of inequalities and many others in the motion of the moon and the other secondary planets; and therefore will be best apprehended by what shall be said in the next chapter.

[a] See Chap. 5. § 9, &c.

[b] In the foregoing page.

 Those

Thoſe other irregularities in the motion of the ſecondary planets have place likewiſe here; but are too minute to be obſervable: becauſe they are produced and rectified alternately, for the moſt part in the time of a ſingle revolution; whereas the motion of the aphelion and nodes, which continually increaſe, become ſenſible in a long ſeries of years. Yet ſome of theſe other inequalities are diſcernible in Jupiter and Saturn, in Saturn chiefly; for when Jupiter, who moves faſter than Saturn, approaches near to a conjunction with him, his action upon Saturn will a little retard the motion of that planet, and by the reciprocal action of Saturn he will himſelf be accelerated. After conjunction, Jupiter will again accelerate Saturn, and be likewiſe retarded in the ſame degree, as before the firſt was retarded and the latter accelerated. Whatever inequalities beſides are produced in the motion of Saturn by the action of Jupiter upon that planet, will be ſufficiently rectified, by placing the focus of Saturn's ellipſis, which ſhould otherwiſe be in the ſun, in the common center of gravity of the ſun and Jupiter. And all the inequalities in the motion of Jupiter, cauſed by Saturn's action upon him, are much leſs conſiderable than the irregularities of Saturn's motion[a].

7. THIS one principle therefore of the planets having a power, as well as the ſun, to cauſe bodies to gravitate towards them, which is proved by the motion of the ſecondary planets to obtain in fact, explains all the irregularities relating to the planets ever obſerved by aſtronomers.

[a] See Newton. Princ. Lib. III. prop. 13.

8. Sir ISAAC NEWTON after this proceeds to make an improvement in aſtronomy by applying this theory to the farther correction of their motions. For as we have here obſerved the planets to poſſeſs a principle of gravitation, as well as the ſun; ſo it will be explained at large hereafter, that the third law of motion, which makes action and reaction equal, is to be applied in this caſe [a]; and that the ſun does not only attract each planet, but is it ſelf alſo attracted by them; the force, wherewith the planet is acted on, bearing to the force, wherewith the ſun it ſelf is acted on at the ſame time, the proportion, which the quantity of matter in the ſun bears to the quantity of matter in the planet. From the action between the ſun and planet being thus mutual Sir ISAAC NEWTON proves that the ſun and planet will deſcribe about their common center of gravity ſimilar ellipſis's; and then that the tranſverſe axis of the ellipſis deſcribed thus about the moveable ſun, will bear to the tranſverſe axis of the ellipſis, which would be deſcribed about the ſun at reſt in the ſame time, the ſame proportion as the quantity of ſolid matter in the ſun and planet together bears to the firſt of two mean proportionals between this quantity and the quantity of matter in the ſun only [b].

9. ABOVE, where I ſhewed how to find a cube, that ſhould bear any proportion to another cube [c], the lines FT and TS are two mean proportionals between EF and FG; and counting from EF, FT is called the firſt, and FS the ſecond of thoſe means. In numbers theſe mean proportionals

[a] Chap. 5. § 10. [b] Princ. Lib. I. prop. 60. [c] Book I. Chap. 2. § 80.

are

are thus found. Suppose A and B two numbers, and it be required to find C the first, and D the second of the two mean proportionals between them. First multiply A by it self, and the product multiply by B; then C will be the number which in arithmetic is called the cubic root of this last product; that is, the number C being multiplied by it self, and the product again multiplied by the same number C, will produce the product above mentioned. In like manner D is the cubic root of the product of B multiplied by it self, and the produce of that multiplication multiplied again by A.

A	C
B	D

10. IT will be asked, perhaps, how this correction can be admitted, when the cause of the motions of the planets was before found by supposing the sun the center of the power, which acted upon them: for according to the present correction this power appears rather to be directed to their common center of gravity. But whereas the sun was at first concluded to be the center, to which the power acting on the planets was directed, because the spaces described round the sun in equal times were found to be equal; so Sir ISAAC NEWTON proves, that if the sun and planet move round their common center of gravity, yet to an eye placed in the planet, the spaces, which will appear to be described about the sun, will have the same relation to the times of their description, as the real spaces would have, if the sun were at rest [a]. I farther asserted, that, supposing the planets to move round the sun at rest,

[a] Princ. philos. Lib. I. prop. 58. coroll. 3.

and to be attracted by a power, which every where ſhould act with degrees of ſtrength reciprocally in the duplicate proportion of the diſtances; then the periods of the planets muſt obſerve the ſame relation to their diſtances, as aſtronomers find them to do. But here it muſt not be ſuppoſed, that the obſervations of aſtronomers abſolutely agree without any the leaſt difference; and the preſent correction will not cauſe a deviation from any one aſtronomer's obſervations, ſo much as they differ from one another. For in Jupiter, where this correction is greateſt, it hardly amounts to the 3000th part of the whole axis.

11. Upon this head I think it not improper to mention a reflection made by our excellent author upon theſe ſmall inequalities in the planets motions; which contains under it a very ſtrong philoſophical argument againſt the eternity of the world. It is this, that theſe inequalities muſt continually increaſe by ſlow degrees, till they render at length the preſent frame of nature unfit for the purpoſes, it now ſerves [a]. And a more convincing proof cannot be deſired againſt the preſent conſtitution's having exiſted from eternity than this, that a certain period of years will bring it to an end. I am aware this thought of our author has been repreſented even as impious, and as no leſs than caſting a reflection upon the wiſdom of the author of nature, for framing a periſhable work. But I think ſo bold an aſſertion ought to have been made with ſingular caution. For if this remark upon the increaſing irregularities of the heavenly motions

[a] Newt. Optics, pag. 378.

be true in fact, as it really is, the imputation muſt return upon the aſſerter, that this does detract from the divine wiſdom. Certainly we cannot pretend to know all the omniſcient Creator's purpoſes in making this world, and therefore cannot undertake to determine how long he deſigned it ſhould laſt. And it is ſufficient, if it endure the time intended by the author. The body of every animal ſhews the unlimited wiſdom of its author no leſs, nay, in many reſpects more, than the larger frame of nature; and yet we ſee, they are all deſigned to laſt but a ſmall ſpace of time.

12. There need nothing more be ſaid of the primary planets; the motions of the ſecondary ſhall be next conſidered.

Chap. III.

Of the motion of the MOON and the other SECONDARY PLANETS.

THE excellency of this philoſophy ſufficiently appears from its extending in the manner, which has been related, to the minuteſt circumſtances of the primary planets motions; which nevertheleſs bears no proportion to the vaſt ſucceſs of it in the motions of the ſecondary; for it not only accounts for all the irregularities, by which their motions were known to be diſturbed, but has diſcovered others ſo complicated, that aſtronomers were never able to diſtinguiſh them, and reduce them under proper heads; but theſe were only to be found

found out from their caufes, which this philofophy has brought to light, and has fhewn the dependence of thefe inequalities upon fuch caufes in fo perfect a manner, that we not only learn from thence in general, what thofe inequalities are, but are able to compute the degree of them. Of this Sir Is. NEWTON has given feveral fpecimens, and has moreover found means to reduce the moon's motion fo completely to rule, that he has framed a theory, from which the place of that planet may at all times be computed, very nearly or altogether as exactly, as the places of the primary planets themfelves, which is much beyond what the greateft aftronomers could ever effect.

2. THE firft thing demonftrated of thefe fecondary planets is, that they are drawn towards their refpective primary in the fame manner as the primary planets are attracted by the fun. That each fecondary planet is kept in its orbit by a power pointed towards the center of the primary planet, about which the fecondary revolves; and that the power, by which the fecondaries of the fame primary are influenced, bears the fame relation to the diftance from the primary, as the power, by which the primary planets are guided, does in regard to the diftance from the fun [a]. This is proved in the fatellites of Jupiter and Saturn, becaufe they move in circles, as far as we can obferve, about their refpective primary with an equable courfe, the refpective primary being the center of each orbit: and by comparing the times, in which the different fatellites of the fame primary perform their periods, they are

[a] Newton. Princ. Lib. III. prop. 1.

found

found to obſerve the ſame relation to the diſtances from their primary, as the primary planets obſerve in reſpect of their mean diſtances from the ſun [a]. Here theſe bodies moving in circles with an equable motion, each ſatellite paſſes over equal parts of its orbit in equal portions of time; conſequently the line drawn from the center of the orbit, that is, from the primary planet, to the ſatellite, will paſs over equal ſpaces along with the ſatellite in equal portions of time; which proves the power, by which each ſatellite is held in its orbit, to be pointed towards the primary as a center [b]. It is alſo manifeſt that the centripetal power, which carries a body in a circle concentrical with the power, acts upon the body at all times with the ſame ſtrength. But Sir ISAAC NEWTON demonſtrates that, when bodies are carried in different circles by centripetal powers directed to the centers of thoſe circles, then the degrees of ſtrength of thoſe powers are to be compared by conſidering the relation between the times, in which the bodies perform their periods through thoſe circles [c]; and in particular he ſhews, that if the periodical times bear that relation, which I have juſt now aſſerted the ſatellites of the ſame primary to obſerve; then the centripetal powers are reciprocally in the duplicate proportion of the ſemidiameters of the circles, or in that proportion to the diſtances of the bodies from the centers [d]. Hence it follows that in the planets Jupiter and Saturn, the centripetal power in each decreaſes with the increaſe of diſtance, in the ſame proportion as the centripetal

[a] Newton. Princ. Lib. III. pag. 390, 391, compared with pag. 393.
[b] Book I, Ch. 2. § 29.
[c] Princ. philoſ. Lib. I. prop. 4.
[d] Ibid. coroll.

power

power appertaining to the ſun decreaſes with the increaſe of diſtance. I do not here mean that this proportion of the centripetal powers holds between the power of Jupiter at any diſtance compared with the power of Saturn at any other diſtance; but only in the change of ſtrength of the power belonging to the ſame planet at different diſtances from him. Moreover what is here diſcovered of the planets Jupiter and Saturn by means of the different ſatellites, which revolve round each of them, appears in the earth by the moon alone; becauſe ſhe is found to move round the earth in an ellipſis after the ſame manner as the primary planets do about the ſun; excepting only ſome ſmall irregularities in her motion, the cauſe of which will be particularly explained in what follows, whereby it will appear, that they are no objection againſt the earth's acting on the moon in the ſame manner as the ſun acts on the primary planets; that is, as the other primary planets Jupiter and Saturn act upon their ſatellites. Certainly ſince theſe irregularities can be otherwiſe accounted for, we ought not to depart from that rule of induction ſo neceſſary in philoſophy, that to like bodies like properties are to be attributed, where no reaſon to the contrary appears. We cannot therefore but aſcribe to the earth the ſame kind of action upon the moon, as the other primary planets Jupiter and Saturn have upon their ſatellites; which is known to be very exactly in the proportion aſſigned by the method of comparing the periodical times and diſtances of all the ſatellites, which move about the ſame planet; this abundantly compenſating our not being near enough to obſerve the exact figure of their orbits. For if the little deviation of the moon's orbit from

orbit from a true permanent ellipſis aroſe from the action of the earth upon the moon not being in the exact reciprocal duplicate proportion of the diſtance, were another moon to revolve about the earth, the proportion between the periodical times of this new moon, and the preſent, would diſcover the deviation from the mentioned proportion much more manifeſtly.

3. BY the number of ſatellites, which move round Jupiter and Saturn, the power of each of theſe planets is meaſured in a great diverſity of diſtance; for the diſtance of the outermoſt ſatellite in each of theſe planets exceeds ſeveral times the diſtance of the innermoſt. In Jupiter the aſtronomers have uſually placed the innermoſt ſatellite at a diſtance from the center of that planet equal to about $5\frac{2}{3}$ of the ſemidiameters of Jupiter's body, and this ſatellite performs its revolution in about 1 day $18\frac{1}{2}$ hours. The next ſatellite, which revolves round Jupiter in about 3 days $13\frac{1}{7}$ hours, they place at the diſtance from Jupiter of about 9 of that planet's ſemidiameters. To the third ſatellite, which performs its period nearly in 7 days $3\frac{3}{4}$ hours, they aſſign the diſtance of about $14\frac{2}{5}$ ſemidiameters. But the outermoſt ſatellite they remove to $25\frac{1}{3}$ ſemidiameters, and this ſatellite makes its period in about 16 days $16\frac{1}{2}$ hours [a]. In Saturn there is ſtill a greater diverſity in the diſtance of the ſeveral ſatellites. By the obſervations of the late CASSINI, a celebrated aſtronomer in France, who firſt diſcovered all theſe ſatellites, except one known before, the innermoſt is diſtant about $4\frac{1}{2}$ of Saturn's ſemidiameters from his center, and re-

[a] Newt. Princ. philoſ. Lib. III. pag. 390.

volves round in about 1 day $21\frac{1}{3}$ hours. The next satellite is distant about $5\frac{3}{4}$ semidiameters, and makes its period in about 2 days $17\frac{2}{3}$ hours. The third is removed to the distance of about 8 semidiameters, and performs its revolution in near 4 days $12\frac{1}{2}$ hours. The fourth satellite discovered first by the great HUYGENS, is near $18\frac{2}{3}$ semidiameters, and moves round Saturn in about 15 days $22\frac{2}{3}$ hours. The outermost is distant 56 semidiameters, and makes its revolution in about 79 days $7\frac{4}{5}$ hours [a]. Besides these satellites, there belongs to the planet Saturn another body of a very singular kind. This is a shining, broad, and flat ring, which encompasses the planet round. The diameter of the outermost verge of this ring is more than double the diameter of Saturn. HUYGENS, who first described this ring, makes the whole diameter thereof to bear to the diameter of Saturn the proportion of 9 to 4. The late reverend Mr. POUND makes the proportion something greater, viz. that of 7 to 3. The distances of the satellites of this planet Saturn are compared by CASSINI to the diameter of the ring. His numbers I have reduced to those above, according to Mr. POUND's proportion between the diameters of Saturn and of his ring. As this ring appears to adhere no where to Saturn, so the distance of Saturn from the inner edge of the ring seems rather greater than the breadth of the ring. The distances, which have here been given, of the several satellites, both for Jupiter and Saturn, may be more depended on in relation to the proportion, which those belonging to the same primary planet

[a] Newt. Princ. philos. Lib. III. pag. 391, 392.

bear

bear one to another, than in reſpect to the very numbers, that have been here ſet down, by reaſon of the difficulty there is in meaſuring to the greateſt exactneſs the diameters of the primary planets; as will be explained hereafter, when we come to treat of teleſcopes [a]. By the obſervations of the forementioned Mr. POUND, in Jupiter the diſtance of the innermoſt ſatellite ſhould rather be about 6 ſemidiameters, of the ſecond $9\frac{1}{2}$, of the third 15, and of the outermoſt $26\frac{2}{3}$ [b]; and in Saturn the diſtance of the innermoſt ſatellite 4 ſemidiameters, of the next $6\frac{1}{4}$, of the third $8\frac{3}{4}$, of the fourth $20\frac{1}{3}$, and of the fifth 59 [c]. However the proportion between the diſtances of the ſatellites in the ſame primary is the only thing neceſſary to the point we are here upon.

4. BUT moreover the force, wherewith the earth acts in different diſtances, is confirmed from the following conſideration, yet more expreſly than by the preceding analogical reaſoning. It will appear, that if the power of the earth, by which it retains the moon in her orbit, be ſuppoſed to act at all diſtances between the earth and moon, according to the forementioned rule; this power will be ſufficient to produce upon bodies, near the ſurface of the earth, all the effects aſcribed to the principle of gravity. This is diſcovered by the following method. Let A (in fig. 94.) repreſent the earth, B the moon, BCD the moon's orbit, which differs little from a circle, of which A is the center. If the moon in B were left to it ſelf to move with the velocity, it has in the point B, it

[a] Book III. Ch. 4. [b] Newt. Princ. philoſ. Lib. III. pag. 391. [c] Ibid. pag. 392.

would leave the orbit, and proceed right forward in the line BE, which touches the orbit in B. Suppofe the moon would upon this condition move from B to E in the fpace of one minute of time. By the action of the earth upon the moon, whereby it is retained in its orbit, the moon will really be found at the end of this minute in the point F, from whence a ftraight line drawn to A fhall make the fpace BFA in the circle equal to the triangular fpace BEA; fo that the moon in the time wherein it would have moved from B to E, if left to it felf, has been impelled towards the earth from E to F. And when the time of the moon's paffing from B to F is fmall, as here it is only one minute, the diftance between E and F fcarce differs from the fpace, through which the moon would defcend in the fame time, if it were to fall directly down from B toward A without any other motion. AB the diftance of the earth and moon is about 60 of the earth's femidiameters, and the moon completes her revolution round the earth in about 27 days 7 hours and 43 minutes: therefore the fpace EF will here be found by computation to be about $16\frac{1}{8}$ feet. Confequently, if the power, by which the moon is retained in its orbit, be near the furface of the earth greater, than at the diftance of the moon in the duplicate proportion of that diftance; the number of feet, a body would defcend near the furface of the earth by the action of this power upon it in one minute of time, would be equal to $16\frac{1}{8}$ multiplied twice into the number 60, that is, equal to 58050. But how faft bodies fall near the furface of the earth may be known by the pendulum[a]; and

[a] See Book I. Ch. 2. § 60, 64.

by

by the exacteſt experiments they are found to deſcend the ſpace of $16\frac{1}{8}$ feet in a ſecond of time; and the ſpaces deſcribed by falling bodies being in the duplicate proportion of the times of their fall [a], the number of feet, a body would deſcribe in its fall near the ſurface of the earth in one minute of time, will be equal to $16\frac{1}{8}$ twice multiplied by 60, the ſame as would be cauſed by the power which acts upon the moon.

5. IN this computation the earth is ſuppoſed to be at reſt, whereas it would have been more exact to have ſuppoſed it to move, as well as the moon, about their common center of gravity; as will eaſily be underſtood, by what has been ſaid in the preceding chapter, where it was ſhewn, that the ſun is ſubjected to the like motion about the common center of gravity of it ſelf and the planets. The action of the ſun upon the moon, which is to be explain'd in what follows, is likewiſe here neglected: and Sir ISAAC NEWTON ſhews, if you take in both theſe conſiderations, the preſent computation will beſt agree to a ſomewhat greater diſtance of the moon and earth, viz. to $60\frac{1}{2}$ ſemidiameters of the earth, which diſtance is more conformable to aſtronomical obſervations.

6. THESE computations afford an additional proof, that the action of the earth obſerves the ſame proportion to the diſtance, which is here contended for. Before I ſaid, it was reaſonable to conclude ſo by induction from the pla-

[a] Book I. Ch. 2. § 17.

nets

nets Jupiter and Saturn; becauſe they act in that manner. But now the ſame thing will be evident by drawing no other conſequence from what is ſeen in thoſe planets, than that the power, by which the primary planets act on their ſecondary, is extended from the primary through the whole interval between, ſo that it would act in every part of the intermediate ſpace. In Jupiter and Saturn this power is ſo far from being confined to a ſmall extent of diſtance, that it not only reaches to ſeveral ſatellites at very different diſtances, but alſo from one planet to the other, nay even through the whole planetary ſyſtem [a]. Conſequently there is no appearance of reaſon, why this power ſhould not act at all diſtances, even at the very ſurfaces of theſe planets as well as farther off. But from hence it follows, that the power, which retains the moon in her orbit, is the ſame, as cauſes bodies near the ſurface of the earth to gravitate. For ſince the power, by which the earth acts on the moon, will cauſe bodies near the ſurface of the earth to deſcend with all the velocity they are found to do, it is certain no other power can act upon them beſides; becauſe if it did, they muſt of neceſſity deſcend ſwifter. Now from all this it is at length very evident, that the power in the earth, which we call gravity, extends up to the moon, and decreaſes in the duplicate proportion of the increaſe of the diſtance from the earth.

7. This finiſhes the diſcoveries made in the action of the primary planets upon their ſecondary. The next thing

[a] See Ch. II. § 6.

to

to be ſhewn is, that the ſun acts upon them likewiſe: for this purpoſe it is to be obſerved, that if to the motion of the ſatellite, whereby it would be carried round its primary at reſt, be ſuperadded the ſame motion both in regard to velocity and direction, as the primary it ſelf has, it will deſcribe about the primary the ſame orbit, with as great regularity, as if the primary was indeed at reſt. The cauſe of this is that law of motion, which makes a body near the ſurface of the earth, when let fall, to deſcend perpendicularly, though the earth be in ſo ſwift a motion, that if the falling body did not partake of it, its deſcent would be remarkably oblique; and that a body projected deſcribes in the moſt regular manner the ſame parabola, whether projected in the direction, in which the earth moves, or in the oppoſite direction, if the projecting force be the ſame [a]. From this we learn, that if the ſatellite moved about its primary with perfect regularity, beſides its motion about the primary, it would participate of all the motion of its primary; have the ſame progreſſive velocity, with which the primary is carried about the ſun; and be impelled with the ſame velocity as the primary towards the ſun, in a direction parallel to that impulſe of its primary. And on the contrary, the want of either of theſe, in particular of the impulſe towards the ſun, will occaſion great inequalities in the motion of the ſecondary planet. The inequalities, which would ariſe from the abſence of this impulſe towards the ſun are

[a] The ſecond of the laws of motion laid down in Book I. Ch. 3.

ſo

ſo great, that by the regularity, which appears in the motion of the ſecondary planets, it is proved, that the ſun communicates the ſame velocity to them by its action, as it gives to their primary at the ſame diſtance. For Sir Isaac Newton informs us, that upon examination he found, that if any of the ſatellites of Jupiter were attracted by the ſun more or leſs, than Jupiter himſelf at the ſame diſtance, the orbit of that ſatellite, inſtead of being concentrical to Jupiter, muſt have its center at a greater or leſs diſtance, than the center of Jupiter from the ſun, nearly in the ſubduplicate proportion of the difference between the ſun's action upon the ſatellite, and upon Jupiter; and therefore if any ſatellite were attracted by the ſun but $\frac{1}{1000}$ part more or leſs, than Jupiter is at the ſame diſtance, the center of the orbit of that ſatellite would be diſtant from the center of Jupiter no leſs than a fifth part of the diſtance of the outermoſt ſatellite from Jupiter [a]; which is almoſt the whole diſtance of the innermoſt ſatellite. By the like argument the ſatellites of Saturn gravitate towards the ſun, as much as Saturn it ſelf at the ſame diſtance; and the moon as much as the earth.

8. Thus is proved, that the ſun acts upon the ſecondary planets, as much as upon the primary at the ſame diſtance: but it was found in the laſt chapter, that the action of the ſun upon bodies is reciprocally in the duplicate proportion of the diſtance; therefore the ſecondary

[a] Newton. Princ. philoſ. Lib. III. prop. 6. pag. 401.

planets

planets being ſometimes nearer to the ſun than the primary, and ſometimes more remote, they are not alway acted upon in the ſame degree with their primary, but when nearer to the ſun, are attracted more, and when farther diſtant, are attracted leſs. Hence ariſe various inequalities in the motion of the ſecondary planets [a].

9. SOME of theſe inequalities would take place, though the moon, if undiſturbed by the ſun, would have moved in a circle concentrical to the earth, and in the plane of the earth's motion; others depend on the elliptical figure, and the oblique ſituation of the moon's orbit. One of the firſt kind is, that the moon is cauſed ſo to move, as not to deſcribe equal ſpaces in equal times, but is continually accelerated, as ſhe paſſes from the quarter to the new or full, and is retarded again by the like degrees in returning from the new and full to the next quarter. Here we conſider not ſo much the abſolute, as the apparent motion of the moon in reſpect to us.

10. THE principles of aſtronomy teach how to diſtinguiſh theſe two motions. Let S (in fig. 95.) repreſent the ſun, A the earth moving in its orbit BC, DEFG the moon's orbit, the place of the moon H. Suppoſe the earth to have moved from A to I. Becauſe it has been ſhewn, that the moon partakes of all the progreſſive motion of the earth; and likewiſe that the ſun attracts both the earth and moon equally, when they are at the ſame diſtance from it, or that the mean action of the ſun upon the moon is equal to its action

[a] Newton's Princ. philoſ. Lib. III. prop. 22, 23.

upon the earth: we muſt therefore conſider the earth as carrying about with it the moon's orbit; ſo that when the earth is removed from A to I, the moon's orbit ſhall likewiſe be removed from its former ſituation into that denoted by KLMN. But now the earth being in I, if the moon were found in O, ſo that OI ſhould be parallel to HA, though the moon would really have moved from H to O, yet it would not have appeared to a ſpectator upon the earth to have moved at all, becauſe the earth has moved as much it ſelf; ſo that the moon would ſtill appear in the ſame place with reſpect to the fixed ſtars. But if the moon be obſerved in P, it will then appear to have moved, its apparent motion being meaſured by the angle under OIP. And if the angle under PIS be leſs than the angle under HAS, the moon will have approached nearer to its conjunction with the ſun.

11. To come now to the explication of the mentioned inequality in the moon's motion: let S (in fig. 96.) repreſent the ſun, A the earth, BCDE the moon's orbit, C the place of the moon, when in the latter quarter. Here it will be nearly at the ſame diſtance from the ſun, as the earth is. In this caſe therefore they will both be equally attracted, the earth in the direction AS, and the moon in the direction CS. Whence as the earth in moving round the ſun is continually deſcending toward it, ſo the moon in this ſituation muſt in any equal portion of time deſcend as much; and therefore the poſition of the line AC in reſpect of AS, and the change, which the moon's motion produces in the angle under CAS, will not be altered by the ſun.

12. But

12. BUT now as ſoon as ever the moon is advanced from the quarter toward the new or conjunction, ſuppoſe to G, the action of the ſun upon it will have a different effect. Here, were the ſun's action upon the moon to be applied in the direction G H parallel to A S, if its action on the moon were equal to its action on the earth, no change would be wrought by the ſun on the apparent motion of the moon round the earth. But the moon receiving a greater impulſe in G than the earth receives in A, were the ſun to act in the direction G H, yet it would accelerate the deſcription of the ſpace D A G, and cauſe the angle under G A D to decreaſe faſter, than otherwiſe it would. The ſun's action will have this effect upon account of the obliquity of its direction to that, in which the earth attracts the moon. For the moon by this means is drawn by two forces oblique to each other, one drawing from G toward A, the other from G toward H, therefore the moon muſt neceſſarily be impelled toward D. Again, becauſe the ſun does not act in the direction G H parallel to S A, but in the direction G S oblique to it, the ſun's action on the moon will by reaſon of this obliquity farther contribute to the moon's acceleration. Suppoſe the earth in any ſhort ſpace of time would have moved from A to I, if not attracted by the ſun; the point I being in the ſtraight line C E, which touches the earth's orbit in A. Suppoſe the moon in the ſame time would have moved in her orbit from G to K, and beſides have partook of all the progreſſive motion of the earth. Then if K L be drawn parallel to A I, and taken equal to it, the moon, if not attracted by the ſun, would be found

in L. But the earth by the ſun's action is removed from I. Suppoſe it were moved down to M in the line IMN parallel to SA, and if the moon were attracted but as much, and in the ſame direction, as the earth is here ſuppoſed to be attracted, ſo as to have deſcended during the ſame time in the line LO, parallel alſo to AS, down as far as P, till LP were equal to IM, the angle under PMN would be equal to that under LIN, that is, the moon will appear advanced no farther forward, than if neither it nor the earth had been ſubject to the ſun's action. But this is upon the ſuppoſition, that the action of the ſun upon the moon and earth were equal; whereas the moon being acted upon more than the earth, did the ſun's action draw the moon in the line LO parallel to AS, it would draw it down ſo far as to make LP greater than IM; whereby the angle under PMN will be rendred leſs, than that under LIN. But moreover, as the ſun draws the earth in a direction oblique to IN, the earth will be found in its orbit ſomewhat ſhort of the point M; however the moon is attracted by the ſun ſtill more out of the line LO, than the earth is out of the line IN; therefore this obliquity of the ſun's action will yet farther diminiſh the angle under PMN.

13. THUS the moon at the point G receives an impulſe from the ſun, whereby her motion is accelerated. And the ſun producing this effect in every place between the quarter and the conjunction, the moon will move from the quarter with a motion continually more and more accelerated; and therefore by acquiring from time to time additional degrees of

of velocity in its orbit, the ſpaces, which are deſcribed in equal times by the line drawn from the earth to the moon, will not be every where equal, but thoſe toward the conjunction will be greater, than thoſe toward the quarter. But now in the moon's paſſage from the conjunction D to the next quarter the ſun's action will again retard the moon, till at the next quarter in E it be reſtored to the firſt velocity, which it had in C.

14. AGAIN as the moon moves from E to the full or oppoſition to the ſun in B, it is again accelerated, the deficiency of the ſun's action upon the moon, from what it has upon the earth, producing here the ſame effect as before the exceſs of its action. Conſider the moon in Q moving from E towards B. Here if the moon were attracted by the ſun in a direction parallel to A S, yet being acted on leſs than the earth, as the earth deſcends toward the ſun, the moon will in ſome meaſure be left behind. Therefore Q F being drawn parallel to S B, a ſpectator on the earth would ſee the moon move, as if attracted from the point Q in the direction Q F with a degree of force equal to that, whereby the ſun's action on the moon falls ſhort of its action on the earth. But the obliquity of the ſun's action has alſo here an effect. In the time the earth would have moved from A to I without the influence of the ſun, let the moon have moved in its orbit from Q to R. Drawing therefore R T parallel to A I, and equal to the ſame, for the like reaſon as before, the moon by the motion of its orbit, if not at all attracted by the ſun, muſt be found in T; and therefore, if attracted in a direction parallel to S A, would be

be in the line TV parallel to AS; ſuppoſe in W. But the moon in Q being farther off the ſun than the earth, it will be leſs attracted, that is, TW will be leſs than IM, and if the line SM be prolonged toward X, the angle under XMW will be leſs than that under XIT. Thus by the ſun's action the moon's paſſage from the quarter to the full would be accelerated, if the ſun were to act on the earth and moon in a direction parallel to AS: and the obliquity of the ſun's action will ſtill more increaſe this acceleration. For the action of the ſun on the moon is oblique to the line SA the whole time of the moon's paſſage from Q to T, and will carry the moon out of the line TV toward the earth. Here I ſuppoſe the time of the moon's paſſage from Q to T ſo ſhort, that it ſhall not paſs beyond the line SA. The earth alſo will come a little ſhort of the line IN, as was ſaid before. From theſe cauſes the angle under XMW will be ſtill farther leſſened.

15. The moon in paſſing from the oppoſition B to the next quarter will be retarded again by the ſame degrees, as it is accelerated before its appulſe to the oppoſition. Becauſe this action of the ſun, which in the moon's paſſage from the quarter to the oppoſition cauſes it to be extraordinarily accelerated, and diminiſhes the angle, which meaſures its diſtance from the oppoſition; will make the moon ſlacken its pace afterwards, and retard the augmentation of the ſame angle in its paſſage from the oppoſition to the following quarter; that is, will prevent that angle from increaſing ſo faſt, as otherwiſe it would. And thus the moon, by the ſun's action upon it, is twice accelerated and twice reſtored to its firſt velocity, every circuit

circuit it makes round the earth. This inequality of the moon's motion about the earth is called by aſtronomers its variation.

16. THE next effect of the ſun upon the moon is, that it gives the orbit of the moon in the quarters a greater degree of curvature, than it would receive from the action of the earth alone; and on the contrary in the conjunction and oppoſition the orbit is leſs inflected.

17. WHEN the moon is in conjunction with the ſun in the point D, the ſun attracting the moon more forcibly than it does the earth, the moon by that means is impelled leſs toward the earth, than otherwiſe it would be, and ſo the orbit is leſs incurvated; for the power, by which the moon is impelled toward the earth, being that, by which it is inflected from a rectilinear courſe, the leſs that power is, the leſs it will be inflected. Again, when the moon is in the oppoſition in B, farther removed from the ſun than the earth is; it follows then, though the earth and moon are both continually deſcending to the ſun, that is, are drawn by the ſun toward it ſelf out of the place they would otherwiſe move into, yet the moon deſcends with leſs velocity than the earth; inſomuch that the moon in any given ſpace of time from its paſſing the point of oppoſition will have leſs approached the earth, than otherwiſe it would have done, that is, its orbit in reſpect of the earth will approach nearer to a ſtraight line. In the laſt place, when the moon is in the quarter in F, and equally diſtant from the ſun as the earth, we obſerved before, that

the

the earth and moon would defcend with equal pace toward the fun, fo as to make no change by that defcent in the angle under FAS; but the length of the line FA muft of neceffity be fhortned. Therefore the moon in moving from F toward the conjunction with the fun will be impelled more toward the earth by the fun's action, than it would have been by the earth alone, if neither the earth nor moon had been acted on by the fun; fo that by this additional impulfe the orbit is rendred more curve, than it would otherwife be. The fame effect will alfo be produced in the other quarter.

18. Another effect of the fun's action, confequent upon this we have now explained, is, that though the moon undifturbed by the fun might move in a circle having the earth for its center; by the fun's action, if the earth were to be in the very middle or center of the moon's orbit, yet the moon would be nearer the earth at the new and full, than in the quarters. In this probably will at firft appear fome difficulty, that the moon fhould come neareft to the earth, where it is leaft attracted to it, and be fartheft off when moft attracted. Which yet will appear evidently to follow from that very caufe, by confidering what was laft fhewn, that the orbit of the moon in the conjunction and oppofition is rendred lefs curve; for the lefs curve the orbit of the moon is, the lefs will the moon have defcended from the place it would move into, without the action of the earth. Now if the moon were to move from any place without farther difturbance from that action, fince it would proceed in the line, which would touch its orbit in that place, it would recede

recede continually from the earth; and therefore if the power of the earth upon the moon, be ſufficient to retain it at the ſame diſtance, this diminution of that power will cauſe the diſtance to increaſe, though in a leſs degree. But on the other hand in the quarters, the moon, being preſſed more towards the earth than by the earth's ſingle action, will be made to approach it; ſo that in paſſing from the conjunction or oppoſition to the quarters the moon aſcends from the earth, and in paſſing from the quarters to the conjunction and oppoſition it deſcends again, becoming nearer in theſe laſt mentioned places than in the other.

19. ALL theſe forementioned inequalities are of different degrees, according as the ſun is more or leſs diſtant from the earth; greater when the earth is neareſt the ſun, and leſs when it is fartheſt off. For in the quarters, the nearer the moon is to the ſun, the greater is the addition to the earth's action upon it by the power of the ſun; and in the conjunction and oppoſition, the difference between the ſun's action upon the earth and upon the moon is likewiſe ſo much the greater.

20. THIS difference in the diſtance between the earth and the ſun produces a farther effect upon the moon's motion; cauſing the orbit to dilate when leſs remote from the ſun, and become greater, than when at a farther diſtance. For it is proved by Sir ISAAC NEWTON, that the action of the ſun, by which it diminiſhes the earth's power over the moon, in the conjunction or oppoſition, is about twice as

great, as the addition to the earth's action by the sun in the quarters [a]; so that upon the whole, the power of the earth upon the moon is diminished by the sun, and therefore is most diminished, when the action of the sun is strongest: but as the earth by its approach to the sun has its influence lessened, the moon being less attracted will gradually recede from the earth; and as the earth in its recess from the sun recovers by degrees its former power, the orbit of the moon must again contract. Two consequences follow from hence: the moon will be most remote from the earth, when the earth is nearest the sun; and also will take up a longer time in performing its revolution through the dilated orbit, than through the more contracted.

21. These irregularities the sun would produce in the moon, if the moon, without being acted on unequally by the sun, would describe a perfect circle about the earth, and in the plane of the earth's motion; but though neither of these suppositions obtain in the motion of the moon, yet the forementioned inequalities will take place, only with some difference in respect to the degree of them; but the moon by not moving in this manner is subject to some other inequalities also. For as the moon describes, instead of a circle concentrical to the earth, an ellipsis, with the earth in one focus, that ellipsis will be subjected to various changes. It can neither preserve constantly the same position, nor yet the same figure; and because the plane of this ellipsis is not the same

[a] Newton. Princ. Lib. I. prop. 66. coroll. 7.

with

with that of the earth's orbit, the ſituation of the plane, wherein the moon moves, will continually change; neither the line in which it interſects the plane of the earth's orbit, nor the inclination of the planes to each other, will remain for any time the ſame. All theſe alterations offer themſelves now to be explained.

22. I SHALL firſt conſider the changes which are made in the plane of the moon's orbit. The moon not moving in the ſame plane with the earth, the ſun is ſeldom in the plane of the moon's orbit, viz. only when the line made by the common interſection of the two planes, if produced, will paſs through the ſun, as is repreſented in fig. 97. where S denotes the ſun; T the earth; A T B the earth's orbit deſcribed upon the plane of this ſcheme; C D E F the moon's orbit, the part C D E being raiſed above, and the part C F E depreſſed under the plane of this ſcheme. Here the line C E, in which the plane of this ſcheme, that is, the plane of the earth's orbit and the plane of the moon's orbit interſect each other, being continued paſſes through the ſun in S. When this happens, the action of the ſun is directed in the plane of the moon's orbit, and cannot draw the moon out of this plane, as will evidently appear to any one that ſhall conſider the preſent ſcheme: for ſuppoſe the moon in G, and let a ſtraight line be drawn from G to S, the ſun draws the moon in the direction of this line from G toward S: but this line lies in the plane of the orbit; and if it be prolonged from S beyond G, the continuation of it will lie on the plane C D E; for the plane itſelf, if ſufficiently extended, will paſs through the ſun.

But in other cafes the obliquity of the fun's action to the plane of the orbit will caufe this plane continually to change.

23. Suppose in the firft place, the line, in which the two planes interfect each other, to be perpendicular to the line which joins the earth and fun. Let T (in fig. 98, 99, 100, 101.) reprefent the earth; S the fun; the plane of this fcheme the plane of the earth's motion, in which both the fun and earth are placed. Let AC be perpendicular to ST, which joins the earth and fun; and let the line AC be that, in which the plane of the moon's orbit interfects the plane of the earth's motion. To the center T defcribe in the plane of the earth's motion the circle ABCD. And in the plane of the moon's orbit defcribe the circle AECF, one half of which AEC will be elevated above the plane of this fcheme, the other half AFC as much depreffed below it.

24. Now fuppofe the moon to fet forth from the point A (in fig. 98.) in the direction of the plane AEC. Here fhe will be continually drawn out of this plane by the action of the fun: for this plane AEC, if extended, will not pafs through the fun, but above it; fo that the fun, by drawing the moon directly toward it felf, will force it continually more and more from that plane towards the plane of the earth's motion, in which it felf is; caufing it to defcribe the line AKGHI, which will be convex to the plane AEC, and concave to the plane of the earth's motion. But here this power of the fun, which is faid to draw the moon toward the plane of the earth's motion, muft be underftood principally of fo much only of the

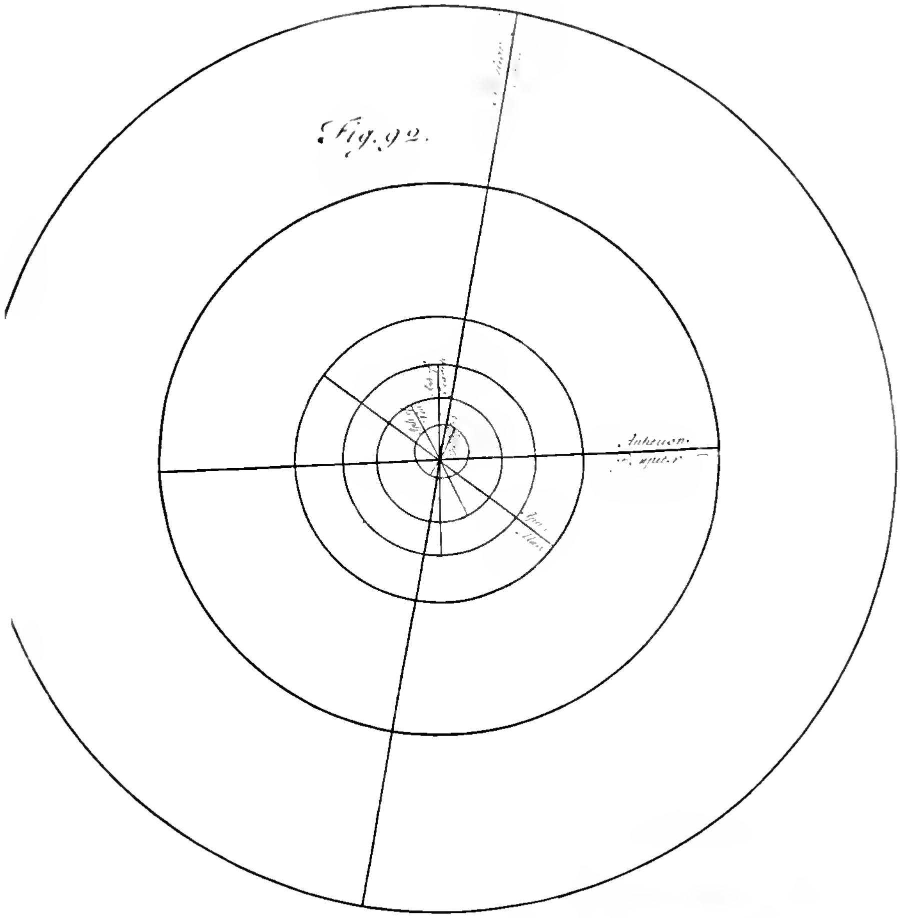


Fig. 92.

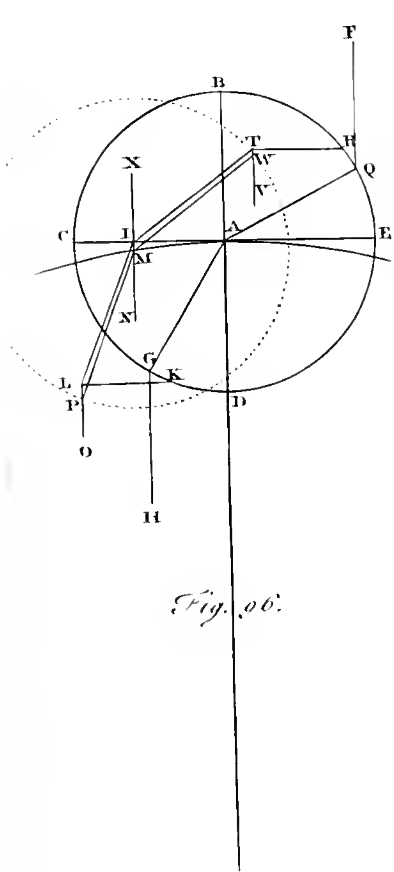


Fig. 96.

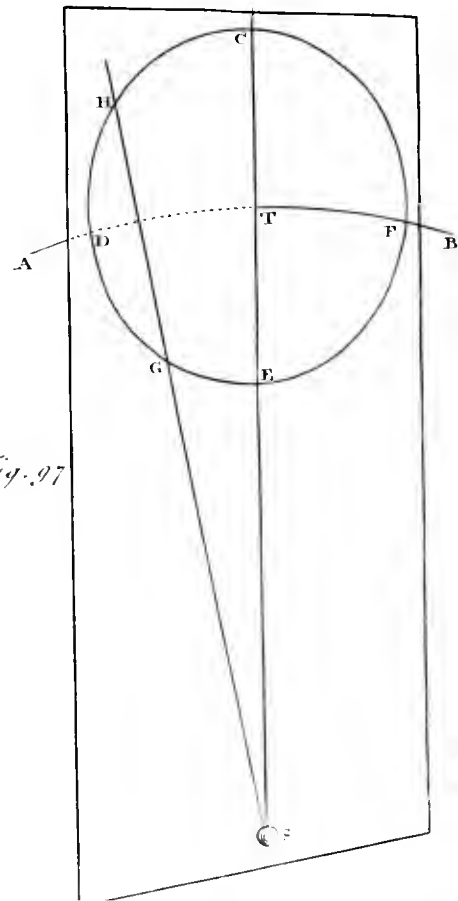


Fig. 97

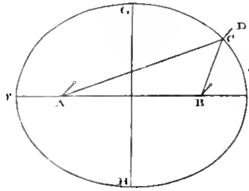


Fig. 91.

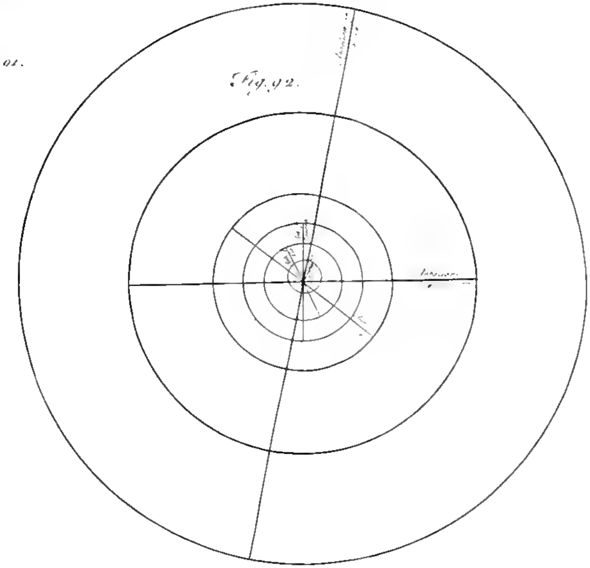


Fig. 92.

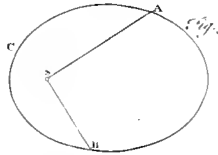


Fig. 93.

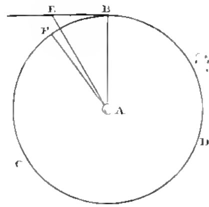


Fig. 94.

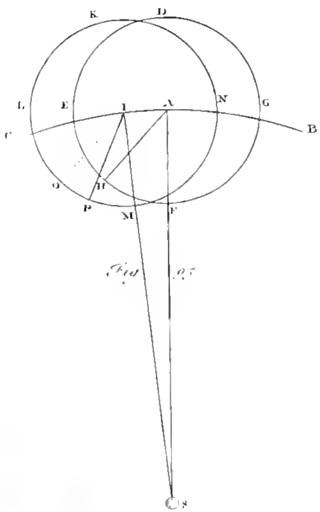


Fig. 95.

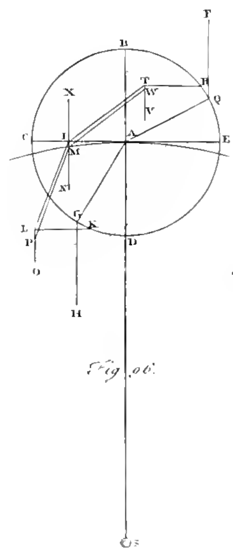


Fig. 96.

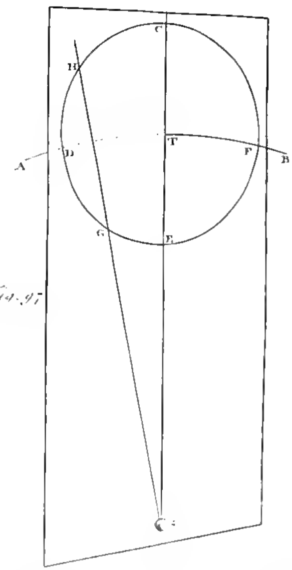


Fig. 97.

the ſun's action upon the moon, as it exceeds the action of the ſame upon the earth. For ſuppoſe the preceding figure to be viewed by the eye, placed in the plane of that ſcheme, and in the line C T A on the ſide of A, the plane A B C D will appear as the ſtraight line D T B, (in fig. 102.) and the plane A E C F as another ſtraight line F E; and the curve line A K G H I under the form of the line T K G H I. Now it is plain, that the earth and moon being both attracted by the ſun, if the ſun's action upon both was equally ſtrong, the earth T, and with it the plane A E C F or line F T E in this ſcheme, would be carried toward the ſun with as great a pace as the moon, and therefore the moon not drawn out of it by the ſun's action, excepting only from the ſmall obliquity of the direction of this action upon the moon to that of the ſun's action upon the earth, which ariſes from the moon's being out of the plane of the earth's motion, and is not very conſiderable; but the action of the ſun upon the moon being greater than upon the earth, all the time the moon is nearer to the ſun than the earth is, it will be drawn from the plane A E C or the line T E by that exceſs, and made to deſcribe the curve line A G I or T G I. But it is the cuſtom of aſtronomers, inſtead of conſidering the moon as moving in ſuch a curve line, to refer its motion continually to the plane, which touches the true line wherein it moves, at the point where at any time the moon is. Thus when the moon is in the point A, its motion is conſidered as being in the plane A E C, in whoſe direction it then eſſaies to move; and when in the point K (in fig. 99.) its motion is referred to the plane, which paſſes through the earth, and touches the line A K G H I in the point K. Thus

the

the moon in paſſing from A to I will continually change the plane of her motion. In what manner this change proceeds, I ſhall now particularly explain.

25. Let the plane, which touches the line AKI in the point K (in fig. 99.) interſect the plane of the earth's orbit in the line LTM. Then, becauſe the line AKI is concave to the plane ABC, it falls wholly between that plane, and the plane which touches it in K; ſo that the plane MKL will cut the plane AEC, before it meets with the plane of the earth's motion; ſuppoſe in the line YT, and the point A will fall between K and L. With a ſemidiameter equal to TY or TL deſcribe the ſemicircle LYM. Now to a ſpectator on the earth the moon, when in A, will appear to move in the circle AECF, and, when in K, will appear to be moving in the ſemicircle LYM. The earth's motion is performed in the plane of this ſcheme, and to a ſpectator on the earth the ſun will appear always moving in that plane. We may therefore refer the apparent motion of the ſun to the circle ABCD, deſcribed in this plane about the earth. But the points where this circle, in which the ſun ſeems to move, interſects the circle in which the moon is ſeen at any time to move, are called the nodes of the moon's orbit at that time. When the moon is ſeen moving in the circle AECD, the points A and C are the nodes of the orbit; when ſhe appears in the ſemicircle LYM, then L and M are the nodes. Now here it appears, from what has been ſaid, that while the moon has moved from A to K, one of the nodes has been carried from A to L, and the other as much from C to M. But the motion from A to L, and from C to

M, is backward in regard to the motion of the moon, which is the other way from A to K, and from thence toward C.

26. FARTHER the angle, which the plane, wherein the moon at any time appears, makes with the plane of the earth's motion, is called the inclination of the moon's orbit at that time. And I ſhall now proceed to ſhew, that this inclination of the orbit, when the moon is in K, is leſs than when ſhe was in A; or, that the plane LYM, which touches the line of the moon's motion in K, makes a leſs angle with the plane of the earth's motion or with the circle ABCD, than the plane AEC makes with the ſame. The ſemicircle LYM interſects the ſemicircle AEC in Y; and the arch AY is leſs than LY, and both together leſs than half a circle. But it is demonſtrated by the writers on that part of aſtronomy, which is called the doctrine of the ſphere, that when a triangle is made, as here, by three arches of circles AL, AY, and YL, the angle under YAB without the triangle is greater than the angle under YLA within, if the two arches AY, YL taken together do not amount to a ſemicircle; if the two arches make a complete ſemicircle, the two angles will be equal; but if the two arches taken together exceed a ſemicircle, the inner angle under YLA is greater than the other [a]. Here therefore the two arches AY and LY together being leſs than a ſemicircle, the angle under ALY is leſs, than the angle under BAE. But from the doctrine of the ſphere it is alſo evident, that the angle under ALY is equal to that, in which the plane of the

[a] Menelai Sphaeric. Lib. I. prop. 10.

circle

circle L Y K M, that is, the plane which touches the line A K G H I in K, is inclined to the plane of the earth's motion A B C; and the angle under B A E is equal to that, in which the plane A E C is inclined to the ſame plane. Therefore the inclination of the former plane is leſs than the inclination of the latter.

27. Suppose now the moon to be advanced to the point G (in fig. 100.) and in this point to be diſtant from its node a quarter part of the whole circle; or in other words, to be in the midway between its two nodes. And in this caſe the nodes will have receded yet more, and the inclination of the orbit be ſtill more diminiſhed: for ſuppoſe the line A K G H I to be touched in the point G by a plane paſſing through the earth T: let the interſection of this plane with the plane of the earth's motion be the line W T O, and the line T P its interſection with the plane L K M. In this plane let the circle N G O be deſcribed with the ſemidiameter T P or N T cutting the other circle L K M in P. Now the line A K G I is convex to the plane L K M, which touches it in K; and therefore the plane N G O, which touches it in G, will interſect the other touching plane between G and K; that is, the point P will fall between thoſe two points, and the plane continued to the plane of the earth's motion will paſs beyond L; ſo that the points N and O, or the places of the nodes, when the moon is in G, will be farther from A and C than L and M, that is, will have moved farther backward. Beſides, the inclination of the plane N G O to the plane of the earth's motion A B C is leſs, than the inclination of the plane L K M to the ſame; for here alſo the two arches L P and N P taken together are leſs than

than a ſemicircle, each of theſe arches being leſs than a quarter of a circle; as appears, becauſe G N, the diſtance of the moon in G from its node N, is here ſuppoſed to be a quarter part of a circle.

28. AFTER the moon is paſſed beyond G, the caſe is altered; for then theſe arches will be greater than quarters of the circle, by which means the inclination will be again increaſed, tho' the nodes ſtill go on to move the ſame way. Suppoſe the moon in H, (in fig. 101.) and that the plane, which touches the line A K G I in H, interſects the plane of the earth's motion in the line Q T R, and the plane N G O in the line T V, and beſides that the circle Q H R be deſcribed in that plane; then, for the ſame reaſon as before, the point V will fall between H and G, and the plane R V Q will paſs beyond the laſt plane O V N, cauſing the points Q and R to fall farther from A and C than N and O. But the arches N V, V Q are each greater than a quarter of a circle, N V the leaſt of them being greater than G N, which is a quarter of a circle; and therefore the two arches N V and V Q together exceed a ſemicircle; conſequently the angle under B Q V will be greater, than that under B N V.

29. IN the laſt place, when the moon is by this attraction of the ſun, drawn at length into the plane of the earth's motion, the node will have receded yet more, and the inclination be ſo much increaſed, as to become ſomewhat more than at firſt: for the line A K G H I being convex to all the planes, which touch it, the part H I will wholly fall between

the plane QVR and the plane ABC; ſo that the point I will fall between B and R; and drawing ITW, the point W will be farther remov'd from A than Q. But it is evident, that the plane, which paſſes through the earth T, and touches the line AGI in the point I, will cut the plane of the earth's motion ABCD in the line ITW, and be inclined to the ſame in the angle under HIB; ſo that the node, which was firſt in A, after having paſſed into L, N and Q, comes at laſt into the point W; as the node which was at firſt in C has paſſed ſucceſſively from thence through the points M, O and R to I: but the angle under HIB, which is now the inclination of the orbit to the plane of the ecliptic, is manifeſtly not leſs than the angle under ECB or EAB, but rather ſomething greater.

30. Thus the moon in the caſe before us, while it paſſes from the plane of the earth's motion in the quarter, till it comes again into the ſame plane, has the nodes of its orbit continually moved backward, and the inclination of its orbit is at firſt diminiſhed, viz. till it comes to G in fig. 100, which is near to its conjunction with the ſun, but afterwards is increaſed again almoſt by the ſame degrees, till upon the moon's arrival again to the plane of the earth's motion, the inclination of the orbit is reſtored to ſomething more than its firſt magnitude, though the difference is not very great, becauſe the points I and C are not far diſtant from each other [a].

[a] Vid. Newt. Princ. Lib. I. prop. 66. coroll. 10.

31. After

31. AFTER the ſame manner, if the moon had departed from the quarter in C, it ſhould have deſcribed the curve line CXW (in fig. 98.) between the planes AFC and ADC, which would be convex to the former of thoſe planes, and concave to the latter; ſo that, here alſo, the nodes ſhould continually recede, and the inclination of the orbit gradually diminiſh more and more, till the moon arrived near its oppoſition to the ſun in X; but from that time the inclination ſhould again increaſe, till it became a little greater than at firſt. This will eaſily appear, by conſidering, that as the action of the ſun upon the moon, by exceeding its action upon the earth, drew it out of the plane AEC towards the ſun, while the moon paſſed from A to I; ſo, during its paſſage from C to W, the moon being all that time farther from the ſun than the earth, it will be attracted leſs; and the earth, together with the plane AECF, will as it were be drawn from the moon, in ſuch ſort, that the path the moon deſcribes ſhall appear from the earth, as it did in the former caſe by the moon's being drawn away.

32. THESE are the changes, which the nodes and the inclination of the moon's orbit undergo, when the nodes are in the quarters; but when the nodes by their motion, and the motion of the ſun together, come to be ſituated between the quarter and conjunction or oppoſition, their motion and the change made in the inclination of the orbit are ſomewhat different.

33. Let AGCH (in fig. 103.) be a circle deſcribed in the plane of the earth's motion, having the earth in T for its center. Let the point oppoſite to the ſun be A, and the point G a fourth part of the circle diſtant from A. Let the nodes of the moon's orbit be ſituated in the line BTD, and B the node, falling between A, the place where the moon would be in the full, and G the place where the moon would be in the quarter. Suppoſe BEDF to be the plane, in which the moon eſſays to move, when it proceeds from the point B. Becauſe the moon in B is more diſtant from the ſun than the earth, it ſhall be leſs attracted by the ſun, and ſhall not deſcend towards the ſun ſo faſt as the earth: conſequently it ſhall quit the plane BEDF, which we ſuppoſe to accompany the earth, and deſcribe the line BIK convex thereto, till ſuch time as it comes to the point K, where it will be in the quarter: but from thenceforth being more attracted than the earth, the moon ſhall change its courſe, and the following part of the path it deſcribes ſhall be concave to the plane BED or BGD, and ſhall continue concave to the plane BGD, till it croſſes that plane in L, juſt as in the preceding caſe. Now I ſay, while the moon is paſſing from B to K, the nodes, contrary to what was found in the foregoing caſe, will proceed forward, or move the ſame way with the moon [a]; and at the ſame time the inclination of the orbit will increaſe [b].

34. When the moon is in the point I, let the plane MIN paſs through the earth T, and touch the path of the

[a] Vid. Newt. Princ. Lib. III. prop. 30. p. 442.   [b] Ibid. Lib. I. prop. 66. coroll. 10.

moon

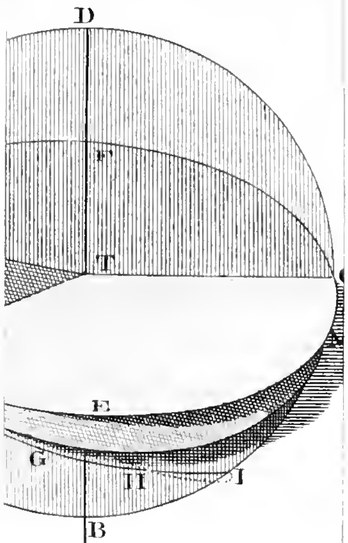
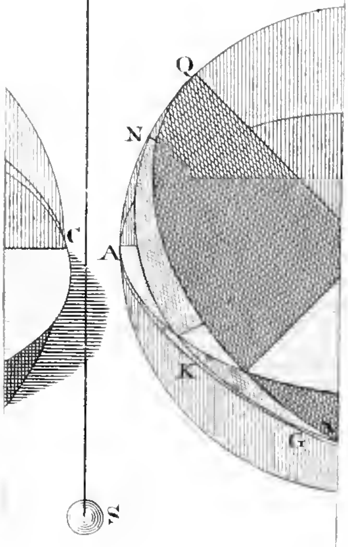


Fig. 99.



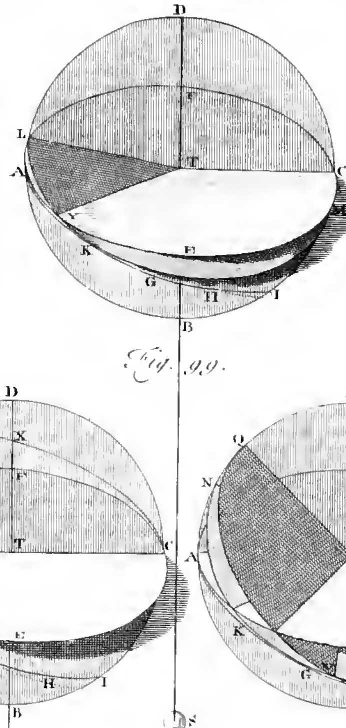


Fig. 99.

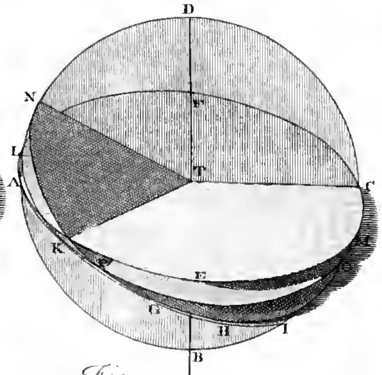


Fig. 100.

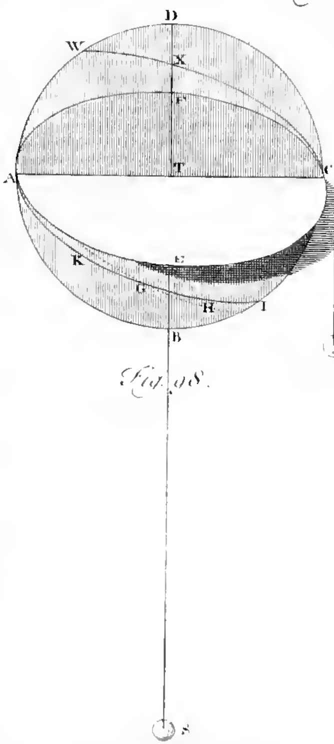


Fig. 98.

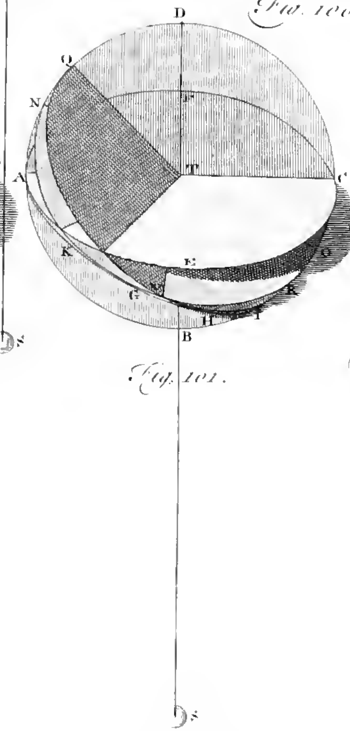


Fig. 101.

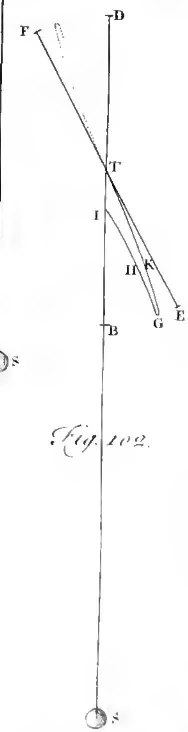


Fig. 102.

moon in I, cutting the plane of the earth's motion in the line MTN, and the plane BED in the line TO. Becauſe the line BIK is convex to the plane BED, which touches it in B, the plane NIM muſt croſs the plane DEB, before it meets the plane CGB; and therefore the point M will fall from B towards G, and the node of the moon's orbit being tranſlated from B to M is moved forward.

35. I SAY farther, the angle under OMG, which the plane MON makes with the plane BGC, is greater than the angle under OBG, which the plane BOD makes with the ſame. This appears from what has been already explained; becauſe the arches BO, OM are each leſs than the quarter of a circle, and therefore taken both together are leſs than a ſemicircle.

36. AGAIN, when the moon is come to the point K in its quarter, the nodes will be advanced yet farther forward, and the inclination of the orbit alſo more augmented. Hitherto the moon's motion has been referred to the plane, which paſſing through the earth touches the path of the moon in the point, where the moon is, according to what was aſſerted at the beginning of this diſcourſe upon the nodes, that it is the cuſtom of aſtronomers ſo to do. But here in the point K no ſuch plane can be found; on the contrary, ſeeing the line of the moon's motion on one ſide the point K is convex to the plane BED, and on the other ſide concave to the ſame, no plane can paſs through the points T and K, but will cut the line BKL in that point. Therefore inſtead

of

of ſuch a touching plane, we muſt here make uſe of what is equivalent, the plane PKQ, with which the line BKL ſhall make a leſs angle than with any other plane; for this plane does as it were touch the line BK in the point K, ſince it ſo cuts it, that no other plane can be drawn ſo, as to paſs between the line BK and the plane PKQ. But now it is evident, that the point P, or the node, is removed from M towards G, that is, has moved yet farther forward; and it is likewiſe as manifeſt, that the angle under KPG, or the inclination of the moon's orbit in the point K, is greater than the angle under IMG, for the reaſon ſo often aſſigned.

37. After the moon has paſſed the quarter, the path of the moon being concave to the plane AGCH, the nodes, as in the preceding caſe, ſhall recede, till the moon arrives at the point L; which ſhews, that conſidering the whole time of the moon's paſſing from B to L, at the end of that time the nodes ſhall be found to have receded, or to be placed backwarder, when the moon is in L, than when it was in B. For the moon takes a longer time in paſſing from K to L, than in paſſing from B to K; and therefore the nodes continue to recede a longer time, than they moved forwards; ſo that their receſs muſt ſurmount their advance.

38. In the ſame manner, while the moon is in its paſſage from K to L, the inclination of the orbit ſhall diminiſh, till the moon comes to the point, in which it is one quarter part of a circle diſtant from its node; ſuppoſe in the point R; and from that time the inclination ſhall again increaſe.

Since

Since therefore the inclination of the orbit increaſes, while the moon is paſſing from B to K, and diminiſhes itſelf again only, while the moon is paſſing from K to R, and then augments again, till the moon arrive in L; while the moon is paſſing from B to L, the inclination of the orbit is much more increaſed than diminiſhed, and will be diſtinguiſhably greater, when the moon is come to L, than when it ſet out from B.

39. IN like manner, while the moon is paſſing from L on the other ſide the plane AGCH, the node ſhall advance forward, as long as the moon is between the point L and the next quarter; but afterwards it ſhall recede, till the moon come to paſs the plane AGCH again in the point V, between B and A: and becauſe the time between the moon's paſſing from L to the next quarter is leſs, than the time between that quarter and the moon's coming to the point V, the node ſhall have more receded than advanced; ſo that the point V will be nearer to A, than L is to C. So alſo the inclination of the orbit, when the moon is in V, will be greater, than when the moon was at L; for this inclination increaſes all the time the moon is between L and the next quarter; it decreaſes only while the moon is paſſing from this quarter to the mid way between the two nodes, and from thence increaſes again during the whole paſſage through the other half of the way to the next node.

40. THUS we have traced the moon from her node in the quarter, and ſhewn, that at every period of the moon the nodes will have receded, and thereby will have approached

toward

toward a conjunction with the ſun. But this conjunction will be much forwarded by the viſible motion of the ſun itſelf. In the laſt ſcheme the ſun will appear to move from S toward W. Suppoſe it appeared to have moved from S to W, while the moon's node has receded from B to V, then drawing the line W T X, the arch V X will repreſent the diſtance of the line drawn between the nodes from the ſun, when the moon is in V; whereas the arch B A repreſented that diſtance, when the moon was in B. This viſible motion of the ſun is much greater, than that of the node; for the ſun appears to revolve quite round each year, and the node is near 19 years in making one revolution. We have alſo ſeen, that when the node was in the quadrature, the inclination of the moon's orbit decreaſed, till the moon came to the conjunction, or oppoſition, according to which node it ſet out from; but that afterwards it again increaſed, till it became at the next node rather greater than at the former. When the node is once removed from the quarter nearer to a conjunction with the ſun, the inclination of the moon's orbit, when the moon comes into the node, is more ſenſibly greater, than it was in the node preceding; the inclination of the orbit by this means more and more increaſing till the node comes into conjunction with the ſun; at which time it has been ſhewn above, that the ſun has no power to change the plane of the moon's motion; and conſequently has no effect either on the nodes, or on the inclination of the orbit.

41. As ſoon as the nodes, by the action of the ſun, are got out of conjunction toward the other quarters, they begin again

again to recede as before; but the inclination of the orbit in the appulse of the moon to each succeeding node is less than at the preceding, till the nodes come again into the quarters. This will appear as follows. Let A (in fig. 104.) represent one of the moon's nodes placed between the point of opposition B and the quarter C. Let the plane A D E pass through the earth T, and touch the path of the moon in A. Let the line A F G H be the path of the moon in her passage from A to H, where she crosses again the plane of the earth's motion. This line will be convex toward the plane A D E, till the moon comes to G, where she is in the quarter; and after this, between G and H, the same line will be concave toward this plane. All the time this line is convex toward the plane A D E, the nodes will recede; and on the contrary proceed, while it is concave to that plane. All this will easily be conceived from what has been before so largely explained. But the moon is longer in passing from A to G, than from G to H; therefore the nodes recede a longer time, than they proceed; consequently upon the whole, when the moon is arrived at H, the nodes will have receded, that is, the point H will fall between B and E. The inclination of the orbit will decrease, till the moon is arrived to the point F, in the middle between A and H. Through the passage between F and G the inclination will increase, but decrease again in the remaining part of the passage from G to H, and consequently at H must be less than at A. The like effects, both in respect to the nodes and inclination of the orbit, will take place in the following passage of the moon on the other side of the plane A B E C, from H, till it comes over that plane again in I.

42. THUS

42. Thus the inclination of the orbit is greateſt, when the line drawn between the moon's nodes will paſs through the ſun; and leaſt, when this line lies in the quarters, eſpecially if the moon at the ſame time be in conjunction with the ſun, or in the oppoſition. In the firſt of theſe caſes the nodes have no motion, in all others, the nodes will each month have receded: and this regreſſive motion will be greateſt, when the nodes are in the quarters; for in that caſe the nodes have no progreſſive motion during the whole month, but in all other caſes the nodes do at ſome times proceed forward, viz. whenever the moon is between either quarter, and the node which is leſs diſtant from that quarter than a fourth part of a circle.

43. It now remains only to explain the irregularities in the moon's motion, which follow from the elliptical figure of the orbit. By what has been ſaid at the beginning of this chapter it appears, that the power of the earth on the moon acts in the reciprocal duplicate proportion of the diſtance: therefore the moon, if undiſturbed by the ſun, would move round the earth in a true ellipſis, and the line drawn from the earth to the moon would paſs over equal ſpaces in equal portions of time. That this deſcription of the ſpaces is altered by the ſun, has been already declared. It has alſo been ſhewn, that the figure of the orbit is changed each month; that the moon is nearer the earth at the new and full, and more remote in the quarters, than it would be without the ſun. Now we muſt paſs by theſe monthly changes, and conſider the effect, which the ſun will have in the different

ent ſituations of the axis of the orbit in reſpect of that luminary.

44. THE action of the ſun varies the force, wherewith the moon is drawn toward the earth; in the quarters the force of the earth is directly increaſed by the ſun; at the new and full the ſame is diminiſhed; and in the intermediate places the influence of the earth is ſometimes aided, and ſometimes leſſened by the ſun. In theſe intermediate places between the quarters and the conjunction or oppoſition, the ſun's action is ſo oblique to the action of the earth on the moon, as to produce that alternate acceleration and retardment of the moon's motion, which I obſerved above to be ſtiled the variation. But beſides this effect, the power, by which the earth attracts the moon toward itſelf, will not be at full liberty to act with the ſame force, as if the ſun acted not at all on the moon. And this effect of the ſun's action, whereby it corroborates or weakens the action of the earth, is here only to be conſidered. And by this influence of the ſun it comes to paſs, that the power, by which the moon is impelled toward the earth, is not perfectly in the reciprocal duplicate proportion of the diſtance. Conſequently the moon will not deſcribe a perfect ellipſis. One particular, wherein the moon's orbit will differ from an ellipſis, conſiſts in the places, where the motion of the moon is perpendicular to the line drawn from itſelf to the earth. In an ellipſis, after the moon ſhould have ſet out in the direction perpendicular to this line drawn from itſelf to the earth, and at its greateſt diſtance from the earth, its motion would

again become perpendicular to this line drawn between itſelf and the earth, and the moon be at its neareſt diſtance from the earth, when it ſhould have performed half its period; after performing the other half of its period its motion would again become perpendicular to the forementioned line, and the moon return into the place whence it ſet out, and have recovered again its greateſt diſtance. But the moon in its real motion, after ſetting out as before, ſometimes makes more than half a revolution, before its motion comes again to be perpendicular to the line drawn from itſelf to the earth, and the moon is at its neareſt diſtance; and then performs more than another half of an intire revolution before its motion can a ſecond time recover its perpendicular direction to the line drawn from the moon to the earth, and the moon arrive again to its greateſt diſtance from the earth. At other times the moon will deſcend to its neareſt diſtance, before it has made half a revolution, and recover again its greateſt diſtance, before it has made an intire revolution. The place, where the moon is at its greateſt diſtance from the earth, is called the moon's apogeon, and the place of the leaſt diſtance the perigeon. This change of the place, where the moon ſucceſſively comes to its greateſt diſtance from the earth, is called the motion of the apogeon. In what manner the ſun cauſes the apogeon to move, I ſhall now endeavour to explain.

45. Our author ſhews, that if the moon were attracted toward the earth by a compoſition of two powers, one of which were reciprocally in the duplicate proportion of the diſtance from the earth, and the other reciprocally in

in the triplicate proportion of the ſame diſtance; then, though the line deſcribed by the moon would not be in reality an ellipſis, yet the moon's motion might be perfectly explained by an ellipſis, whoſe axis ſhould be made to move round the earth; this motion being in conſequence, as aſtronomers expreſs themſelves, that is, the ſame way as the moon itſelf moves, if the moon be attracted by the ſum of the two powers; but the axis muſt move in antecedence, or the contrary way, if the moon be acted on by the difference of theſe powers. What is meant by duplicate proportion has been often explained; namely, that if three magnitudes, as A, B, and C, are ſo related, that the ſecond B bears the ſame proportion to the third C, as the firſt A bears to the ſecond B, then the proportion of the firſt A to the third C, is the duplicate of the proportion of the firſt A to the ſecond B. Now if a fourth magnitude, as D, be aſſumed, to which C ſhall bear the ſame proportion as A bears to B, and B to C, then the proportion of A to D is the triplicate of the proportion of A to B.

46. THE way of repreſenting the moon's motion in this caſe is thus. T denoting the earth (in fig. 105, 106.) ſuppoſe the moon in the point A, its apogeon, or greateſt diſtance from the earth, moving in the direction A F perpendicular to A B, and acted upon from the earth by two ſuch forces as have been named. By that power alone, which is reciprocally in the duplicate proportion of the diſtance, if the moon ſet out from the point A with a proper degree of velocity, the ellipſis A M B may be deſcribed.

ſcribed. But if the moon be acted upon by the ſum of the forementioned powers, and the velocity of the moon in the point A be augmented in a certain proportion [a]; or if that velocity be diminiſhed in a certain proportion, and the moon be acted upon by the difference of thoſe powers; in both theſe caſes the line AE, which ſhall be deſcribed by the moon, is thus to be determined. Let the point M be that, into which the moon would have arrived in any given ſpace of time, had it moved in the ellipſis AMB. Draw MT, and likewiſe CTD in ſuch ſort, that the angle under ATM ſhall bear the ſame proportion to the angle under ATC, as the velocity, with which the ellipſis AMB muſt have been deſcribed, bears to the difference between this velocity, and the velocity, with which the moon muſt ſet out from the point A in order to deſcribe the path AE. Let the angle ATC be taken toward the moon (as in fig. 105.) if the moon be attracted by the ſum of the powers; but the contrary way (as in fig. 106.) if by their difference. Then let the line AB be moved into the poſition CD, and the ellipſis AMB into the ſituation CND, ſo that the point M be tranſlated to L: then the point L ſhall fall upon the path of the moon AE.

47. The angular motion of the line AT, wereby it is removed into the ſituation CT, repreſents the motion of the apogeon; by the means of which the motion of the moon might be fully explicated by the ellipſis AMB, if the action of the ſun upon it was directed to the center of the earth, and

[a] What this proportion is, may be known from Coroll. 2 prop 44. Lib. I. Princ. philoſ. Newton.

3 reci-

reciprocally in the triplicate proportion of the moon's diſtance from it. But that not being ſo, the apogeon will not move in the regular manner now deſcribed. However, it is to be obſerved here, that in the firſt of the two preceding caſes, where the apogeon moves forward, the whole centripetal power increaſes faſter, with the decreaſe of diſtance, than if the intire power were reciprocally in the duplicate proportion of the diſtance; becauſe one part only is in that proportion, and the other part, which is added to this to make up the whole power, increaſes faſter with the decreaſe of diſtance. On the other hand, when the centripetal power is the difference between theſe two, it increaſes leſs with the decreaſe of the diſtance, than if it were ſimply in the reciprocal duplicate proportion of the diſtance. Therefore if we chuſe to explain the moon's motion by an ellipſis (as is moſt convenient for aſtronomical uſes to be done, and by reaſon of the ſmall effect of the ſun's power, the doing ſo will not be attended with any ſenſible error;) we may collect in general, that when the power, by which the moon is attracted to the earth, by varying the diſtance, increaſes in a greater than in the duplicate proportion of the diſtance diminiſhed, a motion in conſequence muſt be aſcribed to the apogeon; but that when the attraction increaſes in a leſs proportion than that named, the apogeon muſt have given to it a motion in antecedence [a]. It is then obſerved by Sir IS. NEWTON, that the firſt of theſe caſes obtains, when the moon is in the conjunction and oppoſition; and the latter, when the moon is in the quarters: ſo that in the firſt the apogeon moves according to the order of the

[a] Princ. Phil. Newt. Lib. I. prop. 45. Coroll. 1.

ſigns;

ſigns; in the other, the contrary way[a]. But, as was ſaid before, the diſturbance given to the action of the earth by the ſun in the conjunction and oppoſition being near twice as great as in the quarters[b], the apogeon will advance with a greater velocity than recede, and in the compaſs of a whole revolution of the moon will be carried in conſequence[c].

48. It is ſhewn in the next place by our author, that when the line A B coincides with that, which joins the earth and the ſun, the progreſſive motion of the apogeon, when the moon is in the conjunction or oppoſition, exceeds the regreſſive in the quadratures more than in any other ſituation of the line A B[d]. On the contrary, when the line A B makes right angles with that, which joins the earth and ſun, the retrograde motion will be more conſiderable[e], nay is found ſo great as to exceed the progreſſive; ſo that in this caſe the apogeon in the compaſs of an intire revolution of the moon is carried in antecedence. Yet from the conſiderations in the laſt paragraph the progreſſive motion exceeds the other; ſo that in the whole the mean motion of the apogeon is in conſequence, according as aſtronomers find. Moreover, the line A B changes its ſituation with that, which joins the earth and ſun, by ſuch ſlow degrees, that the inequalities in the motion of the apogeon ariſing from this laſt conſideration, are much greater than what ariſes from the other[f].

[a] Pr. Phil. Newt. Lib. I. prop. 66. Coroll. 7.
[b] See § 19. of this chapter.
[c] Phil. Nat. Pr. Math Lib. I. prop. 66 cor. 8.
[d] Ibid. Coroll. 8.
[e] Ibid.
[f] Ibid.

49. Far-

49. FARTHER, this unſteady motion in the apogeon is attended with another inequality in the motion of the moon, that it cannot be explained at all times by the ſame ellipſis. The ellipſis in general is called by aſtronomers an eccentric orbit. The point, in which the two axis's croſs, is called the center of the figure; becauſe all lines drawn through this point within the ellipſis, from ſide to ſide, are divided in the middle by this point. But the center, about which the heavenly bodies revolve, lying out of this center of the figure in one focus, theſe orbits are ſaid to be eccentric; and where the diſtance of the focus from this center bears the greateſt proportion to the whole axis, that orbit is called the moſt eccentric: and in ſuch an orbit the diſtance from the focus to the remoter extremity of the axis bears the greateſt proportion to the diſtance of the nearer extremity. Now whenever the apogeon of the moon moves in conſequence, the moon's motion muſt be referred to an orbit more eccentric, than what the moon would deſcribe, if the whole power, by which the moon was acted on in its paſſing from the apogeon, changed according to the reciprocal duplicate proportion of the diſtance from the earth, and by that means the moon did deſcribe an immoveable ellipſis; and when the apogeon moves in antecedence, the moon's motion muſt be referred to an orbit leſs eccentric. In the firſt of the two figures laſt referred to, the true place of the moon L falls without the orbit A M B, to which its motion is referred: whence the orbit A L E, truly deſcribed by the moon, is leſs incurvated in the point A, than is the orbit A M B; therefore the orbit A M B is more oblong, and differs farther from a circle, than the ellipſis would,

whose curvature in A were equal to that of the line ALB, that is, the proportion of the distance of the earth T from the center of the ellipsis to its axis will be greater in the ellipsis AMB, than in the other; but that other is the ellipsis, which the moon would describe, if the power acting upon it in the point A were altered in the reciprocal duplicate proportion of the distance. In the second figure, when the apogeon recedes, the place of the moon L falls within the orbit AMB, and therefore that orbit is less eccentric, than the immoveable orbit which the moon should describe. The truth of this is evident; for, when the apogeon moves forward, the power, by which the moon is influenced in its descent from the apogeon, increases faster with the decrease of distance, than in the duplicate proportion of the distance; and consequently the moon being drawn more forcibly toward the earth, it will descend nearer to it. On the other hand, when the apogeon recedes, the power acting on the moon increases with the decrease of distance in less than the duplicate proportion of the distance; and therefore the moon is less impelled toward the earth, and will not descend so low.

50. Now suppose in the first of these figures, that the apogeon A is in the situation, where it is approaching toward the conjunction or opposition of the sun. In this case the progressive motion of the apogeon is more and more accelerated. Here suppose that the moon, after having descended from A through the orbit AE as far as F, where it is come to its nearest distance from the earth, ascends again up the line FG. Because the motion of the apogeon is here continually more and

more

more accelerating, the cause of its motion is constantly upon the increase; that is, the power, whereby the moon is drawn to the earth, will decrease with the increase of distance, in the moon's ascent from F, in a greater proportion than that wherewith it increased with the decrease of distance in the moon's descent to F. Consequently the moon will ascend higher than to the distance A T, from whence it descended; therefore the proportion of the greatest distance of the moon to the least is increased. And when the moon descends again, the power will yet more increase with the decrease of distance, than in the last ascent it decreased with the augmentation of distance; the moon therefore must descend nearer to the earth than it did before, and the proportion of the greatest distance to the least yet be more increased. Thus as long as the apogeon is advancing toward the conjunction or opposition, the proportion of the greatest distance of the moon from the earth to the least will continually increase; and the elliptical orbit, to which the moon's motion is referred, will be rendered more and more eccentric.

51. As soon as the apogeon is passed the conjunction with the sun or the opposition, the progressive motion thereof abates, and with it the proportion of the greatest distance of the moon from the earth to the least distance will also diminish; and when the apogeon becomes regressive, the diminution of this proportion will be still farther continued on, till the apogeon comes into the quarter; from thence this proportion, and the eccentricity of the orbit will increase again. Thus the orbit of the moon is most eccentric, when the apo-

geon is in conjunction with the ſun, or in oppoſition to it, and leaſt of all when the apogeon is in the quarters.

52. THESE changes in the nodes, in the inclination of the orbit to the plane of the earth's motion, in the apogeon, and in the eccentricity, are varied like the other inequalities in the motion of the moon, by the different diſtance of the earth from the ſun; being greateſt, when their cauſe is greateſt, that is, when the earth is neareſt to the ſun.

53. I ſaid at the beginning of this chapter, that Sir ISAAC NEWTON has computed the very quantity of many of the moon's inequalities. That acceleration of the moon's motion, which is called the variation, when greateſt, removes the moon out of the place, in which it would otherwiſe be found, ſomething more than half a degree [a]. In the phraſe of aſtronomers, a degree is $\frac{1}{360}$ part of the whole circuit of the moon or any planet. If the moon, without diſturbance from the ſun, would have deſcribed a circle concentrical to the earth, the ſun will cauſe the moon to approach nearer to the earth in the conjunction and oppoſition, than in the quarters, nearly in the proportion of 69 to 70 [b]. We had occaſion to mention above, that the nodes perform their period in almoſt 19 years. This the aſtronomers found by obſervation; and our author's computations aſſign to them the ſame period [c]. The inclination of the moon's orbit when leaſt, is an angle about $\frac{1}{18}$ part of that angle, which conſtitutes

[a] Newt. Princ. Lib. III. prop. 29. [b] Ibid. prop. 28. [c] Ibid. prop. 32.

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a perpendicular; and the difference between the greateſt and leaſt inclination of the orbit is determined by our author's computation to be about $\frac{1}{18}$ of the leaſt inclination [a]. And this alſo is agreeable to the obſervations of aſtronomers. The motion of the apogeon, and the changes in the eccentricity, Sir ISAAC NEWTON has not computed. The apogeon performs its revolution in about eight years and ten months. When the moon's orbit is moſt eccentric, the greateſt diſtance of the moon from the earth bears to the leaſt diſtance nearly the proportion of 8 to 7; when the orbit is leaſt eccentric, this proportion is hardly ſo great as that of 12 to 11.

54. Sir ISAAC NEWTON ſhews farther, how, by comparing the periods of the motion of the ſatellites, which revolve round Jupiter and Saturn, with the period of our moon round the earth, and the periods of thoſe planets round the ſun with the period of our earth's motion, the inequalities in the motion of thoſe ſatellites may be derived from the inequalities in the moon's motion; excepting only in regard to that motion of the axis of the orbit, which in the moon makes the motion of the apogeon; for the orbits of thoſe ſatellites, as far as can be diſcerned by us at this diſtance, appearing little or nothing eccentric, this motion, as deduced from the moon, muſt be diminiſhed.

[a] Newt. Princ. pag. 459.

CHAP.

CHAP. IV.

Of COMETS.

IN the former of the two preceding chapters the powers have been explained, which keep in motion thoſe celeſtial bodies, whoſe courſes had been well determined by the aſtronomers. In the laſt chapter we have ſhewn, how thoſe powers have been applied by our author to the making a more perfect diſcovery of the motion of thoſe bodies, the courſes of which were but imperfectly underſtood; for ſome of the inequalities, which we have been deſcribing in the moon's motion, were unknown to the aſtronomers. In this chapter we are to treat of a third ſpecies of the heavenly bodies, the true motion of which was not at all apprehended before our author writ; in ſo much, that here Sir ISAAC NEWTON has not only explained the cauſes of the motion of theſe bodies, but has performed alſo the part of an aſtronomer, by diſcovering what their motions are.

2. THAT theſe bodies are not meteors in our air, is manifeſt; becauſe they riſe and ſet in the ſame manner, as the ſun and ſtars. The aſtronomers had gone ſo far in their inquiries concerning them, as to prove by their obſervations, that they moved in the etherial ſpaces far beyond the moon; but they had no true notion at all of the path, which they deſcribed. The moſt prevailing opinion before our

our author was, that they moved in ſtraight lines; but in what part of the heavens was not determined. DES CARTES [a] removed them far beyond the ſphere of Saturn, as finding the ſtraight motion attributed to them, inconſiſtent with the vortical fluid, by which he explains the motions of the planets, as we have above related [b]. But Sir ISAAC NEWTON diſtinctly proves from aſtronomical obſervation, that the comets paſs through the region of the planets, and are moſtly inviſible at a leſs diſtance, than that of Jupiter [c].

3. AND from hence finding the comets to be evidently within the ſphere of the ſun's action, he concludes they muſt neceſſarily move about the ſun, as the planets do [d]. The planets move in ellipſis's; but it is not neceſſary that every body, which is influenced by the ſun, ſhould move in that particular kind of line. However our author proves, that the power of the ſun being reciprocally in the duplicate proportion of the diſtance, every body acted on by the ſun muſt either fall directly down, or move in ſome conic ſection; of which lines I have above obſerved, that there are three ſpecies, the ellipſis, parabola, and hyperbola [e]. If a body, which deſcends toward the ſun as low as the orbit of any planet, move with a ſwifter motion than the planet does, that body will deſcribe an orbit of a more oblong figure, than that of the planet, and have a longer axis at leaſt. The velocity of the body may be ſo great, that it

[a] In Princ. philoſ. part. 3. § 41.
[b] Chap. 1. § 11.
[c] Newton. Princ. philoſ. Lib. III. Lemm. 4. pag. 478.
[d] Princ. philoſ. Lib. III. prop. 40.
[e] Book I. chap. 2. § 82.

ſhall

ſhall move in a parabola, and having once paſſed about the ſun, ſhall aſcend for ever without returning any more: but the ſun will be placed in the focus of this parabola. With a velocity ſtill greater the body will move in an hyperbola. But it is moſt probable, that the comets move in elliptical orbits, though of a very oblong, or in the phraſe of aſtronomers, of a very eccentric form, ſuch as is repreſented in fig. 107, where S is the ſun, C the comet, and ABDE its orbit, wherein the diſtance of S and D far exceeds that of S and A. Whence it is, that they ſometimes are found at a moderate diſtance from the ſun, and appear within the planetary regions; at other times they aſcend to vaſt diſtances, far beyond the very orbit of Saturn, and ſo become inviſible. That the comets do move in this manner is proved by our author, from computations built upon the obſervations, which aſtronomers had made on many comets. Theſe computations were performed by Sir ISAAC NEWTON himſelf upon the comet, which appeared toward the latter end of the year 1680, and at the beginning of the year following [a]; but the learned Dr. HALLEY proſecuted the like computations more at large in this, and alſo in many other comets [b]. Which computations are made upon propoſitions highly worthy of our author's unparallel'd genius, ſuch as could ſcarce have been diſcovered by any one not poſſeſſed of the utmoſt force of invention;

[a] Princ. philoſ. Lib. III. pag. 499, 500. [b] Ibid. pag. 500, and 510, &c.

4. THOSE

4. THOSE computations depend upon this principle, that the eccentricity of the orbits of the comets is ſo great, that if they are really elliptical, yet they approach ſo near to parabolas in that part of them, where they come under our view, that they may be taken for ſuch without ſenſible error [a]: as in the preceding figure the parabola FAG differs in the lower part of it about A very little from the ellipſis DEAB. Upon which ground our great author teaches a method of finding by three obſervations made upon any comet the parabola, which neareſt agrees with its orbit [b].

5. NOW what confirms this whole theory beyond the leaſt room for doubt is, that the places of the comets computed in the orbits, which the method here mentioned aſſigns them, agree to the obſervations of aſtronomers with the ſame degree of exactneſs, as the computations of the primary planets places uſually do; and this in comets, whoſe motions are very extraordinary [c].

6. OUR author afterwards ſhews how to make uſe of any ſmall deviation from the parabola, that ſhall be obſerved, to determine whether the orbits of the comets are elliptical or not, and ſo to diſcover if the ſame comet returns at certain periods [d]. And upon examining the comet in 1680, by the rule laid down for this purpoſe, he finds its orbit to agree more exactly to an ellipſis than

[a] Princ. Philoſ. Lib. III. prop. 40.
[b] Ibid. prop. 41.
[c] Ibid. pag 522.
[d] Ibid prop. 42.

to a parabola, though the ellipsis be so very eccentric, that the comet cannot perform its period through it in the space of 500 years [a]. Upon this Dr. HALLEY observed, that mention is made in history of a comet, with the like eminent tail as this, having appeared three several times before; the first of which appearances was at the death of JULIUS CESAR, and each appearance was at the distance of 575 years from the next preceding. He therefore computed the motion of this comet in such an elliptic orbit, as would require this number of years for the body to revolve through it; and these computations agree yet more perfectly with the observations made on this comet, than any parabolical orbit will do [b].

7. THE comparing together different appearances of the same comet, is the only way to discover certainly the true form of the orbit: for it is impossible to determine with exactness the figure of an orbit so exceedingly eccentric, from single observations taken in one part of it; and therefore Sir ISAAC NEWTON [c] proposes to compare the orbits, upon the supposition that they are parabolical, of such comets as appear at different times; for if the same orbit be found to be described by a comet at different times, in all probability it will be the same comet which describes it. And here he remarks from Dr. HALLEY, that the same orbit very nearly agrees to two appearances of a comet about the space of 75 years distance [d]; so that

[a] Newt. Princ. philos. edit. 2. p. 464, 465.
[b] Ibid. edit. 3. p 501, 502.
[c] Ibid pag. 519.
[d] Ibid. pag. 524.

if thoſe two appearances were really of the ſame comet, the tranſverſe axis of the orbit of the comet would be near 18 times the axis of the earth's orbit; and the comet, when at its greateſt diſtance from the ſun, will be removed not leſs than 35 times as far as the middle diſtance of the earth.

8. AND this ſeems to be the ſhorteſt period of any of the comets. But it will be farther confirmed, if the ſame comet ſhould return a third time after another period of 75 years. However it is not to be expected, that comets ſhould preſerve the ſame regularity in their periods, as the planets; becauſe the great eccentricity of their orbits makes them liable to ſuffer very conſiderable alterations from the action of the planets, and other comets, upon them.

9. IT is therefore to prevent too great diſturbances in their motions from theſe cauſes, as our author obſerves, that while the planets revolve all of them nearly in the ſame plane, the comets are diſpoſed in very different ones, and diſtributed over all parts of the heavens; that, when in their greateſt diſtance from the ſun, and moving ſloweſt, they might be removed as far as poſſible out of the reach of each other's action [a]. The ſame end is likewiſe farther anſwered in thoſe comets, which by moving ſloweſt in the aphelion, or remoteſt diſtance from the ſun, deſcend neareſt to it, by placing the aphelion of theſe at the greateſt height from the ſun [b].

[a] Newt. Princ. philoſ. p. 525.   [b] Ibid.

10. OUR philoſopher being led by his principles to explain the motions of the comets, in the manner now related, takes occaſion from thence to give us his thoughts upon their nature and uſe. For which end he proves in the firſt place, that they muſt neceſſarily be ſolid and compact bodies, and by no means any ſort of vapour or light ſubſtance exhaled from the planets or ſtars: becauſe at the near diſtance, to which ſome comets approach the ſun, it could not be, but the immenſe heat, to which they are expoſed, ſhould inſtantaneouſly diſperſe and ſcatter any ſuch light volatile ſubſtance [a]. In particular the forementioned comet of 1680 deſcended ſo near the ſun, as to come within a ſixth part of the ſun's diameter from the ſurface of it. In which ſituation it muſt have been expoſed, as appears by computation, to a degree of heat exceeding the heat of the ſun upon our earth no leſs than 28000 times; and therefore might have contracted a degree of heat 2000 times greater, than that of red hot iron [b]. Now a ſubſtance, which could endure ſo intenſe a heat, without being diſperſed in vapor, muſt needs be firm and ſolid.

11. IT is ſhewn likewiſe, that the comets are opake ſubſtances, ſhining by a reflected light, borrowed from the ſun [c]. This is proved from the obſervation, that comets, though they are approaching the earth, yet diminiſh in luſtre, if at the ſame time they recede from

[a] Ibid pag 508. [b] Ibid. [c] Ibid. pag. 484.

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the ſun; and on the contrary, are found to encreaſe daily in brightneſs, when they advance towards the ſun, though at the ſame time they move from the earth [a].

12. THE comets therefore in theſe reſpects reſemble the planets; that both are durable opake bodies, and both revolve about the ſun in conic ſections. But farther the comets, like our earth, are ſurrounded by an atmoſphere. The air we breath is called the earth's atmoſphere; and it is moſt probable, that all the other planets are inveſted with the like fluid. Indeed here a difference is found between the planets and comets. The atmoſpheres of the planets are of ſo fine and ſubtile a ſubſtance, as hardly to be diſcerned at any diſtance, by reaſon of the ſmall quantity of light which they reflect, except only in the planet Mars. In him there is ſome little appearance of ſuch a ſubſtance ſurrounding him, as ſtars which have been covered by him are ſaid to look ſomewhat dim a ſmall ſpace before his body comes under them, as if their light, when he is near, were obſtructed by his atmoſphere. But the atmoſpheres which ſurround the comets are ſo groſs and thick, as to reflect light very copiouſly. They are alſo much greater in proportion to the body they ſurround, than thoſe of the planets, if we may judge of the reſt from our air; for it has been obſerved of comets, that the bright light appearing in the middle of them, which

[a] Ibid. pag. 482, 483.

is

is reflected from the solid body, is scarce a ninth or tenth part of the whole comet.

13. I speak only of the heads of the comets, the most lucid part of which is surrounded by a fainter light, the most lucid part being usually not above a ninth or tenth part of the whole in breadth [a]. Their tails are an appearance very peculiar, nothing of the same nature appertaining in the least degree to any other of the celestial bodies. Of that appearance there are several opinions; our author reduces them to three [b]. The two first, which he proposes, are rejected by him; but the third he approves. The first is, that they arise from a beam of light transmitted through the head of the comet, in like manner as a stream of light is discerned, when the sun shines into a darkened room through a small hole. This opinion, as Sir ISAAC NEWTON observes, implies the authors of it wholly unskilled in the principles of optics; for that stream of light, seen in a darken'd room, arises from the reflection of the sun beams by the dust and motes floating in the air: for the rays of light themselves are not seen, but by their being reflected to the eye from some substance, upon which they fall [c]. The next opinion examined by our author is that of the celebrated DES CARTES, who imagins these tails to be the light of the comet refracted in its passage to us, and thence affording an oblong representation; as the light of the sun does, when refracted

[a] Ibid. pag. 481. [b] Ibib. pag. 509. [c] See the fore-cited place.

by

by the prifm in that noted experiment, which will have a great fhare in the third book of this difcourfe [a]. But this opinion is at once overturned from this confideration only, that the planets could be no more free from this refraction than the comets; nay ought to have larger or brighter tails, than they, becaufe the light of the planets is ftrongeft. However our author has thought proper to add fome farther objections againft this opinion: for inftance, that thefe tails are not variegated with colours, as is the image produced by the prifm, and which is infeparable from that unequal refraction, which produces that difproportioned length of the image. And befides, when the light in its paffage from different comets to the earth defcribes the fame path through the heavens, the refraction of it fhould of neceffity be in all refpects the fame. But this is contrary to obfervation; for the comet in 1680, the 28th day of December, and a former comet in the year 1577, the 29th day of December, appear'd in the fame place of the heavens, that is, were feen adjacent to the fame fixed ftars, the earth likewife being in the fame place at both times; yet the tail of the latter comet deviated from the oppofition to the fun a little to the northward, and the tail of the former comet declined from the oppofition of the fun five times as much fouthward [b].

14. THERE are fome other falfe opinions, though lefs regarded than thefe, which have been advanced upon this

[a] Ibid and Cartef. Princ. Phil. part. 3. § 134, &c. [b] Vid. Phil. Nat. princ. Math. p. 511.

argument.

argument. Theſe our excellent author paſſes over, haſtening to explain, what he takes to be the true cauſe of this appearance. He thinks it is certainly owing to ſteams and vapours exhaled from the body, and groſs atmoſphere of the comets, by the heat of the ſun; becauſe all the appearances agree perfectly to this ſentiment. The tails are but ſmall, while the comet is deſcending to the ſun, but enlarge themſelves to an immenſe degree, as ſoon as ever the comet has paſſed its perihelion; which ſhews the tail to depend upon the degree of heat, which the comet receives from the ſun. And that the intenſe heat to which comets, when neareſt the ſun, are expoſed, ſhould exhale from them a very copious vapour, is a moſt reaſonable ſuppoſition; eſpecially if we conſider, that in thoſe free and empty regions ſteams will more eaſily aſcend, than here upon the ſurface of the earth, where they are ſuppreſſed and hindered from riſing by the weight of the incumbent air: as we find by experiments made in veſſels exhauſted of the air, where upon removal of the air ſeveral ſubſtances will fume and diſcharge ſteams plentifully, which emit none in the open air. The tails of comets, like ſuch a vapour, are always in the plane of the comet's orbit, and oppoſite to the ſun, except that the upper part thereof inclines towards the parts, which the comet has left by its motion; reſembling perfectly the ſmoak of a burning coal, which, if the coal remain fixed, aſcends from it perpendicularly; but, if the coal be in motion, aſcends obliquely, inclining from the motion of the coal. And beſides, the tails of comets may be compared to this ſmoak in another

3 reſpect,

respect, that both of them are denser and more compact on the convex side, than on the concave. The different appearance of the head of the comet, after it has past its perihelion, from what it had before, confirms greatly this opinion of their tails: for smoke raised by a strong heat is blacker and grosser, than when raised by a less; and accordingly the heads of comets, at the same distance from the sun, are observed less bright and shining after the perihelion, than before, as if obscured by such a gross smoke.

15. THE observations of HEVELIUS upon the atmospheres of comets still farther illustrate the same; who relates, that the atmospheres, especially that part of them next the sun, are remarkably contracted when near the sun, and dilated again afterwards.

16. To give a more full idea of these tails, a rule is laid down by our author, whereby to determine at any time, when the vapour in the extremity of the tail first rose from the head of the comet. By this rule it is found, that the tail does not consist of a fleeting vapour, dissipated soon after it is raised, but is of long continuance; that almost all the vapour, which rose about the time of the perihelion from the comet of 1680, continued to accompany it, ascending by degrees, being succeeded constantly by fresh matter, which rendered the tail contiguous to the comet. From this computation the tails are found to participate of another property of ascending vapours, that when they ascend with the greatest velocity, they are least incurvated.

17. THE only objection that can be made againſt this opinion is the difficulty of explaining, how a ſufficient quantity of vapour can be raiſed from the atmoſphere of a comet to fill thoſe vaſt ſpaces, through which their tails are ſometimes extended. This our author removes by the following computation: our air being an elaſtic fluid, as has been ſaid before[a], is more denſe here near the ſurface of the earth, where it is preſſed upon by the whole air above; than it is at a diſtance from the earth, where it has a leſs weight incumbent. I have obſerved, that the denſity of the air is reciprocally proportional to the compreſſing weight. From hence our author computes to what degree of rarity the air muſt be expanded, according to this rule, at an height equal to a ſemidiameter of the earth: and he finds, that a globe of ſuch air, as we breath here on the ſurface of the earth, which ſhall be one inch only in diameter,if it were expanded to the degree of rarity, which the air muſt have at the height now mentioned, would fill all the planetary regions even to the very ſphere of Saturn, and far beyond. Now ſince the air at a greater height will be ſtill immenſly more rarified, and the ſurface of the atmoſpheres of comets is uſually about ten times the diſtance from the center of the comet, as the ſurface of the comet it ſelf, and the tails are yet vaſtly farther removed from the center of the comet; the vapour, which compoſes thoſe tails, may very well be allowed to be ſo expanded, as that a moderate quantity of matter may fill all that ſpace, they are ſeen to take up. Though indeed the atmoſpheres of comets being

[a] Book I. Ch. 4. § 11.

very

very groſs, they will hardly be rarified in their tails to ſo great a degree, as our air under the ſame circumſtances; eſpecially ſince they may be ſomething condenſed, as well by their gravitation to the ſun, as that the parts will gravitate to one another; which will hereafter be ſhewn to be the univerſal property of all matter [a]. The only ſcruple left is, how ſo much light can be reflected from a vapour ſo rare, as this computation implies. For the removal of which our author obſerves, that the moſt refulgent of theſe tails hardly appear brighter, than a beam of the ſun's light tranſmitted into a darkened room through a hole of a ſingle inch diameter; and that the ſmalleſt fixed ſtars are viſible through them without any ſenſible diminution of their luſtre.

18. ALL theſe conſiderations put it beyond doubt, what is the true nature of the tails of comets. There has indeed nothing been ſaid, which will account for the irregular figures, in which thoſe tails are ſometimes reported to have appeared; but ſince none of thoſe appearances have ever been recorded by aſtronomers, who on the contrary aſcribe the ſame likeneſs to the tails of all comets, our author with great judgment refers all thoſe to accidental refractions by intervening clouds, or to parts of the milky way contiguous to the comets [b].

19. THE diſcuſſion of this appearance in comets has led Sir ISAAC NEWTON into ſome ſpeculations relating to their uſe, which I cannot but extreamly admire, as

[a] Ch. 5. [b] All theſe arguments are laid down in Philoſ. Nat. Princ. Lib. III. from p. 509, to 517.

representing in the strongest light imaginable the extensive providence of the great author of nature, who, besides the furnishing this globe of earth, and without doubt the rest of the planets, so abundantly with every thing necessary for the support and continuance of the numerous races of plants and animals, they are stocked with, has over and above provided a numerous train of comets, far exceeding the number of the planets, to rectify continually, and restore their gradual decay, which is our author's opinion concerning them [a]. For since the comets are subject to such unequal degrees of heat, being sometimes burnt with the most intense degree of it, at other times scarce receiving any sensible influence from the sun; it can hardly be supposed, they are designed for any such constant use, as the planets. Now the tails, which they emit, like all other kinds of vapour, dilate themselves as they ascend, and by consequence are gradually dispersed and scattered through all the planetary regions, and thence cannot but be gathered up by the planets, as they pass through their orbs: for the planets having a power to cause all bodies to gravitate towards them, as will in the sequel of this discourse be shewn [b]; these vapours will be drawn in process of time into this or the other planet, which happens to act strongest upon them. And by entering the atmospheres of the earth and other planets, they may well be supposed to contribute to the renovation of the face of things, in particular to supply the diminution caused in the humid parts

[a] Philos. Nat. Princ. Lib. III. p. 515. [b] Ch. 5.

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by vegetation and putrefaction. For vegetables are nouriſhed by moiſture, and by putrefaction are turned in great part into dry earth; and an earthy ſubſtance always ſubſides in fermenting liquors; by which means the dry parts of the planets muſt continually increaſe, and the fluids diminiſh, nay in a ſufficient length of time be exhauſted, if not ſupplied by ſome ſuch means. It is farther our great author's opinion, that the moſt ſubtile and active parts of our air, upon which the life of things chiefly depends, is derived to us, and ſupplied by the comets. So far are they from portending any hurt or miſchief to us, which the natural fears of men are ſo apt to ſuggeſt from the appearance of any thing uncommon and aſtoniſhing.

20. THAT the tails of comets have ſome ſuch important uſe ſeems reaſonable, if we conſider, that thoſe bodies do not ſend out thoſe fumes merely by their near approach to the ſun; but are framed of a texture, which diſpoſes them in a particular manner to fume in that ſort: for the earth, without emitting any ſuch ſteam, is more than half the year at a leſs diſtance from the ſun, than the comet of 1664 and 1665 approached it, when neareſt; likewiſe the comets of 1682 and 1683 never approached the ſun much above a ſeventh part nearer than Venus, and were more than half as far again from the ſun as Mercury; yet all theſe emitted tails.

21. FROM the very near approach of the comet of 1680 our author draws another ſpeculation; for if the

ſun

ſun have an atmoſphere about it, the comet mentioned ſeems to have deſcended near enough to the ſun to enter within it. If ſo, it muſt have been ſomething retarded by the reſiſtance it would meet with, and conſequently in its next deſcent to the ſun will fall nearer than now; by which means it will meet with a greater reſiſtance, and be again more retarded. The event of which muſt be, that at length it will impinge upon the ſun's ſurface, and thereby ſupply any decreaſe, which may have happened by ſo long an emiſſion of light, or otherwiſe. And ſomething like this our author conjectures may be the caſe of thoſe fixed ſtars, which by an additional increaſe of their luſtre have for a certain time become viſible to us, though uſually they are out of ſight. There is indeed a kind of fixed ſtars, which appear and diſappear at regular and equal intervals: here ſome more ſteady cauſe muſt be ſought for; perhaps theſe ſtars turn round their own axis's, as our ſun does [a], and have ſome part of their body more luminous than the other, whereby they are ſeen, when the moſt lucid part is next to us, and when the darker part is turned toward us, they vaniſh out of ſight.

22. WHETHER the ſun does really diminiſh, as has been here ſuggeſted, is difficult to prove; yet that it either does ſo, or that the earth increaſes, if not both, is rendered probable from Dr. HALLEY's obſervation [b], that by comparing

[a] See Ch. 1. § 11.

[b] Newt. Princ. Philoſ. pag. 525, 526. An account of all the ſtars of both theſe kinds, which have appeared within the laſt 150 years may be ſeen in the Philoſophical tranſactions, vol. 29. numb. 346.

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the proportion, which the periodical time of the moon bore to that of the ſun in former times, with the proportion between them at preſent, the moon is found to be ſomething accelerated in reſpect of the ſun. But if the ſun diminiſh, the periods of the primary planets will be lengthened; and if the earth be encreaſed, the period of the moon will be ſhortened: as will appear by the next chapter, wherein it ſhall be ſhewn, that the power of the ſun and earth is the reſult of the ſame power being lodg'd in all their parts, and that this principle of producing gravitation in other bodies is proportional to the ſolid matter in each body.

CHAP. V.

Of the BODIES of the SUN and PLANETS.

OUR author, after having diſcovered that the celeſtial motions are performed by a force extended from the ſun and primary planets, follows this power into the deepeſt receſſes of thoſe bodies themſelves, and proves the ſame to accompany the ſmalleſt particle, of which they are compoſed.

2. PREPARATIVE hereto he ſhews firſt, that each of the heavenly bodies attracts the reſt, and all bodies, with ſuch different degrees of force, as that the force of the ſame at-

tracting body is exerted on others exactly in proportion to the quantity of matter in the body attracted [a].

3. Of this the first proof he brings is from experiments made here upon the earth. The power by which the moon is influenced was above shewn to be the same, with that power here on the surface of the earth, which we call gravity [b]. Now one of the effects of the principle of gravity is, that all bodies descend by this force from the same height in equal times. Which has been long taken notice of; particular methods having been invented to shew that the only cause, why some bodies were observed to fall from the same height sooner than others, was the resistance of the air. This we have above related [c]; and proved from hence, that since bodies resist to any change of their state from rest to motion, or from motion to rest, in proportion to the quantity of matter contained in them; the power that can move different quantities of matter equally, must be proportional to the quantity. The only objection here is, that it can hardly be made certain, whether this proportion in the effect of gravity on different bodies holds perfectly exact or not from these experiments; by reason that the great swiftness, with which bodies fall, prevents our being able to determine the times of their descent with all the exactness requisite. Therefore to remedy this inconvenience, our author substitutes another more certain experiment in the room of these made upon falling bodies. Pen-

[a] Newt. Princ. Philos Nat. Lib. III. prop. 6. [b] Ch. 3. § 6. [c] Book I. Ch. 2. § 24.

dulums are caused to vibrate by the same principle, as makes bodies descend; the power of gravity putting them in motion, as well as the other. But if the ball of any pendulum, of the same length with another, were more or less attracted in proportion to the quantity of solid matter in the ball, that pendulum must accordingly move faster or slower than the other. Now the vibrations of pendulums continue for a great length of time, and the number of vibrations they make may easily be determined without suspicion of error; so that this experiment may be extended to what exactness one pleases: and our author assures us, that he examined in this way several substances, as gold, silver, lead, glass, sand, common salt, wood, water, and wheat; in all which he found not the least deviation from the proportion mentioned, though he made the experiment in such a manner, that in bodies of the same weight a difference in the quantity of their matter less than a thousandth part of the whole would have discovered it self [a]. It appears therefore, that all bodies are made to descend by the power of gravity here, near the surface of the earth, with the same degree of swiftness. We have above observed this descent to be after the rate of 16 $\frac{1}{2}$ feet in the first second of time from the beginning of their fall. Moreover it was also observed, that if any body, which fell here at the surface of the earth after this rate, were to be conveyed up to the height of the moon, it would

[a] Newt. Princ. Lib. III. prop. 6

defcend from thence juft with the fame degree of velocity, as that with which the moon is attracted toward the earth; and therefore the power of the earth upon the moon bears the fame proportion to the power it would have upon thofe bodies at the fame diftance, as the quantity of matter in the moon bears to the quantity in thofe bodies.

4. Thus the affertion laid down is proved in the earth, that the power of the earth on every body it attracts is, at the fame diftance from the earth, proportional to the quantity of folid matter in the body acted on. As to the fun, it has been fhewn, that the power of the fun's action upon the fame primary planet is reciprocally in the duplicate proportion of the diftance; and that the power of the fun decreafes throughout in the fame proportion, the motion of comets traverfing the whole planetary region teftifies. This proves, that if any planet were removed from the fun to any other diftance whatever, the degree of its acceleration toward the fun would yet remain reciprocally in the duplicate proportion of its diftance. But it has likewife been fhewn, that the degree of acceleration, which the fun gives to every one of the planets, is reciprocally in the duplicate proportion of their refpective diftances. All which compared together puts it out of doubt, that the power of the fun upon any planet, removed into the place of any other, would give it the fame velocity of defcent, as it gives that other; and confequently, that the fun's action upon different planets at the fame diftance would be proportional to the quantity of matter in each. It has farther

been

been ſhewn, that the ſun attracts the primary planets, and their reſpective ſecondary, when at the ſame diſtance, ſo as to communicate to both the ſame degree of velocity; and therefore the force, wherewith the ſun acts on the ſecondary planet, bears the ſame proportion to the force, wherewith at the ſame diſtance it attracts the primary, as the quantity of ſolid matter in the ſecondary planet bears to the quantity of matter in the primary.

5. This property therefore is proved of both kinds of planets, in reſpect of the ſun. Therefore the ſun poſſeſſes the quality found in the earth, of acting on bodies with a degree of force proportional to the quantity of matter in the body, which receives the influence.

6. That the power of attraction, with which the other planets are endued, ſhould differ from that of the earth, can hardly be ſuppoſed, if we conſider the ſimilitude between thoſe bodies; and that it does not in this reſpect, is farther proved from the ſatellites of Saturn and Jupiter, which are attracted by their reſpective primary according to the ſame law, that is, in the ſame proportion to their diſtances, as the primary are attracted by the ſun: ſo that what has been concluded of the ſun in relation to the primary planets, may be juſtly concluded of theſe primary in reſpect of their ſecondary, and in conſequence of that, in regard likewiſe to all other bodies, viz. that they will attract every body in proportion to the quantity of ſolid matter it contains.

7. HENCE it follows, that this attraction extends itſelf to every particle of matter in the attracted body: and that no portion of matter whatever is exempted from the influence of thoſe bodies, to which we have proved this attractive power to belong.

8. BEFORE we proceed farther, we may here remark, that this attractive power both of the ſun and planets now appears to be quite of the ſame nature in all; for it acts in each in the ſame proportion to the diſtance, and in the ſame manner acts alike upon every particle of matter. This power therefore in the ſun and other planets is not of a different nature from this power in the earth; which has been already ſhewn to be the ſame with that, which we call gravity [a].

9. AND this lays open the way to prove, that the attracting power lodged in the ſun and planets, belongs likewiſe to every part of them: and that their reſpective powers upon the ſame body are proportional to the quantity of matter, of which they are compoſed; for inſtance, that the force with which the earth attracts the moon, is to the force, with which the ſun would attract it at the ſame diſtance, as the quantity of ſolid matter contained in the earth, to the quantity contained in the ſun [b].

10. THE firſt of theſe aſſertions is a very evident conſequence from the latter. And before we proceed to the proof,

[a] Ch. 3. § 6. [b] Newt. Princ. philoſ. Lib. III. prop. 7. cor. 1.

it

it muſt firſt be ſhewn, that the third law of motion, which makes action and reaction equal, holds in theſe attractive powers. The moſt remarkable attractive force, next to the power of gravity, is that, by which the loadſtone attracts iron. Now if a loadſtone were laid upon water, and ſupported by ſome proper ſubſtance, as wood or cork, ſo that it might ſwim; and if a piece of iron were cauſed to ſwim upon the water in like manner: as ſoon as the loadſtone begins to attract the iron, the iron ſhall move toward the ſtone, and the ſtone ſhall alſo move toward the iron; when they meet, they ſhall ſtop each other, and remain fixed together without any motion. This ſhews, that the velocities, wherewith they meet, are reciprocally proportional to the quantities of ſolid matter in each; and that by the ſtone's attracting the iron, the ſtone itſelf receives as much motion, in the ſtrict philoſophic ſenſe of that word[a], as it communicates to the iron: for it has been declared above to be an effect of the percuſſion of two bodies, that if they meet with velocities reciprocally proportional to the reſpective bodies, they ſhall be ſtopped by the concourſe, unleſs their elaſticity put them into freſh motion; but if they meet with any other velocities, they ſhall retain ſome motion after meeting[b]. Amber, glaſs, ſealing-wax, and many other ſubſtances acquire by rubbing a power, which from its having been remarkable, particularly in amber, is called electrical. By this power they will for ſome time after

[a] See Book I. Ch. 1. § 25. [b] Ibid. § 5, 6.

rubbing

rubbing attract light bodies, that ſhall be brought within the ſphere of their activity. On the other hand Mr. Boyle found, that if a piece of amber be hung in a perpendicular poſition by a ſtring, it ſhall be drawn itſelf toward the body whereon it was rubbed, if that body be brought near it. Both in the loadſtone and in electrical bodies we uſually aſcribe the power to the particular body, whoſe preſence we find neceſſary for producing the effect. The loadſtone and any piece of iron will draw each other, but in two pieces of iron no ſuch effect is ordinarily obſerved; therefore we call this attractive power the power of the loadſtone: though near a loadſtone two pieces of iron will alſo draw each other. In like manner the rubbing of amber, glaſs, or any ſuch body, till it is grown warm, being neceſſary to cauſe any action between thoſe bodies and other ſubſtances, we aſcribe the electrical power to thoſe bodies. But in all theſe caſes if we would ſpeak more correctly, and not extend the ſenſe of our expreſſions beyond what we ſee; we can only ſay that the neighbourhood of a loadſtone and a piece of iron is attended with a power, whereby the loadſtone and the iron are drawn toward each other; and the rubbing of electrical bodies gives riſe to a power, whereby thoſe bodies and other ſubſtances are mutually attracted. Thus we muſt alſo underſtand in the power of gravity, that the two bodies are mutually made to approach by the action of that power. When the ſun draws any planet, that planet alſo draws the ſun; and the motion, which the planet receives from the ſun, bears the ſame proportion to the motion, which the ſun it ſelf receives, as

the quantity of ſolid matter in the ſun bears to the quantity of ſolid matter in the planet. Hitherto, for brevity ſake in ſpeaking of theſe forces, we have generally aſcribed them to the body, which is leaſt moved; as when we called the power, which exerts itſelf between the ſun and any planet, the attractive power of the ſun; but to ſpeak more correctly, we ſhould rather call this power in any caſe the force, which acts between the ſun and earth, between the ſun and Jupiter, between the earth and moon, &c. for both the bodies are moved by the power acting between them, in the ſame manner, as when two bodies are tied together by a rope, if that rope ſhrink by being wet, or otherwiſe, and thereby cauſe the bodies to approach, by drawing both, it will communicate to both the ſame degree of motion, and cauſe them to approach with velocities reciprocally proportional to the reſpective bodies. From this mutual action between the ſun and planet it follows, as has been obſerved above [a], that the ſun and planet do each move about their common center of gravity. Let A (in fig. 108.) repreſent the ſun, B a planet, C their common center of gravity. If theſe bodies were once at reſt, by their mutual attraction they would directly approach each other with ſuch velocities, that their common center of gravity would remain at reſt, and the two bodies would at length meet in that point. If the planet B were to receive an impulſe, as in the direction of the line DF, this would prevent the two bodies from falling together;

[a] Chap. 2. § 8.

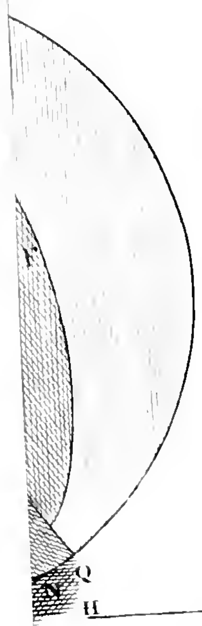
but

but their common center of gravity would be put into motion in the direction of the line CF equidiſtant from BE. In this caſe Sir ISAAC NEWTON proves[a], that the ſun and planet would deſcribe round their common center of gravity ſimilar orbits, while that center would proceed with an uniform motion in the line CF; and ſo the ſyſtem of the two bodies would move on with the center of gravity without end. In order to keep the ſyſtem in the ſame place, it is neceſſary, that when the planet received its impulſe in the direction BE, the ſun ſhould alſo receive ſuch an impulſe the contrary way, as might keep the center of gravity C without motion; for if theſe began once to move without giving any motion to their common center of gravity, that center would always remain fixed.

11. By this may be underſtood in what manner the action between the ſun and planets is mutual. But farther, we have ſhewn above[b], that the power, which acts between the ſun and primary planets, is altogether of the ſame nature with that, which acts between the earth and the bodies at its ſurface, or between the earth and its parts, and with that which acts between the primary planets and their ſecondary; therefore all theſe actions muſt be aſcribed to the ſame cauſe[c]. Again, it has been already proved, that in different planets the force of the ſun's action upon each at the ſame diſtance would be proportional to the quantity of ſolid matter in the planet[d]; therefore the reaction of each planet

[a] Newt. Princ. Lib. I. prop. 63. [b] § 8. [c] See Introd. § 23. [d] § 4, 5.

on



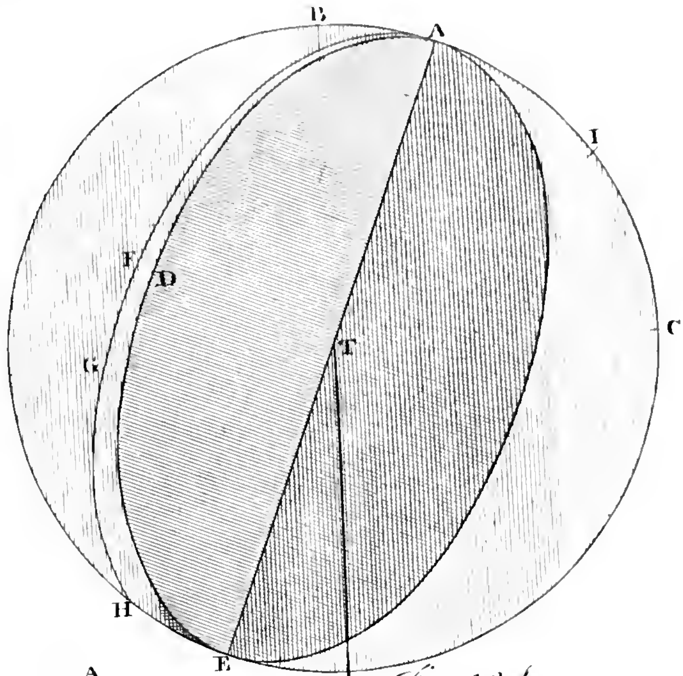


Fig. 104.

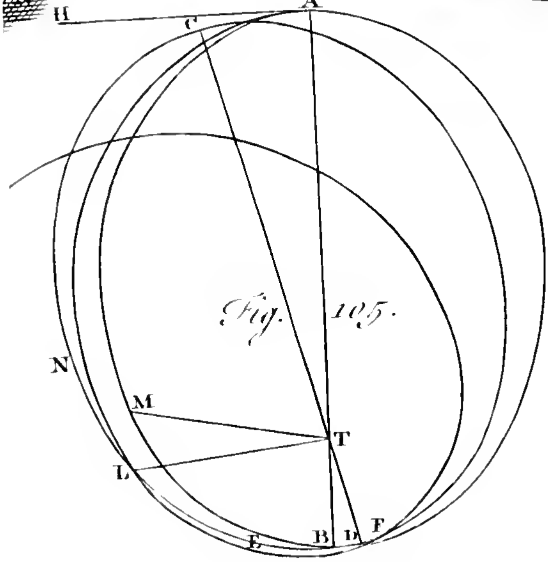


Fig. 105.

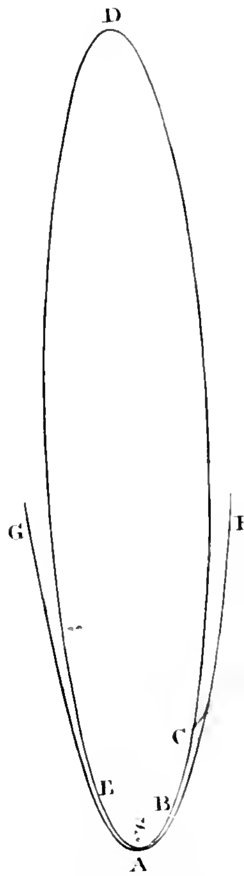


Fig. 107.

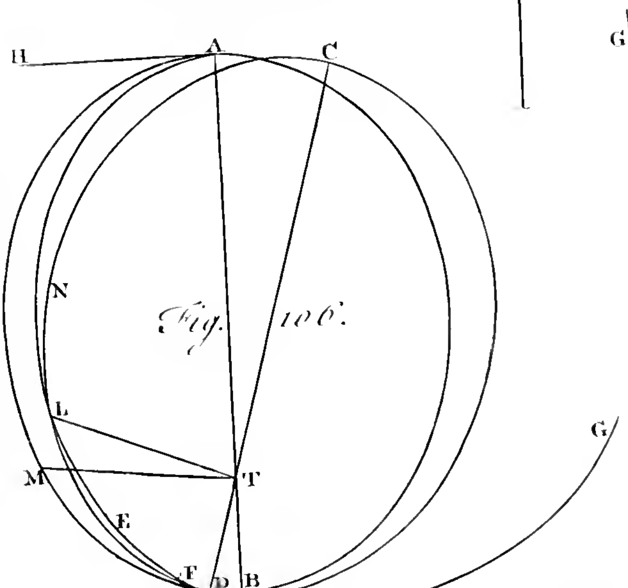


Fig. 106.

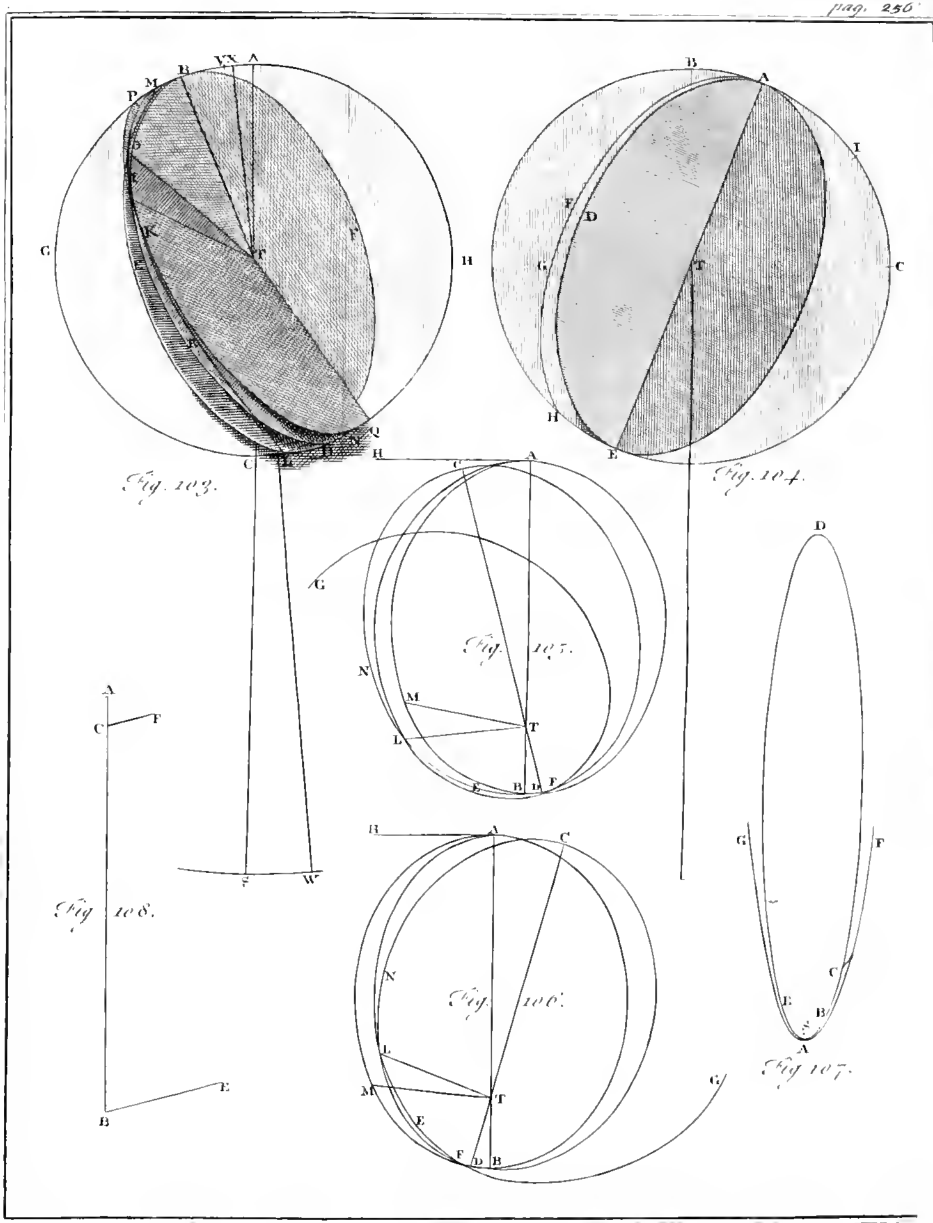


Fig. 103.

Fig. 104.

Fig. 105.

Fig. 106.

Fig. 107.

Fig. 108.

on the ſun at the ſame diſtance, or the motion, which the ſun would receive from each planet, would alſo be proportional to the quantity of matter in the planet; that is, theſe planets at the ſame diſtance would act on the ſame body with degrees of ſtrength proportional to the quantity of ſolid matter in each.

12. IN the next place, from what has been now proved, our great author has deduced this farther conſequence, no leſs ſurprizing than elegant; that each of the particles, out of which the bodies of the ſun and planets are framed, exert their power of gravitation by the ſame law, and in the ſame proportion to the diſtance, as the great bodies which they compoſe. For this purpoſe he firſt demonſtrates, that if a globe were compounded of particles, which will attract the particles of any other body reciprocally in the duplicate proportion of their diſtances, the whole globe will attract the ſame in the reciprocal duplicate proportion of their diſtances from the center of the globe; provided the globe be of uniform denſity throughout [a]. And from this our author deduces the reverſe, that if a globe acts upon diſtant bodies by the law juſt now ſpecified, and the power of the globe is derived from its being compoſed of attractive particles; each of thoſe particles will attract after the ſame proportion [b]. The manner of deducing this is not ſet down at large by our author, but is as follows. The globe is

[a] Newt. Princ. philoſ. Lib. I. prop. 74.

[b] Ibid. coroll. 3.

ſuppoſed to act upon the particles of a body without it conſtantly in the reciprocal duplicate proportion of their diſtances from its center; and therefore at the ſame diſtance from the globe, on which ſide ſoever the body be placed, the globe will act equally upon it. Now becauſe, if the particles, of which the globe is compoſed, acted upon thoſe without in the reciprocal duplicate proportion of their diſtances, the whole globe would act upon them in the ſame manner as it does; therefore, if the particles of the globe have not all of them that property, ſome muſt act ſtronger than in that proportion, while others act weaker: and if this be the condition of the globe, it is plain, that when the body attracted is in ſuch a ſituation in reſpect of the globe, that the greater number of the ſtrongeſt particles are neareſt to it, the body will be more forcibly attracted; than when by turning the globe about, the greater quantity of weak particles ſhould be neareſt, though the diſtance of the body ſhould remain the ſame from the center of the globe. Which is contrary to what was at firſt remarked, that the globe on all ſides of it acts with the ſame ſtrength at the ſame diſtance. Whence it appears, that no other conſtitution of the globe can agree to it.

13. From theſe propoſitions it is farther collected, that if all the particles of one globe attract all the particles of another in the proportion ſo often mentioned, the attracting globe will act upon the other in the ſame proportion to the diſtance between the center of the globe which attracts, and the center of that which is attracted [a]: and farther, that this

[a] Lib. I. Prop. 75. and Lib. III. prop. 8.

proportion

proportion holds true, though either or both the globes be compoſed of diſſimilar parts, ſome rarer and ſome more denſe; provided only, that all the parts in the ſame globe equally diſtant from the center be homogeneous [a]. And alſo, if both the globes attract each other [b]. All which place it beyond contradiction, that this proportion obtains with as much exactneſs near and contiguous to the ſurface of attracting globes, as at greater diſtances from them.

14. THUS our author, without the pompous pretence of explaining the cauſe of gravity, has made one very important ſtep toward it, by ſhewing that this power in the great bodies of the univerſe, is derived from the ſame power being lodged in every particle of the matter which compoſes them: and conſequently, that this property is no leſs than univerſal to all matter whatever, though the power be too minute to produce any viſible effects on the ſmall bodies, wherewith we converſe, by their action on each other [c]. In the fixed ſtars indeed we have no particular proof that they have this power; for we find no apperance to demonſtrate that they either act, or are acted upon by it. But ſince this power is found to belong to all bodies, whereon we can make obſervation; and we ſee that it is not to be altered by any change in the form of bodies, but always accompanies them in every ſhape without diminution, remaining ever proportional to the quantity of ſolid matter in each; ſuch a power muſt without doubt belong univerſally to all matter.

[a] Lib. I Prop. 76. [b] Ibid. cor. 5. [c] Vid Lib. III. Prop. 7. coroll. 1.

15. This therefore is the univerſal law of matter; which recommends it ſelf no leſs for its great plainneſs and ſimplicity, than for the ſurprizing diſcoveries it leads us to. By this principle we learn the different weight, which the ſame body will have upon the ſurfaces of the ſun and of diverſe planets; and by the ſame we can judge of the compoſition of thoſe celeſtial bodies, and know the denſity of each; which is formed of the moſt compact, and which of the moſt rare ſubſtance. Let the adverſaries of this philoſophy reflect here, whether loading this principle with the appellation of an occult quality, or perpetual miracle, or any other reproachful name, be ſufficient to diſſuade us from cultivating it; ſince this quality, which they call occult, leads to the knowledge of ſuch things, that it would have been reputed no leſs than madneſs for any one, before they had been diſcovered, even to have conjectured that our faculties ſhould ever have reached ſo far.

16. See how all this naturally follows from the foregoing principles in thoſe planets, which have ſatellites moving about them. By the times, in which theſe ſatellites perform their revolutions, compared with their diſtances from their reſpective primary, the proportion between the power, with which one primary attracts his ſatellites, and the force with which any other attracts his will be known; and the proportion of the power with which any planet attracts its ſecondary, to the power with which it attracts a body at its ſurface is found, by comparing the diſtance of the ſecondary planet from the center of the primary, to the

the diftance of the primary planet's furface from the fame: and from hence is deduced the proportion between the power of gravity upon the furface of one planet, to the gravity upon the furface of another. By the like method of comparing the periodical time of a primary planet about the fun, with the revolution of a fatellite about its primary, may be found the proportion of gravity, or of the weight of any body upon the furface of the fun, to the gravity, or to the weight of the fame body upon the furface of the planet, which carries about the fatellite.

17. By thefe kinds of computation it is found, that the weight of the fame body upon the furface of the fun will be about 23 times as great, as here upon the furface of the earth; about $10\frac{3}{5}$ times as great, as upon the furface of Jupiter; and near 19 times as great, as upon the furface of Saturn [a].

18. The quantity of matter, which compofes each of thefe bodies, is proportional to the power it has upon a body at a given diftance. By this means it is found, that the fun contains 1067 times as much matter as Jupiter; Jupiter $158\frac{2}{3}$ times as much as the earth, and $2\frac{5}{6}$ times as much as Saturn [b]. The diameter of the fun is about 92 times, that of Jupiter about 9 times, and that of Saturn about 7 times the diameter of the earth.

[a] Newt. Princ. Lib. III. prop. 8. coroll. 1. [b] Ibid. coroll. 2.

19. By

19. By making a comparison between the quantity of matter in these bodies and their magnitudes, to be found from their diameters, their respective densities are readily deduced; the density of every body being measured by the quantity of matter contained under the same bulk, as has been above remarked [a]. Thus the earth is found $4\frac{1}{4}$ times more dense than Jupiter; Saturn has between $\frac{2}{3}$ and $\frac{3}{4}$ of the density of Jupiter; but the sun has one fourth part only of the density of the earth [b]. From which this observation is drawn by our author; that the sun is rarified by its great heat, and that of the three planets named, the more dense is nearer the sun than the more rare; as was highly reasonable to expect, the densest bodies requiring the greatest heat to agitate and put their parts in motion; as on the contrary, the planets which are more rare, would be rendered unfit for their office, by the intense heat to which the denser are exposed. Thus the waters of our seas, if removed to the distance of Saturn from the sun, would remain perpetually frozen; and if as near the sun as Mercury, would constantly boil [c].

20. The densities of the three planets Mercury, Venus, and Mars, which have no satellites, cannot be expresly assigned; but from what is found in the others, it is very probable, that they also are of such different degrees of density, that universally the planet which is nearest to the sun, is formed of the most compact substance.

[a] Book I. Ch. 4. § 2. [b] Newt. Princ. Lib. III. prop. 8. coroll. 3. [c] Ibid. coroll. 4.

CHAP.

CHAP. VI.

Of the FLUID PARTS of the PLANETS.

THIS globe, that we inhabit, is compofed of two parts; the folid earth, which affords us a foundation to dwell upon; and the feas and other waters, that furnifh rains and vapours neceffary to render the earth fruitful, and productive of what is requifite for the fupport of life. And that the moon, though but a fecondary planet, is compofed in like manner, is generally thought, from the different degrees of light which appear on its furface; the parts of that planet, which reflect a dim light, being fuppofed to be fluid, and to imbibe the fun's rays, while the folid parts reflect them more copioufly. Some indeed do not allow this to be a conclufive argument: but whether we can diftinguifh the fluid part of the moon's furface from the reft or not; yet it is moft probable that there are two fuch different parts, and with ftill greater reafon we may afcribe the like to the other primary planets, which yet more nearly refemble our earth. The earth is alfo encompaffed by another fluid the air, and we have before remarked, that probably the reft of the planets are furrounded by the like. Thefe fluid parts in particular engage our author's attention, both by reafon of fome remarkable appearances peculiar to them, and likewife of fome effects they have upon the whole bodies to which they belong.

2. FLUIDS have been already treated of in general, with respect to the effect they have upon solid bodies moving in them [a]; now we must consider them in reference to the operation of the power of gravity upon them. By this power they are rendered weighty, like all other bodies, in proportion to the quantity of matter, which is contained in them. And in any quantity of a fluid the upper parts press upon the lower as much, as any solid body would press on another, whereon it should lie. But there is an effect of the pressure of fluids on the bottom of the vessel, wherein they are contained, which I shall particularly explain. The force supported by the bottom of such a vessel is not simply the weight of the quantity of the fluid in the vessel, but is equal to the weight of that quantity of the fluid, which would be contained in a vessel of the same bottom and of equal width throughout, when this vessel is filled up to the same height, as that to which the vessel proposed is filled. Suppose water were contained in the vessel ABCD (in fig. 109.) filled up to EF. Here it is evident, that if a part of the bottom, as GH, which is directly under any part of the space EF, be considered separately; it will appear at once, that this part sustains the weight of as much of the fluid, as stands perpendicularly over it up to the height of EF; that is, the two perpendiculars GI and HK being drawn, the part GH of the bottom will sustain the whole weight of the fluid included between these two perpendiculars. Again, I say, every other part of the bottom equally broad with this, will sustain as great a pressure. Let the part LM be of the

[a] Book I Ch. 4.

same

ſame breadth with G H. Here the perpendiculars L O and M N being drawn, the quantity of water contained between theſe perpendiculars is not ſo great, as that contained between the perpendiculars G I and H K; yet, I ſay, the preſſure on L M will be equal to that on G H. This will appear by the following conſiderations. It is evident, that if the part of the veſſel between O and N were removed, the water would immediately flow out, and the ſurface E F would ſubſide; for all parts of the water being equally heavy, it muſt ſoon form itſelf to a level ſurface, if the form of the veſſel, which contains it, does not prevent. Therefore ſince the water is prevented from riſing by the ſide N O of the veſſel, it is manifeſt, that it muſt preſs againſt N O with ſome degree of force. In other words, the water between the perpendiculars L O and M N endeavours to extend itſelf with a certain degree of force; or more correctly, the ambient water preſſes upon this, and endeavours to force this pillar or column of water into a greater length. But ſince this column of water is ſuſtained between N O and L M, each of theſe parts of the veſſel will be equally preſſed againſt by the power, wherewith this column endeavours to extend. Conſequently L M bears this force over and above the weight of the column of water between L O and M N. To know what this expanſive force is, let the part O N of the veſſel be removed, and the perpendiculars L O and M N be prolonged; then by means of ſome pipe fixed over N O let water be filled between theſe perpendiculars up to P Q an equal height with E F. Here the water between the perpendiculars L P and M Q is of an equal height with the higheſt part of the water in the veſſel; therefore the water in the

veſſel cannot by its preſſure force it up higher, nor can the water in this column ſubſide; becauſe, if it ſhould, it would raiſe the water in the veſſel to a greater height than itſelf. But it follows from hence, that the weight of water contained between PO and QN is a juſt balance to the force, wherewith the column between LO and MN endeavours to extend. So the part LM of the bottom, which ſuſtains both this force and the weight of the water between LO and MN, is preſſed upon by a force equal to the united weight of the water between LO and MN, and the weight of the water between PO and QN; that is, it is preſſed on by a force equal to the weight of all the water contained between LP and MQ. And this weight is equal to that of the water contained between GI and HK, which is the weight ſuſtained by the part GH of the bottom. Now this being true of every part of the bottom BC, it is evident, that if another veſſel RSTV be formed with a bottom RV equal to the bottom BC, and be throughout its whole height of one and the ſame breadth; when this veſſel is filled with water to the ſame height, as the veſſel ABCD is filled, the bottoms of theſe two veſſels ſhall be preſſed upon with equal force. If the veſſel be broader at the top than at the bottom, it is evident, that the bottom will bear the preſſure of ſo much of the fluid, as is perpendicularly over it, and the ſides of the veſſel will ſupport the reſt. This property of fluids is a corollary from a propoſition of our author [a]; from whence alſo he deduces the effects of the preſſure of fluids on bodies reſting in them.

[a] Lib. II. prop. 20. cor. 2.

Theſe

Theſe are, that any body heavier than a fluid will ſink to the bottom of the veſſel, wherein the fluid is contained, and in the fluid will weigh as much as its own weight exceeds the weight of an equal quantity of the fluid; any body uncompreſſible of the ſame denſity with the fluid, will reſt any where in the fluid without ſuffering the leaſt change either in its place or figure from the preſſure of ſuch a fluid, but will remain as undiſturbed as the parts of the fluid themſelves; but every body of leſs denſity than the fluid will ſwim on its ſurface, a part only being received within the fluid. Which part will be equal in bulk to a quantity of the fluid, whoſe weight is equal to the weight of the whole body; for by this means the parts of the fluid under the body will ſuffer as great a preſſure as any other parts of the fluid as much below the ſurface as theſe.

3. IN the next place, in relation to the air, we have above made mention, that the air ſurrounding the earth being an elaſtic fluid, the power of gravity will have this effect on it, to make the lower parts near the ſurface of the earth more compact and compreſſed together by the weight of the air incumbent, than the higher parts, which are preſſed upon by a leſs quantity of the air, and therefore ſuſtain a leſs weight [a]. It has been alſo obſerved, that our author has laid down a rule for computing the exact degree of denſity in the air at all heights from the earth [b]. But there is a farther effect from the air's being compreſſed by

[a] Chap. 4. § 17. [b] Ibid.

the power of gravity, which he has diſtinctly conſidered. The air being elaſtic and in a ſtate of compreſſion, any tremulous body will propagate its motion to the air, and excite therein vibrations, which will ſpread from the body that occaſions them to a great diſtance. This is the efficient cauſe of ſound: for that ſenſation is produced by the air, which, as it vibrates, ſtrikes againſt the organ of hearing. As this ſubject was extremely difficult, ſo our great author's ſucceſs is ſurprizing.

4. Our author's doctrine upon this head I ſhall endeavour to explain ſomewhat at large. But preliminary thereto muſt be ſhewn, what he has delivered in general of preſſure propagated through fluids; and alſo what he has ſet down relating to that wave-like motion, which appears upon the ſurface of water, when agitated by throwing any thing into it, or by the reciprocal motion of the finger, &c.

5. Concerning the firſt, it is proved, that preſſure is ſpread through fluids, not only right forward in a ſtreight line, but alſo laterally, with almoſt the ſame eaſe and force. Of which a very obvious exemplification by experiment is propoſed: that is, to agitate the ſurface of water by the reciprocal motion of the finger forwards and backwards only; for though the finger have no circular motion given it, yet the waves excited in the water will diffuſe themſelves on each hand of the direction of the motion, and ſoon ſurround the finger. Nor is what we obſerve in ſounds unlike to this, which do not proceed in ſtraight lines only, but are heard though a

mountain intervene, and when they enter a room in any part of it, they ſpread themſelves into every corner; not by reflection from the walls, as ſome have imagined, but as far as the ſenſe can judge, directly from the place where they enter.

6. How the waves are excited in the ſurface of ſtagnant water, may be thus conceived. Suppoſe in any place, the water raiſed above the reſt in form of a ſmall hillock; that water will immediately ſubſide, and raiſe the circumambient water above the level of the parts more remote, to which the motion cannot be communicated under longer time. And again, the water in ſubſiding will acquire, like all falling bodies, a force, which will carry it below the level ſurface, till at length the preſſure of the ambient water prevailing, it will riſe again, and even with a force like to that wherewith it deſcended, which will carry it again above the level. But in the mean time the ambient water before raiſed will ſubſide, as this did, ſinking below the level; and in ſo doing, will not only raiſe the water, which firſt ſubſided, but alſo the water next without itſelf. So that now beſide the firſt hillock, we ſhall have a ring inveſting it, at ſome diſtance raiſed above the plain ſurface likewiſe; and between them the water will be ſunk below the reſt of the ſurface. After this, the firſt hillock, and the new made annular riſing, will deſcend; raiſing the water between them, which was before depreſſed, and likewiſe the adjacent part of the ſurface without. Thus will theſe annular waves be ſucceſſively ſpread more and more. For, as the hillock ſubſiding produces one ring, and that ring ſub-

ſiding

ſiding raiſes again the hillock, and a ſecond ring; ſo the hillock and ſecond ring ſubſiding together raiſe the firſt ring, and a third; then this firſt and third ring ſubſiding together raiſe the firſt hillock, the ſecond ring, and a fourth; and ſo on continually, till the motion by degrees ceaſes. Now it is demonſtrated, that theſe rings aſcend and deſcend in the manner of a pendulum; deſcending with a motion continually accelerated, till they become even with the plain ſurface of the fluid, which is half the ſpace they deſcend; and then being retarded again by the ſame degrees as thoſe, whereby they were accelerated, till they are depreſſed below the plain ſurface, as much as they were before raiſed above it: and that this augmentation and diminution of their velocity proceeds by the ſame degrees, as that of a pendulum vibrating in a cycloid, and whoſe length ſhould be a fourth part of the diſtance between any two adjacent waves: and farther, that a new ring is produced every time a pendulum, whoſe length is four times the former, that is, equal to the interval between the ſummits of two waves, makes one oſcillation or ſwing [a].

7. THIS now opens the way for underſtanding the motion conſequent upon the tremors of the air, excited by the vibrations of ſonorous bodies: which we muſt conceive to be performed in the following manner.

8. LET A, B, C, D, E, F, G, H (in fig. 110.) repreſent a ſeries of the particles of the air, at equal diſtances from each other. I K L a muſical chord, which I ſhall uſe for the tre-

[a] Vid. Newt. Princ. Lib. II. prop. 46.

mulous

muleus and ſonorous body, to make the conception as ſimple as may be. Suppoſe this chord ſtretched upon the points I and L, and forcibly drawn into the ſituation I K L, ſo that it become contiguous to the particle A in its middle point K: and let the chord from this ſituation begin to recoil, preſſing againſt the particle A, which will thereby be put into motion towards B: but the particles A, B, C being equidiſtant, the elaſtic power, by which B avoids A, is equal to, and balanced by the power, by which it avoids C; therefore the elaſtic force, by which B is repelled from A, will not put B into any degree of motion, till A is by the motion of the chord brought nearer to B, than B is to C: but as ſoon as that is done, the particle B will be moved towards C; and being made to approach C, will in the next place move that; which will upon that advance, put D likewiſe into motion, and ſo on: therefore the particle A being moved by the chord, the following particles of the air B, C, D, &c. will ſucceſſively be moved. Farther, if the point K of the chord moves forward with an accelerated velocity, ſo that the particle A ſhall move againſt B with an advancing pace, and gain ground of it, approaching nearer and nearer continually; A by approaching will preſs more upon B, and give it a greater velocity likewiſe, by reaſon that as the diſtance between the particles diminiſhes, the elaſtic power, by which they fly each other, increaſes. Hence the particle B, as well as A, will have its motion gradually accelerated, and by that means will more and more approach to C. And from the ſame cauſe C will more and more approach D; and ſo of the reſt. Suppoſe now, ſince the agitation of theſe particles has been ſhewn to

be

be ſucceſſive, and to follow one another, that E be the remoteſt particle moved, while the chord is moving from its curve ſituation I K L into that of a ſtreight line, as I k L; and F the firſt which remains unaffected, though juſt upon the point of being put into motion. Then ſhall the particles A, B, C, D, E, F, G, when the point K is moved into k, have acquired the rangement repreſented by the adjacent points a, b, c, d, e, f, g: in which a is nearer to b than b to c, and b nearer to c than c to d, and c nearer to d than d to e, and d nearer to e than e to f, and laſtly e nearer to f than f to g.

9. But now the chord having recovered its rectilinear ſituation I k L, the following motion will be changed, for the point K, which before advanced with a motion more and more accelerated, though by the force it has acquired it will go on to move the ſame way as before, till it has advanced near as far forwards, as it was at firſt drawn backwards; yet the motion of it will henceforth be gradually leſſened. The effect of which upon the particles a, b, c, d, e, f, g will be, that by the time the chord has made its utmoſt advance, and is upon the return, theſe particles will be put into a contrary rangement; ſo that f ſhall be nearer to g, than e to f, and e nearer to f than d to e; and the like of the reſt, till you come to the firſt particles a, b, whoſe diſtance will then be nearly or quite what it was at firſt. All which will appear as follows. The preſent diſtance between a and b is ſuch, that the elaſtic power, by which a repels b, is ſtrong enough to maintain that diſtance, though a advance with the velocity, with which the ſtring reſumes its rectilinear figure; and the motion

motion of the particle *a* being afterwards ſlower, the preſent elaſticity between *a* and *b* will be more than ſufficient to preſerve the diſtance between them. Therefore while it accelerates *b* it will retard *a*. The diſtance *bc* will ſtill diminiſh, till *b* come about as near to *c*, as it is from *a* at preſent; for after the diſtances *ab* and *bc* are become equal, the particle *b* will continue its velocity ſuperior to that of *c* by its own power of inactivity, till ſuch time as the increaſe of elaſticity between *b* and *c* more than ſhall be between *a* and *b* ſhall ſuppreſs its motion: for as the power of inactivity in *b* made a greater elaſticity neceſſary on the ſide of *a* than on the ſide of *c* to puſh *b* forward, ſo what motion *b* has acquired it will retain by the ſame power of inactivity, till it be ſuppreſſed by a greater elaſticity on the ſide of *c*, than on the ſide of *a*. But as ſoon as *b* begins to ſlacken its pace the diſtance of *b* from *c* will widen as the diſtance *ab* has already done. Now as *a* acts on *b*, ſo will *b* on *c*, *c* on *d*, &c. ſo that the diſtances between all the particles *a*, *b*, *c*, *d*, *e*, *f*, *g* will be ſucceſſively contracted into the diſtance of *a* from *b*, and then dilated again. Now becauſe the time, in which the chord deſcribes this preſent half of its vibration, is about equal to that it took up in deſcribing the former; the particles *a*, *b* will be as long in dilating their diſtance, as before in contracting it, and will return nearly to their original diſtance. And farther, the particles *b*, *c*, which did not begin to approach ſo ſoon as *a*, *b*, are now about as much longer, before they begin to recede; and likewiſe the particles *c*, *d*, which began to approach after *b*, *c*, begin to ſeparate later. Whence it appears that the particles, whoſe diſtance began to be leſſened, when

that of a, b was firſt enlarged, viz. the particles f, g, ſhould be about their neareſt diſtance, when a and b have recovered their prime interval. Thus will the particles a, b, c, d, e, f, g have changed their ſituation in the manner aſſerted. But farther, as the particles f, g or F, G gradually approach each other, they will move by degrees the ſucceeding particles to as great a length, as the particles A, B did by a like approach. So that, when the chord has made its greateſt advance, being arrived into the ſituation I κ L, the particles moved by it will have the rangement noted by the points α, β, γ, δ, ϵ, ζ, η, θ, λ, μ, ν, ξ. Where α, β are at the original diſtance of the particles in the line AH; ζ, η are the neareſt of all, and the diſtance $\nu\xi$ is equal to that between α and β.

10. By this time the chord I κ L begins to return, and the diſtance between the particles α and β being enlarged to its original magnitude, α has loſt all that force it had acquired by its motion, being now at reſt; and therefore will return with the chord, making the diſtance between α and β greater than the natural; for β will not return ſo ſoon, becauſe its motion forward is not yet quite ſuppreſſed, the diſtance $\beta\gamma$ not being already enlarged to its prime dimenſion: but the receſs of α, by diminiſhing the preſſure upon β by its elaſticity, will occaſion the motion of β to be ſtopt in a little time by the action of γ, and then ſhall β begin to return: at which time the diſtance between γ and δ ſhall by the ſuperior action of δ above β be enlarged to the dimenſion of the diſtance $\beta\gamma$, and therefore ſoon after to that of $\alpha\beta$. Thus it appears, that each of theſe particles goes on to move forward, till its diſtance from the

the preceding one be equal to its original diſtance; the whole chain $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta$, having an undulating motion forward, which is ſtopt gradually by the exceſs of the expanſive power of the preceding parts above that of the hinder. Thus are theſe parts ſucceſſively ſtopt, as before they were moved; ſo that when the chord has regained its rectilinear ſituation, the expanſion of the parts of the air will have advanced ſo far, that the interval between $\zeta\,\eta$, which at preſent is moſt contracted, will then be reſtored to its natural ſize: the diſtances between η and θ, θ and λ, λ and μ, μ and ν, ν and ξ, being ſucceſſively contracted into the preſent diſtance of ζ from η, and again enlarged; ſo that the ſame effect ſhall be produced upon the parts beyond $\zeta\,\eta$, by the enlargement of the diſtance between thoſe two particles, as was occaſioned upon the particles $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \lambda, \mu, \nu, \xi$, by the enlargement of the diſtance $\alpha\,\beta$ to its natural extent. And therefore the motion in the air will be extended half as much farther as at preſent, and the diſtance between ν and ξ contracted into that, which is at preſent between ζ and η, all the particles of the air in motion taking the rangement expreſſed in figure 111. by the points $\alpha, \beta, \gamma, \delta, \epsilon, \zeta, \eta, \theta, \lambda, \mu, \nu, \xi, \pi, \rho, \sigma, \tau, \phi$; wherein the particles from α to ξ have their diſtances from each other gradually diminiſhed, the diſtances between the particles ν, ξ being contracted the moſt from the natural diſtance between thoſe particles, and the diſtance between α, β as much augmented, and the diſtance between the middle particles ζ, η becoming equal to the natural. The particles $\pi, \rho, \sigma, \tau, \phi$, which follow ξ, have their diſtances gradually greater

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and greater, the particles ν, ξ, π, ρ, σ, τ, φ being ranged like the particles *a*, *b*, *c*, *d*, *e*, *f*, *g*, or like the particles ζ, η, θ, λ, μ, ν, ξ in the former figure. Here it will be underſtood, by what has been before explained, that the particles ζ, η being at their natural diſtance from each other, the particle ζ is at reſt, the particles ε, δ, γ, β, α between them and the ſtring being in motion backward, and the reſt of the particles η, θ, λ, μ, ν, ξ, π, ρ, σ, τ in motion forward: each of the particles between η and ξ moving faſter than that, which immediately follows it; but of the particles from ξ to φ, on the contrary, thoſe behind moving on faſter than thoſe, which precede.

11. But now the ſtring having recovered its rectilinear figure, though it ſhall go on recoiling, till it return near to its firſt ſituation IKL, yet there will be a change in its motion; ſo that whereas it returned from the ſituation IκL with an accelerated motion, its motion ſhall from hence be retarded again by the ſame degrees, as accelerated before. The effect of which change upon the particles of the air will be this. As by the accelerated motion of the chord α contiguous to it moved faſter than β, ſo as to make the interval $\alpha\beta$ greater than the interval $\beta\gamma$, and from thence β was made likewiſe to move faſter than γ, and the diſtance between β and γ rendered greater than the diſtance between γ and δ, and ſo of the reſt; now the motion of α being diminiſhed, β ſhall overtake it, and the diſtance between α and β be reduced into that, which is at preſent between β and γ, the interval between β and γ being inlarged into the preſent

ſent diſtance between α and β; but when the interval $\beta\gamma$ is increaſed to that, which is at preſent between α and β, the diſtance between γ and δ ſhall be enlarged to the preſent diſtance between γ and β, and the diſtance between δ and ϵ inlarged into the preſent diſtance between γ and δ; and the ſame of the reſt. But the chord more and more ſlackening its pace, the diſtance between α and β ſhall be more and more diminiſhed; and in conſequence of that the diſtance between β and γ ſhall be again contracted, firſt into its preſent dimenſion, and afterwards into a narrower ſpace; while the interval $\gamma\delta$ ſhall dilate into that at preſent between α and β, and as ſoon as it is ſo much enlarged, it ſhall contract again. Thus by the reciprocal expanſion and contraction of the air between α and ζ, by that time the chord is got into the ſituation I K L, the interval $\zeta\eta$ ſhall be expanded into the preſent diſtance between α and β; and by that time likewiſe the preſent diſtance of α from β will be contracted into their natural interval: for this diſtance will be about the ſame time in contracting it ſelf, as has been taken up in its dilatation; ſeeing the ſtring will be as long in returning from its rectilinear figure, as it has been in recovering it from its ſituation I κ L. This is the change which will be made in the particles between α and ζ. As for thoſe between ζ and ξ, becauſe each preceding particle advances faſter than that, which immediately follows it, their diſtances will ſucceſſively be dilated into that, which is at preſent between ζ and η. And as ſoon as any two particles are arrived at their natural diſtance, the hindermoſt of them ſhall be ſtopt, and immediately after return;

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the diſtances between the returning particles being greater than the natural. And this dilatation of theſe diſtances ſhall extend ſo far, by that time the chord is returned into its firſt ſituation IKL, that the particles $\nu\xi$ ſhall be removed to their natural diſtance. But the dilatation of $\nu\,\xi$ ſhall contract the interval $\tau\varphi$ into that at preſent between ν and ξ, and the contraction of the diſtance between thoſe two particles τ and φ will agitate a part of the air beyond; ſo that when the chord is returned into the ſituation IKL, having made an intire vibration, the moved particles of the air will take the rangement expreſſed by the points, $l, m, n, o, p, q, r, s, t, u, w, x, y, z$, 1, 2, 3, 4, 5, 6, 7, 8: in which lm, are at the natural diſtance of the particles, the diſtance mn greater than lm, and no greater than mn, and ſo on, till you come to qr, the wideſt of all: and then the diſtances gradually diminiſh not only to the natural diſtance, as wx, but till they are contracted as much as $\xi\,\tau$ was before; which falls out in the points 2, 3, from whence the diſtances augment again, till you come to the part of the air untouched.

12. THIS is the motion, into which the air is put, while the chord makes one vibration, and the whole length of air thus agitated in the time of one vibration of the chord our author calls the length of one pulſe. When the chord goes on to make another vibration, it will not only continue to agitate the air at preſent in motion, but ſpread the pulſation of the air as much farther, and by the ſame degrees, as before. For when the chord returns into its rectilinear ſituation IkL, lm ſhall be brought into its moſt contracted ſtate

ſtate, *qr* now in the ſtate of greateſt dilatation ſhall be reduced to its natural diſtance, the points *w*, *x* now at their natural diſtance ſhall be at their greateſt diſtance, the points 2, 3 now moſt contracted enlarged to their natural diſtance, and the points 7, 8 reduced to their moſt contracted ſtate: and the contraction of them will carry the agitation of the air as far beyond them, as that motion was carried from the chord, when it firſt moved out of the ſituation IKL into its rectilinear figure. When the chord is got into the ſituation I*k*L, *lm* ſhall recover its natural dimenſions, *qr* be reduced to its ſtate of greateſt contraction, *wx* brought to its natural dimenſion, the diſtance 2 3 enlarged to the utmoſt, and the points 7, 8 ſhall have recovered their natural diſtance; and by thus recovering themſelves they ſhall agitate the air to as great a length beyond them, as it was moved beyond the chord, when it firſt came into the ſituation I*k*L. When the chord is returned back again into its rectilinear ſituation, *lm* ſhall be in its utmoſt dilatation, *qr* reſtored again to its natural diſtance, *wx* reduced into its ſtate of greateſt contraction, 2 3 ſhall recover its natural dimenſion, and 7 8 be in its ſtate of greateſt dilatation. By which means the air ſhall be moved as far beyond the points 7, 8, as it was moved beyond the chord, when it before made its return back to its rectilinear ſituation; for the particles 7, 8 have been changed from their ſtate of reſt and their natural diſtance into a ſtate of contraction, and then have proceeded to the recovery of their natural diſtance, and after that to a dilatation of it, in the ſame manner as the particles contiguous to the chord were agitated before. In

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the laſt place, when the chord is returned into the ſituation IKL, the particles of air from l to λ ſhall acquire their preſent rangement, and the motion of the air be extended as much farther. And the like will happen after every compleat vibration of the ſtring.

13. Concerning this motion of ſound, our author ſhews how to compute the velocity thereof, or in what time it will reach to any propoſed diſtance from the ſonorous body. For this he requires to know the height of air, having the ſame denſity with the parts here at the ſurface of the earth, which we breath, that would be equivalent in weight to the whole incumbent atmoſphere. This is to be found by the barometer, or common weatherglaſs. In that inſtrument quickſilver is included in a hollow glaſs cane firmly cloſed at the top. The bottom is open, but immerged into quickſilver contained in a veſſel open to the air. Care is taken when the lower end of the cane is immerged, that the whole cane be full of quickſilver, and that no air inſinuate itſelf. When the inſtrument is thus fixed, the quickſilver in the cane being higher than that in the veſſel, if the top of the cane were open, the fluid would ſoon ſink out of the glaſs cane, till it came to a level with that in the veſſel. But the top of the cane being cloſed up, ſo that the air, which has free liberty to preſs on the quickſilver in the veſſel, cannot bear at all on that, which is within the cane, the quickſilver in the cane will be ſuſpended to ſuch a height, as to balance the preſſure of the air on the quickſilver in the veſſel. Here it is evident, that the

weight of the quickſilver in the glaſs cane is equivalent to the preſſure of ſo much of the air, as is perpendicularly over the hollow of the cane; for if the cane be opened that the air may enter, there will be no farther uſe of the quickſilver to ſuſtain the preſſure of the air without; for the quickſilver in the cane, as has already been obſerved, will then ſubſide to a level with that without. Hence therefore if the proportion between the denſity of quickſilver and of the air we breath be known, we may know what height of ſuch air would form a column equal in weight to the column of quickſilver within the glaſs cane. When the quickſilver is ſuſtained in the barometer at the height of 30 inches, the height of ſuch a column of air will be about 29725 feet; for in this caſe the air has about $\frac{1}{870}$ of the denſity of water, and the denſity of quickſilver exceeds that of water about $13\frac{2}{3}$ times, ſo that the denſity of quickſilver exceeds that of the air about 11890 times; and ſo many times 30 inches make 29725 feet. Now Sir ISAAC NEWTON determines, that while a pendulum of the length of this column ſhould make one vibration or ſwing, the ſpace, which any ſound will have moved, ſhall bear to this length the ſame proportion, as the circumference of a circle bears to the diameter thereof; that is, about the proportion of 355 to 113 [a]. Only our author here conſiders ſingly the gradual progreſs of ſound in the air from particle to particle in the manner we have explained, without taking into conſideration the magnitude of thoſe particles. And though there requires time for the motion to be propagated from one par-

[a] Princ. philoſ. Lib. II. prop. 49.

ticle to another, yet it is communicated to the whole of the ſame particle in an inſtant: therefore whatever proportion the thickneſs of theſe particles bears to their diſtance from each other, in the ſame proportion will the motion of ſound be ſwifter. Again the air we breath is not ſimply compoſed of the elaſtic part, by which ſound is conveyed, but partly of vapours, which are of a different nature; and in the computation of the motion of ſound we ought to find the height of a column of this pure air only, whoſe weight ſhould be equal to the weight of the quickſilver in the cane of the barometer, and this pure air being a part only of that we breath, the column of this pure air will be higher than 29725 feet. On both theſe accounts the motion of ſound is found to be about 1142 feet in one ſecond of time, or near 13 miles in a minute, whereas by the computation propoſed above, it ſhould move but 979 feet in one ſecond.

14. We may obſerve here, that from theſe demonſtrations of our author it follows, that all ſounds whether acute or grave move equally ſwift, and that ſound is ſwifteſt, when the quickſilver ſtands higheſt in the barometer.

15. Thus much of the appearances, which are cauſed in theſe fluids from their gravitation toward the earth. They alſo gravitate toward the moon; for in the laſt chapter it has been proved, that the gravitation between the earth and moon is mutual, and that this gravitation of the whole bodies ariſes from that power acting in all their parts; ſo that every

every particle of the moon gravitates toward the earth, and every particle of the earth toward the moon. But this gravitation of these fluids toward the moon produces no sensible effect, except only in the sea, where it causes the tides.

16. THAT the tides depend upon the influence of the moon has been the receiv'd opinion of all antiquity; nor is there indeed the least shadow of reason to suppose otherwise, considering how steadily they accompany the moon's course. Though how the moon caused them, and by what principle it was enabled to produce so distinguish'd an appearance, was a secret left for this philosophy to unfold: which teaches, that the moon is not here alone concerned, but that the sun likewise has a considerable share in their production; though they have been generally ascribed to the other luminary, because its effect is greatest, and by that means the tides more immediately suit themselves to its motion; the sun discovering its influence more by enlarging or restraining the moon's power, than by any distinct effects. Our author finds the power of the moon to bear to the power of the sun about the proportion of $4\frac{1}{2}$ to 1. This he deduces from the observations made at the mouth of the river Avon, three miles from Bristol, by Captain STURMEY, and at Plymouth by Mr. COLEPRESSE, of the height to which the water is raised in the conjunction and opposition of the luminaries, compared with the elevation of it, when the moon is in either quarter; the first being caused

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by the united actions of the ſun and moon, and the other by the difference of them, as ſhall hereafter be ſhewn.

17. THAT the ſun ſhould have a like effect on the ſea, as the moon, is very manifeſt; ſince the ſun likewiſe attracts every ſingle particle, of which this earth is compoſed. And in both luminaries ſince the power of gravity is reciprocally in the duplicate proportion of the diſtance, they will not draw all the parts of the waters in the ſame manner; but muſt act upon the neareſt parts ſtronger, than upon the remoteſt, producing by this inequality an irregular motion. We ſhall now attempt to ſhew how the actions of the ſun and moon on the waters, by being combined together, produce all the appearances obſerved in the tides.

18. TO begin therefore, the reader will remember what has been ſaid above, that if the moon without the ſun would have deſcribed an orbit concentrical to the earth, the action of the ſun would make the orbit oval, and bring the moon nearer to the earth at the new and full, than at the quarters[a]. Now our excellent author obſerves, that if inſtead of one moon, we ſuppoſe a ring of moons, contiguous and occupying the whole orbit of the moon, his demonſtration would ſtill take place, and prove that the parts of this ring in paſſing from the quarter to the conjunction or oppoſition would be accelerated, and be retarded again in paſſing from the conjunction or oppoſition to the next quarter. And as this effect does not de-

[a] Chap. 3. § 18.

pend

pend on the magnitude of the bodies, whereof the ring is compoſed, the ſame would hold, though the magnitude of theſe moons were ſo far to be diminiſhed, and their number increaſed, till they ſhould form a fluid [a]. Now the earth turns round continually upon its own center, cauſing thereby the alternate change of day and night, while by this revolution each part of the earth is ſucceſſively brought toward the ſun, and carried off again in the ſpace of 24 hours. And as the ſea revolves round along with the earth itſelf in this diurnal motion, it will repreſent in ſome ſort ſuch a fluid ring.

19. But as the water of the ſea does not move round with ſo much ſwiftneſs, as would carry it about the center of the earth in the circle it now deſcribes, without being ſupported by the body of the earth; it will be neceſſary to conſider the water under three diſtinct caſes. The firſt caſe ſhall ſuppoſe the water to move with the degree of ſwiftneſs, required to carry a body round the center of the earth diſingaged from it in a circle at the diſtance of the earth's ſemidiameter, like another moon. The ſecond caſe is, that the waters make but one turn about the axis of the earth in the ſpace of a month, keeping pace with the moon; ſo that all parts of the water ſhould preſerve continually the ſame ſituation in reſpect of the moon. The third caſe ſhall be the real one of the waters moving with a velocity between theſe two, neither ſo ſwift as the firſt caſe requires, nor ſo ſlow as the ſecond.

[a] Newt. Princ. philoſ. Lib. I. prop. 66. coroll 18.

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20. In the firſt caſe the waters, like the body which they equalled in velocity, by the action of the moon would be brought nearer the center under and oppoſite to the moon, than in the parts in the middle between theſe eaſtward or weſtward. That ſuch a body would ſo alter its diſtance by the moon's action upon it, is clear from what has been mentioned of the like changes in the moon's motion cauſed by the ſun [a]. And computation ſhews, that the difference between the greateſt and leaſt diſtance of ſuch a body would not be much above $4\frac{1}{2}$ feet. But in the ſecond caſe, where all the parts of the water preſerve the ſame ſituation continually in reſpect of the moon, the weight of thoſe parts under and oppoſite to the moon will be diminiſhed by the moon's action, and the parts in the middle between theſe will have their weight increaſed: this being effected juſt in the ſame manner, as the ſun diminiſhes the attraction of the moon towards the earth in the conjunction and oppoſition, but increaſes that attraction in the quarters. For as the firſt of theſe conſequences from the ſun's action on the moon is occaſioned by the moon's being attracted by the ſun in the conjunction more than the earth, and in the oppoſition leſs than it, and therefore in the common motion of the earth and moon, the moon is made to advance toward the ſun in one caſe too faſt, and in the other is left as it were behind; ſo the earth will not have its middle parts drawn towards the moon ſo ſtrongly as the nearer parts, and yet more forcibly than the remoteſt: and therefore ſince the earth and moon move each

[a] § 8.

month

month round their common center of gravity[a], while the earth moves round this center, the ſame effect will be produced, on the parts of the water neareſt to that center or to the moon, as the moon feels from the ſun when in conjunction, and the water on the contrary ſide of the earth will be affected by the moon, as the moon is by the ſun, when in oppoſition[b]; that is, in both caſes the weight of the water, or its propenſity towards the center of the earth, will be diminiſhed. The parts in the middle between theſe will have their weight increaſed, by being preſſed towards the center of the earth through the obliquity of the moon's action upon them to its action upon the earth's center, juſt as the ſun increaſes the gravitation of the moon in the quarters from the ſame cauſe[c]. But now it is manifeſt, that where the weight of the ſame quantity of water is leaſt, there it will be accumulated; while the parts, which have the greateſt weight, will ſubſide. Therefore in this caſe there would be no tide or alternate riſing and falling of the water, but the water would form it ſelf into an oblong figure, whoſe axis prolonged would paſs through the moon. By Sir ISAAC NEWTON's computation the exceſs of this axis above the diameters perpendicular to it, that is, the height of the waters under and oppoſite to the moon above their height in the middle between theſe places eaſtward or weſtward cauſed by the moon, is about $8\frac{2}{3}$ feet.

[a] Ch. 3. § 5. [b] Ch. 3. § 17. [c] Ibid.

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21. Thus the difference of height in this latter ſuppoſition is little ſhort of twice that difference in the preceding. But the caſe of the ſea is a middle between theſe two: for a body, which ſhould revolve round the center of the earth at the diſtance of a ſemidiameter without preſſing on the earth's ſurface, muſt perform its period in leſs than an hour and half, whereas the earth turns round but once in a day; and in the caſe of the waters keeping pace with the moon it ſhould turn round but once in a month: ſo that the real motion of the water is between the motions required in theſe two caſes. Again, if the waters moved round as ſwiftly as the firſt caſe required, their weight would be wholly taken off by their motion; for this caſe ſuppoſes the body to move ſo, as to be kept revolving in a circle round the earth by the power of gravity without preſſing on the earth at all, ſo that its motion juſt ſupports its weight. But if the power of gravity had been only $\frac{1}{289}$ part of what it is, the body could have moved thus without preſſing on the earth, and have been as long in moving round, as the earth it ſelf is. Conſequently the motion of the earth takes off from the weight of the water in the middle between the poles, where its motion is ſwifteſt, $\frac{1}{289}$ part of its weight and no more. Since therefore in the firſt caſe the weight of the waters muſt be intirely taken off by their motion, and by the real motion of the earth they loſe only $\frac{1}{289}$ part thereof, the motion of the water will ſo little diminiſh their weight, that their figure will much nearer reſemble the caſe of their keeping pace with the moon than the other. Upon the whole, if the waters moved with the

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velocity neceſſary to carry a body round the center of the earth at the diſtance of the earth's ſemidiameter without bearing on its ſurface, the water would be loweſt under the moon, and riſe gradually as it moved on with the earth eaſtward, till it came half way toward the place oppoſite to the moon; from thence it would ſubſide again, till it came to the oppoſition, where it would become as low as at firſt; afterwards it would riſe again, till it came half way to the place under the moon; and from hence it would ſubſide, till it came a ſecond time under the moon. But in caſe the water kept pace with the moon, it would be higheſt where in the other caſe it is loweſt, and loweſt where in the other it is higheſt; therefore the diurnal motion of the earth being between the motions of theſe two caſes, it will cauſe the higheſt place of the water to fall between the places of the greateſt height in theſe two caſes. The water as it paſſes from under the moon ſhall for ſome time riſe, but deſcend again before it arrives half way to the oppoſite place, and ſhall come to its leaſt height before it becomes oppoſite to the moon; then it ſhall riſe again, continuing ſo to do till it has paſſed the place oppoſite to the moon, but ſubſide before it comes to the middle between the places oppoſite to and under the moon; and laſtly it ſhall come to its loweſt, before it comes a ſecond time under the moon. If A (in fig. 112, 113, 114.) repreſent the moon, B the center of the earth, the oval CDEF in fig. 112. will repreſent the ſituation of the water in the firſt caſe; but if the water kept pace with the moon, the line CDEF in fig. 113. would repreſent the ſitua-

tion of the water; but the line CDEF in fig. 114. will represent the ſame in the real motion of the water, as it accompanies the earth in its diurnal rotation: in all theſe figures C and E being the places where the water is loweſt, and D and F the places where it is higheſt. Purſuant to this determination it is found, that on the ſhores, which lie expoſed to the open ſea, the high water uſually falls out about three hours after the moon has paſſed the meridian of each place.

22. Let this ſuffice in general for explaining the manner, in which the moon acts upon the ſeas. It is farther to be noted, that theſe effects are greateſt, when the moon is over the earth's equator [a], that is, when it ſhines perpendicularly upon the parts of the earth in the middle between the poles. For if the moon were placed over either of the poles, it could have no effect upon the water to make it aſcend and deſcend. So that when the moon declines from the equator toward either pole, it's action muſt be ſomething diminiſhed, and that the more, the farther it declines. The tides likewiſe will be greateſt, when the moon is neareſt to the earth, it's action being then the ſtrongeſt.

23. Thus much of the action of the moon. That the ſun ſhould produce the very ſame effects, though in a leſs degree, is too obvious to require a particular explanation: but as was remarked before, this action of the

[a] See below § 44.

ſun

ſun being weaker than that of the moon, will cauſe the tides to follow more nearly the moon's courſe, and principally ſhew it ſelf by heightening or diminiſhing the effects of the other luminary. Which is the occaſion, that the higheſt tides are found about the conjunction and oppoſition of the luminaries, being then produced by their united action, and the weakeſt tides about the quarters of the moon; becauſe the moon in this caſe raiſing the water where the ſun depreſſes it, and depreſſing it where the ſun raiſes it, the ſtronger action of the moon is in part retunded and weakened by that of the ſun. Our author computes that the ſun will add near two feet to the height of the water in the firſt caſe, and in the other take from it as much. However the tides in both comply with the ſame hour of the moon. But at other times, between the conjunction or oppoſition and quarters, the time deviates from that forementioned, towards the hour in which the ſun would make high water, though ſtill it keeps much nearer to the moon's hour than to the ſun's.

24. Again the tides have ſome farther varieties from the ſituation of the places where they happen northward or ſouthward. Let *p* P (in fig. 115.) repreſent the axis, on which the earth daily revolves, let *h p* H P repreſent the figure of the water, and let *n* B N D be a globe inſcribed within this figure. Suppoſe the moon to be advanced from the equator toward the north pole, ſo that *h* H the axis of the figure of the water *p* A H P E *h* ſhall decline towards the north pole N; take any place G nearer to

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the north pole than to the south, and from the center of the earth C draw CGF; then will GF denote the altitude to which the water is raised by the tide, when the moon is above the horizon: in the space of twelve hours, the earth having turned half round its axis, the place G will be removed to *g*; but the axis *h* H will have kept its place preserving its situation in respect of the moon, at least will have moved no more than the moon has done in that time, which it is not necessary here to take into consideration. Now in this case the height of the water will be equal to *g f*, which is not so great as GF. But whereas GF is the altitude at high water, when the moon is above the horizon, *g f* will be the altitude of the same, when the moon is under the horizon. The contrary happens toward the south pole, for KL is less than *k l*. Hence is proved, that when the moon declines from the equator, in those places, which are on the same side of the equator as the moon, the tides are greater, when the moon is above the horizon, than when under it; and the contrary happens on the other side of the equator.

25. Now from these principles may be explained all the known appearances in the tides; only by the assistance of this additional remark, that the fluctuating motion, which the water has in flowing and ebbing, is of a durable nature, and would continue for some time, though the action of the luminaries should cease; for this prevents the difference between the tide when the moon is above

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the horizon, and the tide when the moon is below it from being ſo great, as the rule laid down requires. This likewiſe makes the greateſt tides not exactly upon the new and full moon, but to be a tide or two after; as at Briſtol and Plymouth they are found the third after.

26. THIS doctrine farther ſhews us, why not only the ſpring tides fall out about the new and full moon, and the neap tides about the quarters; but likewiſe how it comes to paſs, that the greateſt ſpring tides happen about the equinoxes; becauſe the luminaries are then one of them over the equator, and the other not far from it. It appears too, why the neap tides, which accompany theſe, are the leaſt of all; for the ſun ſtill continuing over the equator continues to have the greateſt power of leſſening the moon's action, and the moon in the quarters being far removed toward one of the poles, has its power thereby weakned.

27. MOREOVER the action of the moon being ſtronger, when near the earth, than when more remote; if the moon, when new ſuppoſe, be at its neareſt diſtance from the earth, it ſhall when at the full be fartheſt off; whence it is, that two of the very largeſt ſpring tides do never immediately ſucceed each other.

28. BECAUSE the ſun in its paſſage from the winter ſolſtice to the ſummer recedes from the earth, and paſſing from the ſummer ſolſtice to the winter approaches it, and is therefore nearer the earth before the vernal equinox than after,

after, but nearer after the autumnal equinox than before; the greateſt tides oftner precede the vernal equinox than follow it, and in the autumnal equinox on the contrary they oftner follow it than come before it.

29. The altitude, to which the water is raiſed in the open ocean, correſponds very well to the forementioned calculations; for as it was ſhewn, that the water in ſpring tides ſhould riſe to the height of 10 or 11 feet, and the neap tides to 6 or 7; accordingly in the Pacific, Atlantic and Ethiopic oceans in the parts without the tropics, the water is obſerved to riſe about 6, 9, 12 or 15 feet. In the Pacific ocean this elevation is ſaid to be greater than in the other, as it ought to be by reaſon of the wide extent of that ſea. For the ſame reaſon in the Ethiopic ocean between the tropics the aſcent of the water is leſs than without, by reaſon of the narrowneſs of the ſea between the coaſts of Africa and the ſouthern parts of America. And iſlands in ſuch narrow ſeas, if far from ſhore, have leſs tides than the coaſts. But now in thoſe ports where the water flows in with great violence upon fords and ſhoals, the force it acquires by that means will carry it to a much greater height, ſo as to make it aſcend and deſcend to 30, 40 or even 50 feet and more; inſtances of which we have at Plymouth, and in the Severn near Chepſtow; at St. Michael's and Auranches in Normandy; at Cambay and Pegu in the Eaſt Indies.

30. Again the tides take a conſiderable time in paſſing through long ſtraits, and ſhallow places. Thus the tide,

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which is made on the weſt coaſt of Ireland and on the coaſt of Spain at the third hour after the moon's coming to the meridian, in the ports eaſtward toward the Britiſh channel falls out later, and as the flood paſſes up that channel ſtill later and later, ſo that the tide takes up full twelve hours in coming up to London bridge.

31. In the laſt place tides may come to the ſame port from different ſeas, and as they may interfere with each other, they will produce particular effects. Suppoſe the tide from one ſea come to a port at the third hour after the moon's paſſing the meridian of the place, but from another ſea to take up ſix hours more in its paſſage. Here one tide will make high water, when by the other it ſhould be loweſt; ſo that when the moon is over the equator, and the two tides are equal, there will be no riſing and falling of the water at all; for as much as the water is carried off by one tide, it will be ſupplied by the other. But when the moon declines from the equator, the ſame way as the port is ſituated, we have ſhewn that of the two tides of the ocean, which are made each day, that tide, which is made when the moon is above the horizon, is greater than the other. Therefore in this caſe, as four tides come to this port each day the two greateſt will come on the third, and on the ninth hour after the moon's paſſing the meridian, and the two leaſt at the fifteenth and at the twenty firſt hour. Thus from the third to the ninth hour more water will be in this port by the two greateſt tides than from the ninth to the fifteenth, or from the twenty firſt to the

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following third hour, where the water is brought by one great and one ſmall tide; but yet there will be more water brought by theſe tides, than what will be found between the two leaſt tides, that is, between the fifteenth and twenty firſt hour. Therefore in the middle between the third and ninth hour, or about the moon's ſetting, the water will be at its greateſt height; in the middle between the ninth and fifteenth, as alſo between the twenty firſt and following third hour it will have its mean height; and be loweſt in the middle between the fifteenth and twenty firſt hour, that is, at the moon's riſing. Thus here the water will have but one flood and one ebb each day. When the moon is on the other ſide of the equator, the flood will be turned into ebb, and the ebb into flood; the high water falling out at the riſing of the moon, and the low water at the ſetting. Now this is the caſe of the port of Batſham in the kingdom of Tunquin in the Eaſt Indies; to which port there are two inlets, one between the continent and the iſlands which are called the Manillas, and the other between the continent and Borneo.

32. The next thing to be conſidered is the effect, which theſe fluids of the planets have upon the ſolid part of the bodies to which they belong. And in the firſt place I ſhall ſhew, that it was neceſſary upon account of theſe fluid parts to form the bodies of the planets into a figure ſomething different from that of a perfect globe. Becauſe the diurnal rotation, which our earth performs about its axis, and the like motion we ſee in ſome of the other planets, which

(which is an ample conviction that they all do the like) will diminiſh the force, with which bodies are attracted upon all the parts of their ſurfaces, except at the very poles, upon which they turn. Thus a ſtone or other weighty ſubſtance reſting upon the ſurface of the earth, by the force which it receives from the motion communicated to it by the earth, if its weight prevented not, would continue that motion in a ſtraight line from the point where it received it, and according to the direction, in which it was given, that is, in a line which touches the ſurface at that point; inſomuch that it would move off from the earth in the ſame manner, as a weight faſten'd to a ſtring and whirled about endeavours continually to recede from the center of motion, and would forthwith remove it ſelf to a greater diſtance from it, if looſed from the ſtring which retains it. And farther, as the centrifugal force, with which ſuch a weight preſſes from the center of its motion, is greater, by how much greater the velocity is, with which it moves; ſo ſuch a body, as I have been ſuppoſing to lie on the earth, would recede from it with the greater force, the greater the velocity is, with which the part of the earth's ſurface it reſts upon is moved, that is, the farther diſtant it is from the poles. But now the power of gravity is great enough to prevent bodies in any part of the earth from being carried off from it by this means; however it is plain that bodies having an effort contrary to that of gravity, though much weaker than it, their weight, that is, the degree of force, with which they are preſſed to the earth, will be diminiſhed thereby, and be the more diminiſhed,

the greater this contrary effort is; or in other words, the ſame body will weigh heavier at either of the poles, than upon any other part of the earth; and if any body be removed from the pole towards the equator, it will loſe of its weight more and more, and be lighteſt of all at the equator, that is, in the middle between the poles.

33. THIS now is eaſily applied to the waters of the ſeas, and ſhews that the water under the poles will preſs more forcibly to the earth, than at or near the equator: and conſequently that which preſſes leaſt, muſt give place, till by aſcending it makes room for receiving a greater quantity, which by its additional weight may place the whole upon a ballance. To illuſtrate this more particularly I ſhall make uſe of fig. 116 In which let ACBD be a circle, by whoſe revolution about the diameter AB a globe ſhould be formed, repreſenting a globe of ſolid earth. Suppoſe this globe covered on all ſides with water to the ſame height, ſuppoſe that of EA or BF, at which diſtance the circle EGFH ſurrounds the circle ACBD; then it is evident, if the globe of earth be at reſt, the water which ſurrounds it will reſt in that ſituation. But if the globe be turned inceſſantly about its axis AB, and the water have likewiſe the ſame motion, it is alſo evident, from what has been explained, that the water between the circles EHFG and ADBC will remain no longer in the preſent ſituation, the parts of it between H and D, and between G and C being by this rotation become lighter, than the parts between E and A and between B and F; ſo that the water over the poles A and B muſt of neceſſity ſubſide, and the

water

water be accumulated over D and C, till the greater quantity in theſe latter places ſupply the defect of its weight. This would be the caſe, were the globe all covered with water. And the ſame figure of the ſurface would alſo be preſerved, if ſome part of the water adjoining to the globe in any part of it were turned into ſolid earth, as is too evident to need any proof; becauſe the parts of the water remaining at reſt, it is the ſame thing, whether they continue in the ſtate of being eaſily ſeparable, which denominates them fluid, or were to be conſolidated together, ſo as to make a hard body: and this, though the water ſhould in ſome places be thus conſolidated, even to the ſurface of it. Which ſhews that the form of the ſolid part of the earth makes no alteration in the figure the water will take: and by conſequence in order to the preventing ſome parts of the earth from being entirely overflowed, and other parts quite deſerted, the ſolid parts of the earth muſt have given them much the ſame figure, as if the whole earth were covered on all ſides with water.

34. FARTHER, I ſay, this figure of the earth is the ſame, as it would receive, were it entirely a globe of water, provided that water were of the ſame denſity as the ſubſtance of the globe. For ſuppoſe the globe ACBD to be liquified, and that the globe EHFG, now entirely water, by its rotation about its axis ſhould receive ſuch a figure as we have been deſcribing, and then the globe ACBD ſhould be conſolidated again, the figure of the water would plainly not be altered, by ſuch a conſolidation.

 35. BUT

35. But from this laſt obſervation our author is enabled to determine the proportion between the axis of the earth drawn from pole to pole, and the diameter of the equator, upon the ſuppoſition that all the parts of the earth are of equal denſity; which he does by computing in the firſt place the proportion of the centrifugal force of the parts under the equator to the power of gravity; and then by conſidering the earth as a ſpheroid, made by the revolution of an ellipſis about its leſſer axis, that is, ſuppoſing the line MILK to be an exact ellipſis, from which it can differ but little, by reaſon that the difference between the leſſer axis ML and the greater IK is but very ſmall. From this ſuppoſition, and what was proved before, that all the particles which compoſe the earth have the attracting power explained in the preceding chapter, he finds at what diſtance the parts under the equator ought to be removed from the center, that the force, with which they ſhall be attracted to the center, diminiſhed by their centrifugal force, ſhall be ſufficient to keep thoſe parts in a ballance with thoſe which lie under the poles. And upon the ſuppoſition of all the parts of the earth having the ſame degree of denſity, the earth's ſurface at the equator muſt be above 17 miles more diſtant from the center, than at the poles[a].

36. After this it is ſhewn, from the proportion of the equatorial diameter of the earth to its axis, how the ſame may be determined of any other planet, whoſe denſity in

[a] Newton Princ. Lib. III prop. 19.

compariſon

comparifon of the denfity of the earth, and the time of its revolution about its axis, are known. And by the rule delivered for this, it is found, that the diameter of the equator in Jupiter fhould bear to its axis about the proportion of 10 to 9 [a], and accordingly this planet appears of an oval form to the aftronomers. The moft confiderable effects of this fpheroidical figure our author takes likewife into confideration; one of which is that bodies are not equally heavy in all diftances from the poles; but near the equator, where the diftance from the center is greateft, they are lighter than towards the poles: and nearly in this proportion, that the actual power, by which they are drawn to the center, refulting from the difference between their abfolute gravity and centrifugal force, is reciprocally as the diftance from the center. That this may not appear to contradict what has before been faid of the alteration of the power of gravity, in proportion to the change of the diftance from the center, it is proper carefully to remark, that our author has demonftrated three things relating hereto: the firft is, that decreafe of the power of gravity as we recede from the center, which has been fully explained in the laft chapter, upon fuppofition that the earth and planets are perfect fpheres, from which their difference is by many degrees too little to require notice for the purpofes there intended: the next is, that whether they be perfect fpheres, or exactly fuch fpheroids as have now been mentioned, the power of gravity, as we defcend in the fame line to the center, is at all diftances as the diftance from the center, the parts of the

[a] Lib III prop. 19.

earth

earth above the body by drawing the body towards them lessening its gravitation towards the center [a]; and both these assertions relate to gravity alone: the third is what we mentioned in this place, that the actual force on different parts of the surface, with which bodies are drawn to the center, is in the proportion here assigned [b].

38. The next effect of this figure of the earth is an obvious consequence of the former: that pendulums of the same length do not in different distances from the pole make their vibrations in the same time; but towards the poles, where the gravity is strongest, they move quicker than near the equator, where they are less impelled to the center; and accordingly pendulums, that measure the same time by their vibrations, must be shorter near the poles than at a greater distance. Both which deductions are found true in fact; of which our author has recounted particularly several experiments, in which it was found, that clocks exactly adjusted to the true measure of time at Paris, when transported nearer to the equator, became erroneous and moved too slow, but were reduced to their true motion by contracting their pendulums. Our author is particular in remarking, how much they lost of their motion, while the pendulums remained unaltered; and what length the observers are said to have shortened them, to bring them to time. And the experiments, which appear to be most carefully made, shew the earth to be raised in the middle between the poles, as much as our author found it by his computation [c].

[a] Lib. I. prop. 73. [b] Lib. III. prop. 20. [c] Ibid.

39. These

39. THESE experiments on the pendulum our author has been very exact in examining, inquiring particularly how much the extension of the rod of the pendulum by the great heats in the torrid zone might make it necessary to shorten it. For by an experiment made by PICART, and another made by DE LA HIRE, heat, though not very intense, was found to increase the length of rods of iron. The experiment of PICART was made with a rod one foot long, which in winter, at the time of frost, was found to increase in length by being heated at the fire. In the experiment of DE LA HIRE a rod of six foot in length was found, when heated by the summer sun only, to grow to a greater length, than it had in the aforesaid cold season. From which observations a doubt has been raised, whether the rod of the pendulums in the aforementioned experiments was not extended by the heat of those warm climates to all that excess of length, the observers found themselves obliged to lessen them by. But the experiments now mentioned shew the contrary. For in the first of them the rod of a foot long was lengthened no more than $\frac{1}{9}$ part of what the pendulum under the equator must be diminished; and therefore a rod of the length of the pendulum would not have been extended above $\frac{1}{3}$ of that length. In the experiment of DE LA HIRE, where the heat was less, the rod of six foot long was extended no more than $\frac{3}{10}$ of what the pendulum must be shortened; so that a rod of the length of the pendulum would not have gained above $\frac{3}{20}$ or $\frac{1}{7}$ of that length. And the heat in this latter experiment, though less than in the former, was yet greater than the rod of a pendulum can ordinarily

dinarily contract in the hotteſt country; for metals receive a great heat when expoſed to the open ſun, certainly much greater than that of a human body. But pendulums are not uſually ſo expoſed, and without doubt in theſe experiments were kept cool enough to appear ſo to the touch; which they would do in the hotteſt place, if lodged in the ſhade. Our author therefore thinks it enough to allow about $\frac{1}{10}$ of the difference obſerved upon account of the greater warmth of the pendulum.

40. There is a third effect, which the water has on the earth by changing its figure, that is taken notice of by our author; for the explaining of which we ſhall firſt prove, that bodies deſcend perpendicularly to the ſurface of the earth in all places. The manner of collecting this from obſervation, is as follows. The ſurfaces of all fluids reſt parallel to that part of the ſurface of the ſea, which is in the ſame place with them, to the figure of which, as has been particularly ſhewn, the figure of the whole earth is formed. For if any hollow veſſel, open at the bottom, be immerſed into the ſea; it is evident, that the ſurface of the ſea within the veſſel will retain the ſame figure it had, before the veſſel incloſed it; ſince its communication with the external water is not cut off by the veſſel. But all the parts of the water being at reſt, it is as clear, that if the bottom of the veſſel were cloſed, the figure of the water could receive no change thereby, even though the veſſel were raiſed out of the ſea; any more than from the inſenſible alteration of the power of gravity, conſequent upon the augmentation of the

the diſtance from the center. But now it is clear, that bodies deſcend in lines perpendicular to the ſurfaces of quieſcent fluids; for if the power of gravity did not act perpendicularly to the ſurface of fluids, bodies which ſwim on them could not reſt, as they are ſeen to do; becauſe, if the power of gravity drew ſuch bodies in a direction oblique to the ſurface whereon they lay, they would certainly be put in motion, and be carried to the ſide of the veſſel, in which the fluid was contained, that way the action of gravity inclined.

41. HENCE it follows, that as we ſtand, our bodies are perpendicular to the ſurface of the earth. Therefore in going from north to ſouth our bodies do not keep in a parallel direction. Now in all diſtances from the pole the ſame length gone on the earth will not make the ſame change in the poſition of our bodies, but the nearer we are to the poles, we muſt go a greater length to cauſe the ſame variation herein. Let MILK (in fig. 117) repreſent the figure of the earth, M, L the poles, I, K two oppoſite points in the middle between theſe poles. Let TV and PO be two arches, TV being moſt remote from the pole L; draw TW, VX, PQ, OR, each perpendicular to the ſurface of the earth, and let TW, VX meet in Y, and PQ, OR in S. Here it is evident, that in paſſing from V to T the poſition of a man's body would be changed by the angle under TYV, for at V he would ſtand in the line YV continued upward, and at T in the line YT; but in paſſing from O to P the poſition of his body would be changed by

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the angle under OSP. Now I ſay, if theſe two angles are equal the arch OP is longer than TV: for the figure MILK being oblong, and IK longer than ML, the figure will be more incurvated toward I than toward L; ſo that the lines TW and VX will meet in Y before they are drawn out to ſo great a length as the lines PQ and OR muſt be continued to, before they will meet in S. Since therefore YT and YV are ſhorter than PS and SV, TV muſt be leſs than OP. If theſe angles under TYV and OSP are each $\frac{1}{90}$ part of the angle made by a perpendicular line, they are ſaid each to contain one degree. And the unequal length of theſe arches OP and VT gives occaſion to the aſſertion, that in paſſing from north to ſouth the degrees on the earth's ſurface are not of an equal length, but thoſe near the pole longer than thoſe toward the equator. For the length of the arch on the earth lying between the two perpendiculars, which make an angle of a degree with each other, is called the length of a degree on the earth's ſurface.

42. THIS figure of the earth has ſome effect on eclipſes. It has been obſerved above, that ſometimes the nodes of the moon's orbit lie in a ſtraight line drawn from the ſun to the earth; in which caſe the moon will croſs the plane of the earth's motion at the new and full. But whenever the moon paſſes near the plane at the full, ſome part of the earth will intercept the ſun's light, and the moon ſhining only with light borrow'd from the ſun, when that light is prevented from falling on any part of the moon, ſo much of her body will be darkened. Alſo when the moon at the new

new is near the plane of the earth's motion, the inhabitants on ſome part of the earth will ſee the moon come under the ſun, and the ſun thereby be covered from them either wholly or in part. Now the figure, which we have ſhewn to belong to the earth, will occaſion the ſhadow of the earth on the moon not to be perfectly round, but cauſe the diameter from eaſt to weſt to be ſomewhat longer than the diameter from north to ſouth. In eclipſes of the ſun this figure of the earth will make ſome little difference in the place, where the ſun ſhall appear wholly or in any given part covered. Let ABCD (in fig. 118.) repreſent the earth, AC the axis whereon it turns daily, E the center. Let FAGC repreſent a perfect globe inſcribed within the earth. Let HI be a line drawn through the centers of the ſun and moon, croſſing the ſurface of the earth in K, and the ſurface of the globe inſcribed in L. Draw EL, which will be perpendicular to the ſurface of the globe in L: and draw likewiſe KM, ſo that it ſhall be perpendicular to the ſurface of the earth in K. Now whereas the eclipſe would appear central at L, if the earth were the globe AGCF, and does really appear ſo at K; I ſay, the latitude of the place K on the real earth is different from the latitude of the place L on the globe FAGC. What is called the latitude of any place is determined by the angle which the line perpendicular to the ſurface of the earth at that place makes with the axis; the difference between this angle, and that made by a perpendicular line or ſquare being called the latitude of each place. But it might here be proved, that the angle which KM makes with MC is leſs, than the angle made between LE and EC: conſe-

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quently the latitude of the place K is greater, than the latitude, which the place L would have.

43. The next effect, which follows from this figure of the earth, is that gradual change in the diſtance of the fixed ſtars from the equinoctial points, which aſtronomers obſerve. But before this can be explained, it is neceſſary to ſay ſomething more particular, than has yet been done, concerning the manner of the earth's motion round the ſun.

44. It has already been ſaid, that the earth turns round each day on its own axis, while its whole body is carried round the ſun once in a year. How theſe two motions are joined together may be conceived in ſome degree by the motion of a bowl on the ground, where the bowl in rouling on continually turns upon its axis, and at the ſame time the whole body thereof is carried ſtraight on. But to be more expreſs let A (in fig. 119) repreſent the ſun BCDE four different ſituations of the earth in its orbit moving about the ſun. In all theſe let FG repreſent the axis, about which the earth daily turns. The points F, G are called the poles of the earth; and this axis is ſuppoſed to keep always parallel to it ſelf in every ſituation of the earth; at leaſt that it would do ſo, were it not for a minute deviation, the cauſe whereof will be explained in what follows. When the earth is in B, the half HIK will be illuminated by the ſun, and the other half HLK will be in darkneſs. Now if on the globe any point be taken

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in the middle between the poles, this point ſhall deſcribe by the motion of the globe the circle MN, half of which is in the enlightened part of the globe, and half in the dark part. But the earth is ſuppoſed to move round its axis with an equable motion; therefore on this point of the globe the ſun will be ſeen juſt half the day, and be inviſible the other half. And the ſame will happen to every point of this circle, in all ſituations of the earth during its whole revolution round the ſun. This circle MN is called the equator, of which we have before made mention.

45. NOW ſuppoſe any other point taken on the ſurface of the globe toward the pole F, which in the diurnal revolution of the globe ſhall deſcribe the circle OP. Here it appears that more than half this circle is enlightned by the ſun, and conſequently that in any particular point of this circle the ſun will be longer ſeen than lie hid, that is the day will be longer than the night. Again if we conſider the ſame circle OP on the globe ſituated in D the oppoſite part of the orbit from B, we ſhall ſee, that here in any place of this circle the night will be as much longer than the day.

46. IN theſe ſituations of the globe of earth a line drawn from the ſun to the center of the earth will be obliquely inclined toward the axis FG. Now ſuppoſe, that ſuch a line drawn from the ſun to the center of the earth, when in C or E, would be perpendicular to the axis FG;

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in which cases the ſun will ſhine perpendicularly upon the equator, and conſequently the line drawn from the center of the earth to the ſun will croſs the equator, as it paſſes through the ſurface of the earth; whereas in all other ſituations of the globe this line will paſs through the ſurface of the globe at a diſtance from the equator either northward or ſouthward. Now in both theſe caſes half the circle O P will be in the light, and half in the dark; and therefore to every place in this circle the day will be equal to the night. Thus it appears, that in theſe two oppoſite ſituations of the earth the day is equal to the night in all parts of the globe; but in all other ſituations this equality will only be found in places ſituated in the very middle between the poles, that is, on the equator.

47. The times, wherein this univerſal equality between the day and night happens, are called the equinoxes. Now it has been long obſerved by aſtronomers, that after the earth hath ſet out from either equinox, ſuppoſe from E (which will be the ſpring equinox, if F be the north pole) the ſame equinox ſhall again return a little before the earth has made a compleat revolution round the ſun. This return of the equinox preceding the intire revolution of the earth is called the preceſſion of the equinox, and is cauſed by the protuberant figure of the earth.

48. Since the ſun ſhines perpendicularly upon the equator, when the line drawn from the ſun to the center of the earth is perpendicular to the earth's axis, in this caſe the

the plane, which ſhould cut through the earth at the equator, may be extended to paſs through the ſun; but it will not do ſo in any other poſition of the earth. Now let us conſider the prominent part of the earth about the equator, as a ſolid ring moving with the earth round the ſun. At the time of the equinoxes, this ring will have the ſame kind of ſituation in reſpect of the ſun, as the orbit of the moon has, when the line of the nodes is directed to the ſun; and at all other times will reſemble the moon's orbit in other ſituations. Conſequently this ring, which otherwiſe would keep throughout its motion parallel to it ſelf, will receive ſome change in its poſition from the action of the ſun upon it, except only at the time of the equinox. The manner of this change may be underſtood as follows. Let A B C D (in fig. 120) repreſent this ring, E the center of the earth, S the ſun, A F C G a circle deſcribed in the plane of the earth's motion to the center E. Here A and C are the two points, in which the earth's equator croſſes the plane of the earth's motion; and the time of the equinox falls out, when the ſtraight line A C continued would paſs through the ſun. Now let us recollect what was ſaid above concerning the moon, when her orbit was in the ſame ſituation with this ring. From thence it will be underſtood, if a body were ſuppoſed to be moving in any part of this circle A B C D, what effect the action of the ſun on the body would have toward changing the poſition of the line A C. In particular H I being drawn perpendicular to S E, if the body be in any part of this circle between A and H, or between C and I, the line A C

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would be ſo turned, that the point A ſhall move toward B, and the point C toward D; but if it were in any other part of the circle, either between H and C, or between I and A, the line AC would be turned the contrary way. Hence it follows, that as this ſolid ring turns round the center of the earth, the parts of this ring between A and H, and between C and I, are ſo influenced by the ſun, that they will endeavour, ſo to change the ſituation of the line AC as to cauſe the point A to move toward B, and the point C to move toward D; but all the parts of the ring between H and C, and between I and A, will have the oppoſite tendency, and diſpoſe the line AC to move the contrary way. And ſince theſe laſt named parts are larger than the other, they will prevail over the other, ſo that by the action of the ſun upon this ring, the line AC will be ſo turned, that A ſhall continually be more and more moving toward D, and C toward B. Thus no ſooner ſhall the ſun in its viſible motion have departed from A, but the motion of the line AC ſhall haſten its meeting with C, and from thence the motion of this line ſhall again haſten the ſun's ſecond conjunction with A; for as this line ſo turns, that A is continually moving toward D, ſo the ſun's viſible motion is the ſame way as from S toward T.

49. The moon will have on this ring the like effect as the ſun, and operate on it more ſtrongly, in the ſame proportion as its force on the ſea exceeded that of the ſun on the ſame. But the effect of the action of both luminaries will be greatly diminiſhed by reaſon of this ring's being connect-

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ed to the reſt of the earth; for by this means the ſun and moon have not only this ring to move, but likewiſe the whole globe of the earth, upon whoſe ſpherical part they have no immediate influence. Beſide the effect is alſo rendred leſs, by reaſon that the prominent part of the earth is not collected all under the equator, but ſpreads gradually from thence toward both poles. Upon the whole, though the ſun alone carries the nodes of the moon through an intire revolution in about 19 years, the united force of both luminaries on the prominent parts of the earth will hardly carry round the equinox in a leſs ſpace of time than 26000 years.

50. To this motion of the equinox we muſt add another conſequence of this action of the ſun and moon upon the elevated parts of the earth, that this annular part of the earth about the equator, and conſequently the earth's axis, will twice a year and twice a month change its inclination to the plane of the earth's motion, and be again reſtored, juſt as the inclination of the moon's orbit by the action of the ſun is annually twice diminiſhed, and as often recovers its original magnitude. But this change is very inſenſible.

51. I ſhall now finiſh the preſent chapter with our great author's inquiry into the figure of the ſecondary planets, particularly of our moon, upon the figure of which its fluid parts will have an influence. The moon turns always the ſame ſide towards the earth, and conſequently revolves but once round its axis in the ſpace of an entire month;

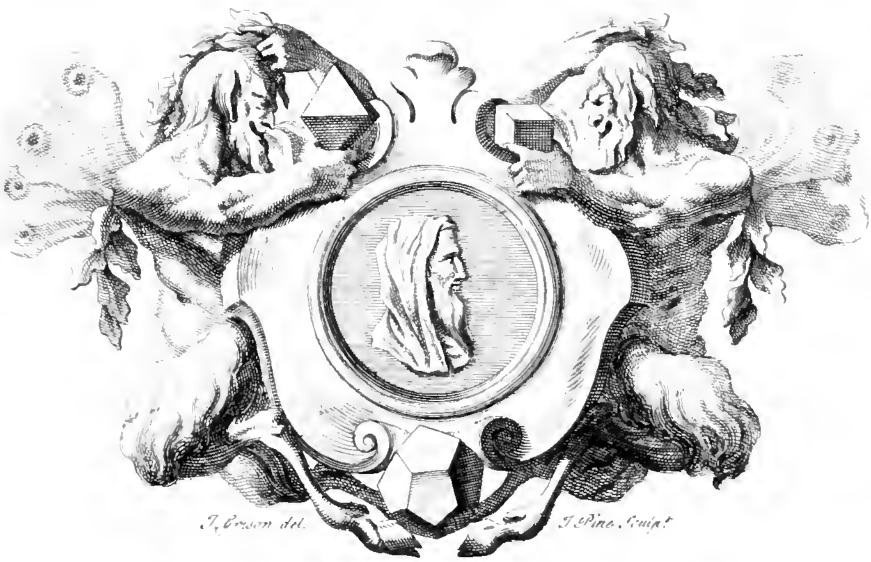
 for

for a ſpectator placed without the circle, in which the moon moves, would in that time obſerve all the parts of the moon ſucceſſively to paſs once before his view and no more, that is, that the whole globe of the moon has turned once round. Now the great ſlowneſs of this motion will render the centrifugal force of the parts of the waters very weak, ſo that the figure of the moon cannot, as in the earth, be much affected by this revolution upon its axis: but the figure of thoſe waters are made different from ſpherical by another cauſe, viz. the action of the earth upon them; by which they will be reduced to an oblong oval form, whoſe axis prolonged would paſs through the earth; for the ſame reaſon, as we have above obſerved, that the waters of the earth would take the like figure, if they had moved ſo ſlowly, as to keep pace with the moon. And the ſolid part of the moon muſt correſpond with this figure of the fluid part: but this elevation of the parts of the moon is nothing near ſo great as is the protuberance of the earth at the equator, for it will not exceed 93 engliſh feet.

52. The waters of the moon will have no tide, except what will ariſe from the motion of the moon round the earth. For the converſion of the moon about her axis is equable, whereby the inequality in the motion round the earth diſcovers to us at ſome times ſmall parts of the moon's ſurface towards the eaſt or weſt, which at other times lie hid; and as the axis, whereon the moon turns, is oblique to her motion round the earth, ſometimes ſmall parts of her

ſurface

ſurface toward the north, and ſometimes the like toward the ſouth are viſible, which at other times are out of ſight. Theſe appearances make what is called the libration of the moon, diſcovered by HEVELIUS. But now as the axis of the oval figure of the waters will be pointed towards the earth, there muſt ariſe from hence ſome fluctuation in them; and beſide, by the change of the moon's diſtance from the earth, they will not always have the very ſame height.



I Grifon del J. Pine Sculp.

BOOK III.

CHAP. I.

Concerning the cause of COLOURS inherent in the LIGHT.

AFTER this view which has been taken of Sir ISAAC NEWTON's mathematical principles of philofophy, and the ufe he has made of them, in explaining the fyftem of the world, &c. the courfe of my defign directs us to turn our eyes to that other philofophical work, his treatife of Optics, in which we fhall find our great author's inimitable genius difcovering it felf no lefs, than in the

the former; nay perhaps even more, ſince this work gives as many inſtances of his ſingular force of reaſoning, and of his unbounded invention, though unaſſiſted in great meaſure by thoſe rules and general precepts, which facilitate the invention of mathematical theorems. Nor yet is this work inferior to the other in uſefulneſs; for as that has made known to us one great principle in nature, by which the celeſtial motions are continued, and by which the frame of each globe is preſerved; ſo does this point out to us another principle no leſs univerſal, upon which depends all thoſe operations in the ſmaller parts of matter, for whoſe ſake the greater frame of the univerſe is erected; all thoſe immenſe globes, with which the whole heavens are filled, being without doubt only deſign'd as ſo many convenient apartments for carrying on the more noble operations of nature in vegetation and animal life. Which ſingle conſideration gives abundant proof of the excellency of our author's choice, in applying himſelf carefully to examine the action between light and bodies, ſo neceſſary in all the varieties of theſe productions, that none of them can be ſucceſsfully promoted without the concurrence of heat in a greater or leſs degree.

2. 'TIS true, our author has not made ſo full a diſcovery of the principle, by which this mutual action between light and bodies is cauſed; as he has in relation to the power, by which the planets are kept in their courſes: yet he has led us to the very entrance upon it, and pointed out the path ſo plainly which muſt be followed to reach it; that one may be

be bold to ſay, whenever mankind ſhall be bleſſed with this improvement of their knowledge, it will be derived ſo directly from the principles laid down by our author in this book, that the greateſt ſhare of the praiſe due to the diſcovery will belong to him.

3. In ſpeaking of the progreſs our author has made, I ſhall diſtinctly purſue three things, the two firſt relating to the colours of natural bodies: for in the firſt head ſhall be ſhewn, how thoſe colours are derived from the properties of the light itſelf; and in the ſecond upon what properties of the bodies they depend: but the third head of my diſcourſe ſhall treat of the action of bodies upon light in refracting, reflecting, and inflecting it.

4. The firſt of theſe, which ſhall be the buſineſs of the preſent chapter, is contained in this one propoſition: that the ſun's direct light is not uniform in reſpect of colour, not being diſpoſed in every part of it to excite the idea of whiteneſs, which the whole raiſes; but on the contrary is a compoſition of different kinds of rays, one ſort of which if alone would give the ſenſe of red, another of orange, a third of yellow, a fourth of green, a fifth of light blue, a ſixth of indigo, and a ſeventh of a violet purple; that all theſe rays together by the mixture of their ſenſations impreſs upon the organ of ſight the ſenſe of whiteneſs, though each ray always imprints there its own colour; and all the difference between the colours of bodies when viewed in open day light ariſes from this, that coloured bodies do

do not reflect all the forts of rays falling upon them in equal plenty, but fome forts much more copioufly than others; the body appearing of that colour, of which the light coming from it is moft compofed.

5. That the light of the fun is compounded, as has been faid, is proved by refracting it with a prifm. By a prifm I here mean a glafs or other body of a triangular form, fuch as is reprefented in fig. 121. But before we proceed to the illuftration of the propofition we have juft now laid down, it will be neceffary to fpend a few words in explaining what is meant by the refraction of light; as the defign of our prefent labour is to give fome notion of the fubject, we are engaged in, to fuch as are not verfed in the mathematics.

6. It is well known, that when a ray of light paffing through the air falls obliquely upon the furface of any tranfparent body, fuppofe water or glafs, and enters it, the ray will not pafs on in that body in the fame line it defcribed through the air, but be turned off from the furface, fo as to be lefs inclined to it after paffing it, than before. Let ABCD (in fig. 122.) reprefent a portion of water, or glafs, AB the furface of it, upon which the ray of light EF falls obliquely; this ray fhall not go right on in the courfe delineated by the line FG, but be turned off from the furface AB into the line FH, lefs inclined to the furface AB than the line EF is, in which the ray is incident upon that furface.

7. On the other hand, when the light paſſes out of any ſuch body into the air, it is inflected the contrary way, being after its emergence rendred more oblique to the ſurface it paſſes through, than before. Thus the ray FH, when it goes out of the ſurface CD, will be turned up towards that ſurface, going out into the air in the line HI.

8. This turning of the light out of its way, as it paſſes from one tranſparent body into another is called its refraction. Both theſe caſes may be tried by an eaſy experiment with a baſon and water. For the firſt caſe ſet an empty baſon in the ſunſhine or near a candle, making a mark upon the bottom at the extremity of the ſhadow caſt by the brim of the baſon, then by pouring water into the baſon you will obſerve the ſhadow to ſhrink, and leave the bottom of the baſon enlightned to a good diſtance from the mark. Let ABC (in fig. 123.) denote the empty baſon, EAD the light ſhining over the brim of it, ſo that all the part ABD be ſhaded. Then a mark being made at D, if water be poured into the baſon (as in fig. 124.) to FG, you ſhall obſerve the light, which before went on to D, now to come much ſhort of the mark D, falling on the bottom in the point H, and leaving the mark D a good way within the enlightened part; which ſhews that the ray EA, when it enters the water at I, goes no longer ſtraight forwards, but is at that place incurvated, and made to go nearer the perpendicular. The other caſe may be tryed by putting any ſmall body into an empty baſon, placed lower than your eye, and then receding from the baſon, till you can but juſt ſee

ſee the body over the brim. After which, if the baſon be filled with water, you ſhall preſently obſerve the body to be viſible, though you go farther off from the baſon. Let ABC (in fig. 125.) denote the baſon as before, D the body in it, E the place of your eye, when the body is ſeen juſt over the edge A, while the baſon is empty. If it be then filled with water, you will obſerve the body ſtill to be viſible, though you take your eye farther off. Suppoſe you ſee the body in this caſe juſt over the brim A, when your eye is at F, it is plain that the rays of light, which come from the body to your eye have not come ſtraight on, but are bent at A, being turned downwards, and more inclined to the ſurface of the water, between A and your eye at F, than they are between A and the body D.

9. THIS we hope is ſufficient to make all our readers apprehend, what the writers of optics mean, when they mention the refraction of the light, or ſpeak of the rays of light being refracted. We ſhall therefore now go on to prove the aſſertion advanced in the forementioned propoſition, in relation to the different kinds of colours, that the direct light of the ſun exhibits to our ſenſe: which may be done in the following manner.

10. IF a room be darkened, and the ſun permitted to ſhine into it through a ſmall hole in the window ſhutter, and be made immediately to fall upon a glaſs priſm, the beam of light ſhall in paſſing through ſuch a priſm be parted into rays, which exhibit all the forementioned colours. In this man-

ner if AB (in fig. 126) represent the window shutter; C the hole in it; DEF the prism; ZY a beam of light coming from the sun, which passes through the hole, and falls upon the prism at Y, and if the prism were removed would go on to X, but in entring the surface EF of the glass it shall be turned off, as has been explained, into the course YW falling upon the second surface of the prism DF in W, going out of which into the air it shall be again farther inflected. Let the light now, after it has passed the prism, be received upon a sheet of paper held at a proper distance, and it shall paint upon the paper the picture, image, or spectrum LM of an oblong figure, whose length shall much exceed its breadth; though the figure shall not be oval, the ends L and M being semicircular and the sides straight. But now this figure will be variegated with colours in this manner. From the extremity M to some length, suppose to the line *no*, it shall be of an intense red; from *no* to *pq* it shall be an orange; from *pq* to *rs* it shall be yellow; from thence to *tu* it shall be green; from thence to *wx* blue; from thence to *yz* indigo; and from thence to the end violet.

11. THUS it appears that the sun's white light by its passage through the prism, is so changed as now to be divided into rays, which exhibit all these several colours. The question is, whether the rays while in the sun's beam before this refraction possessed these properties distinctly; so that some part of that beam would without the rest have given a red colour, and another part alone have given an orange,

Fig. 109.

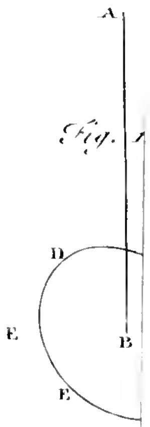
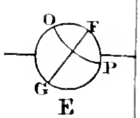
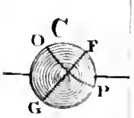
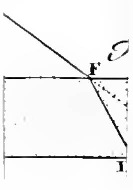


Fig.



9. ○ A





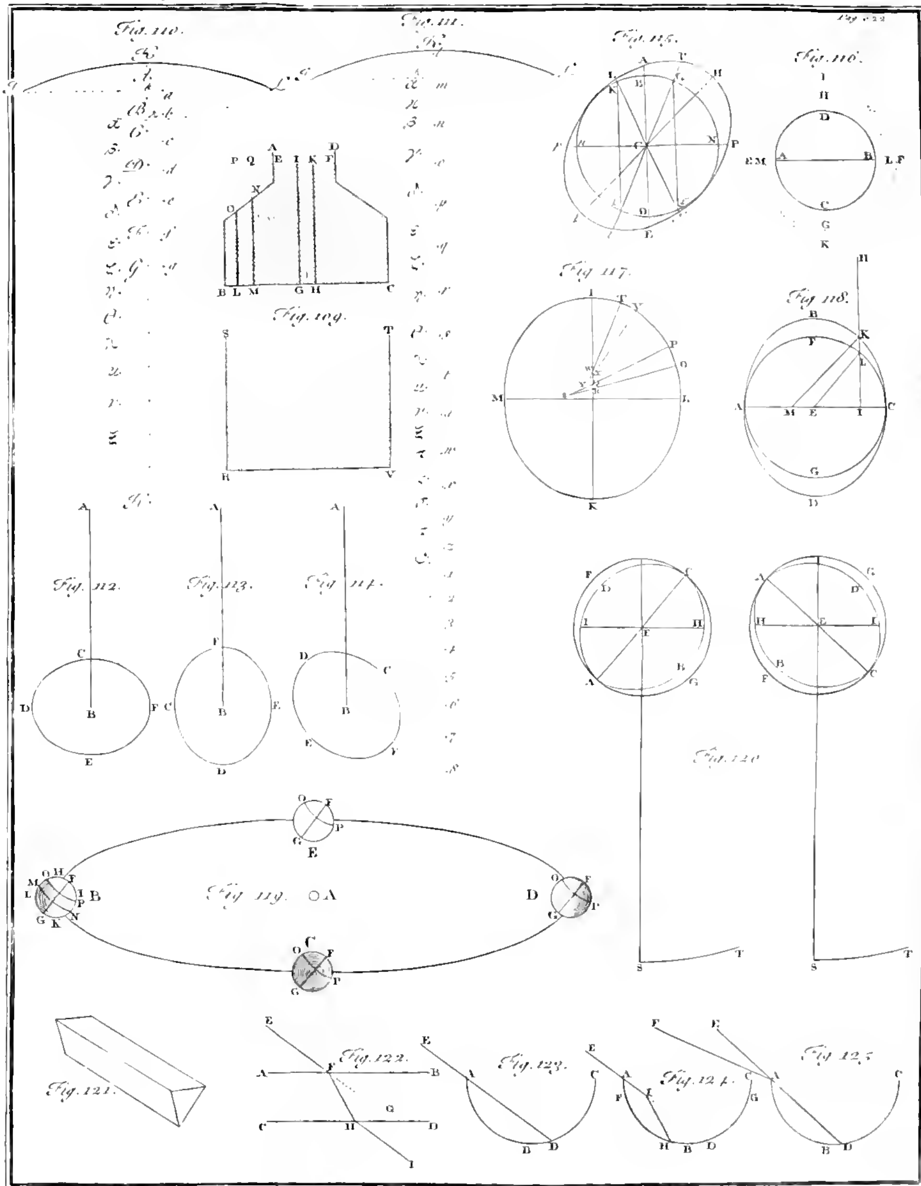


Fig. 110.
Fig. 111.
Fig. 115.
Fig. 116.
Fig. 109.
Fig. 117.
Fig. 118.
Fig. 112.
Fig. 113.
Fig. 114.
Fig. 120
Fig. 119.
Fig. 121.
Fig. 122.
Fig. 123.
Fig. 124.
Fig. 125

orange, &c. That this is poſſible to be the caſe, appears from hence; that if a convex glaſs be placed between the paper and the priſm, which may collect all the rays proceeding out of the priſm into its focus, as a burning glaſs does the ſun's direct rays; and if that focus fall upon the paper, the ſpot formed by ſuch a glaſs upon the paper ſhall appear white, juſt like the ſun's direct light. The reſt remaining as before, let PQ (in fig. 127.) be the convex glaſs, cauſing the rays to meet upon the paper HGIK in the point N, I ſay that point or rather ſpot of light ſhall appear white, without the leaſt tincture of any colour. But it is evident that into this ſpot are now gathered all thoſe rays, which before when ſeparate gave all thoſe different colours; which ſhews that whiteneſs may be made by mixing thoſe colours: eſpecially if we conſider, it can be proved that the glaſs PQ does not alter the colour of the rays which paſs through it. Which is done thus: if the paper be made to approach the glaſs PQ, the colours will manifeſt themſelves as far as the magnitude of the ſpectrum, which the paper receives, will permit. Suppoſe it in the ſituation *hgik*, and that it then receive the ſpectrum *lm*, this ſpectrum ſhall be much ſmaller, than if the glaſs PQ were removed, and therefore the colours cannot be ſo much ſeparated; but yet the extremity *m* ſhall manifeſtly appear red, and the other extremity *l* ſhall be blue; and theſe colours as well as the intermediate ones ſhall diſcover themſelves more perfectly, the farther the paper is removed from N, that is, the larger the ſpectrum is: the ſame thing happens, if the paper be removed farther off from PQ than N. Sup-

poſe into the poſition $\theta\gamma\eta\kappa$, the ſpectrum $\lambda\mu$ painted upon it ſhall again diſcover its colours, and that more diſtinctly, the farther the paper is removed, but only in an inverted order: for as before, when the paper was nearer the convex glaſs, than at N, the upper part of the image was blue, and the under red; now the upper part ſhall be red, and the under blue: becauſe the rays croſs at N.

12. NAY farther that the whiteneſs at the focus N, is made by the union of the colours may be proved without removing the paper out of the focus, by intercepting with any opake body part of the light near the glaſs; for if the under part, that is the red, or more properly the red-making rays, as they are ſtyled by our author, are intercepted, the ſpot ſhall take a bluiſh hue; and if more of the inferior rays are cut off, ſo that neither the red-making nor orange-making rays, and if you pleaſe the yellow-making rays likewiſe, ſhall fall upon the ſpot; then ſhall the ſpot incline more and more to the remaining colours. In like manner if you cut off the upper part of the rays, that is the violet coloured or indigo-making rays, the ſpot ſhall turn reddiſh, and become more ſo, the more of thoſe oppoſite colours are intercepted

13. THIS I think abundantly proves that whiteneſs may be produced by a mixture of all the colours of the ſpectrum. At leaſt there is but one way of evading the preſent arguments, which is, by aſſerting that the rays of light after paſſing the priſm have no different properties to exhibit this or the other colour, but are in that reſpect perfectly

fectly homogeneal, ſo that the rays which paſs to the under and red part of the image do not differ in any properties whatever from thoſe, which go to the upper and violet part of it; but that the colours of the ſpectrum are produced only by ſome new modifications of the rays, made at their incidence upon the paper by the different terminations of light and ſhadow: if indeed this aſſertion can be allowed any place, after what has been ſaid; for it ſeems to be ſufficiently obviated by the latter part of the preceding experiment, that by intercepting the inferior part of the light, which comes from the priſm, the white ſpot ſhall receive a bluiſh caſt, and by ſtopping the upper part the ſpot ſhall turn red, and in both caſes recover its colour, when the intercepted light is permitted to paſs again; though in all theſe trials there is the like termination of light and ſhadow. However our author has contrived ſome experiments expreſly to ſhew the abſurdity of this ſuppoſition; all which he has explained and enlarged upon in ſo diſtinct and expreſſive a manner, that it would be wholly unneceſſary to repeat them in this place[a]. I ſhall only mention that of them, which may be tried in the experiment before us. If you draw upon the paper HGIK, and through the ſpot N, the ſtraight line *wx* parallel to the horizon, and then if the paper be much inclined into the ſituation *rsvt* the line *wx* ſtill remaining parallel to the horizon, the ſpot N ſhall loſe its whiteneſs and receive a blue tincture; but if it be inclined as much the contrary way, the ſame ſpot ſhall exchange its white colour for a reddiſh dye.

[a] Opt. B. I. part 2. prop. 1.

All

All which can never be accounted for by any difference in the termination of the light and ſhadow, which here is none at all; but are eaſily explained by ſuppoſing the upper part of the rays, whenever they enter the eye, diſpoſed to give the ſenſation of the dark colours blue, indigo and violet; and that the under part is fitted to produce the bright colours yellow, orange and red: for when the paper is in the ſituation *r s t u*, it is plain that the upper part of the light falls more directly upon it, than the under part, and therefore thoſe rays will be moſt plentifully reflected from it; and by their abounding in the reflected light will cauſe it to incline to their colour. Juſt ſo when the paper is inclined the contrary way, it will receive the inferior rays moſt directly, and therefore ting the light it reflects with their colour.

14. It is now to be proved that theſe diſpoſitions of the rays of light to produce ſome one colour and ſome another, which manifeſt themſelves after their being refracted, are not wrought by any action of the priſm upon them, but are originally inherent in thoſe rays; and that the priſm only affords each ſpecies an occaſion of ſhewing its diſtinct quality by ſeparating them one from another, which before, while they were blended together in the direct beam of the ſun's light, lay conceal'd. But that this is ſo, will be proved, if it can be ſhewn that no priſm has any power upon the rays, which after their paſſage through one priſm are rendered uncompounded and contain in them but one colour, either to divide that colour into ſeveral, as the ſun's light

light is divided, or ſo much as to change it into any other colour. This will be proved by the following experiment [a]. The ſame thing remaining, as in the firſt experiment, let another priſm NO (in fig. 128.) be placed either immediately, or at ſome diſtance after the firſt, in a perpendicular poſture, ſo that it ſhall refract the rays iſſuing from the firſt ſideways. Now if this priſm could divide the light falling upon it into coloured rays, as the firſt has done, it would divide the ſpectrum breadthwiſe into colours, as before it was divided lengthwiſe; but no ſuch thing is obſerved. If LM were the ſpectrum, which the firſt priſm DEF would paint upon the paper HGIK; PQ lying in an oblique poſture ſhall be the ſpectrum projected by the ſecond, and ſhall be divided lengthwiſe into colours correſponding to the colours of the ſpectrum LM, and occaſioned like them by the refraction of the firſt priſm, but its breadth ſhall receive no ſuch diviſion; on the contrary each colour ſhall be uniform from ſide to ſide, as much as in the ſpectrum LM, which proves the whole aſſertion.

15. THE ſame is yet much farther confirmed by another experiment. Our author teaches that the colours of the ſpectrum LM in the firſt experiment are yet compounded, though not ſo much as in the ſun's direct light. He ſhews therefore how, by placing the priſm at a diſtance from the hole, and by the uſe of a convex glaſs, to ſeparate the colours of the ſpectrum, and make them uncompounded to any degree of exactneſs [b]. And he ſhews when this

[a] Newt. Opt. B. 1. part 1 experim. 5. [b] Ibid prop. 4.

is

is done sufficiently, if you make a small hole in the paper whereon the spectrum is received, through which any one sort of rays may pass, and then let that coloured ray fall so upon a prism, as to be refracted by it, it shall in no case whatever change its colour; but shall always retain it perfectly as at first, however it be refracted [a].

16. Nor yet will these colours after this full separation of them suffer any change by reflection from bodies of different colours; on the other hand they make all bodies placed in these colours appear of the colour which falls upon them [b]: for minium in red light will appear as in open day light; but in yellow light will appear yellow; and which is more extraordinary, in green light will appear green, in blue, blue; and in the violet-purple coloured light will appear of a purple colour; in like manner verdigrease, or blue bise, will put on the appearance of that colour, in which it is placed: so that neither bise placed in the red light shall be able to give that light the least blue tincture, or any other different from red; nor shall minium in the indigo or violet light exhibit the least appearance of red, or any other colour distinct from that it is placed in. The only difference is, that each of these bodies appears most luminous and bright in the colour, which corresponds with that it exhibits in the day light, and dimmest in the colours most remote from that; that is, though minium and bise placed in blue light shall both appear blue, yet the bise shall appear of a bright blue, and the minium of a dusky and obscure blue: but

[a] Newt. Opt. B. 1. part 2. exper. 5. [b] Ibid exper. 6.

if minium and biſe be compared together in red light, the minium ſhall afford a brisk red, the biſe a duller colour, though of the ſame ſpecies.

17. AND this not only proves the immutability of all theſe ſimple and uncompounded colours; but likewiſe unfolds the whole myſtery, why bodies appear in open daylight of ſuch different colours, it conſiſting in nothing more than this, that whereas the white light of the day is compoſed of all ſorts of colours, ſome bodies reflect the rays of one ſort in greater abundance than the rays of any other [a]. Though it appears by the forecited experiment, that almoſt all theſe bodies reflect ſome portion of the rays of every colour, and give the ſenſe of particular colours only by the predominancy of ſome ſorts of rays above the reſt. And what has before been explained of compoſing white by mingling all the colours of the ſpectrum together ſhews clearly, that nothing more is required to make bodies look white, than a power to reflect indifferently rays of every colour. But this will more fully appear by the following method: if near the coloured ſpectrum in our firſt experiment a piece of white paper be ſo held, as to be illuminated equally by all the parts of that ſpectrum, it ſhall appear white; whereas if it be held nearer to the red end of the image, than to the other, it ſhall turn reddiſh; if nearer the blue end, it ſhall ſeem bluiſh [b].

[a] Newton Opt. B. I. prop. 10. [b] Ibid exp. 9.

18. Our indefatigable and circumfpect author farther examined his theory by mixing the powders which painters ufe of feveral colours, in order if poffible to produce a white powder by fuch a compofition [a]. But in this he found fome difficulties for the following reafons. Each of thefe coloured powders reflects but part of the light, which is caft upon them; the red powders reflecting little green or blue, and the blue powders reflecting very little red or yellow, nor the green powders reflecting near fo much of the red or indigo and purple, as of the other colours: and befides, when any of thefe are examined in homogeneal light, as our author calls the colours of the prifm, when well feparated, though each appears more bright and luminous in its own day-light colour, than in any other; yet white bodies, fuppofe white paper for inftance, in thofe very colours exceed thefe coloured bodies themfelves in brightnefs; fo that white bodies reflect not only more of the whole light than coloured bodies do in the day-light, but even more of that very colour which they reflect moft copioufly. All which confiderations make it manifeft that a mixture of thefe will not reflect fo great a quantity of light, as a white body of the fame fize; and therefore will compofe fuch a colour as would refult from a mixture of white and black, fuch as are all grey and dun colours, rather than a ftrong white. Now fuch a colour he compounded of certain ingredients, which he particularly fets down, in fo much that when the compofition was ftrongly illuminated by the fun's direct beams, it would appear much whiter than even white pa-

[a] Newt. Opt. B. I. part 1. exp 15.

per,

per, if conſiderably ſhaded. Nay he found by trials how to proportion the degree of illumination of the mixture and paper, ſo that to a ſpectator at a proper diſtance it could not well be determined which was the more perfect colour; as he experienced not only by himſelf, but by the concurrent opinion of a friend, who chanced to viſit him while he was trying this experiment. I muſt not here omit another method of trying the whiteneſs of ſuch a mixture, propoſed in one of our author's letters on this ſubject [a]: which is to enlighten the compoſition by a beam of the ſun let into a darkened room, and then to receive the light reflected from it upon a piece of white paper, obſerving whether the paper appears white by that reflection; for if it does, it gives proof of the compoſition's being white; becauſe when the paper receives the reflection from any coloured body, it looks of that colour. Agreeable to this is the trial he made upon water impregnated with ſoap, and agitated into a froth [b]: for when this froth after ſome ſhort time exhibited upon the little bubbles, which compoſed it, a great variety of colours, though theſe colours to a ſpectator at a ſmall diſtance diſcover'd themſelves diſtinctly; yet when the eye was ſo far removed, that each little bubble could no longer be diſtinguiſhed, the whole froth by the mixture of all theſe colours appeared intenſly white.

19. OUR author having fully ſatisfied himſelf by theſe and many other experiments, what the reſult is of mixing

[a] Philoſ. Tranſact. N. 88, p. 5099. [b] Opt. B. I. par. 2. exp. 14.

together all the prifmatic colours; he proceeds in the next place to examine, whether this appearance of whitenefs be raifed by the rays of thefe different kinds acting fo, when they meet, upon one another, as to caufe each of them to imprefs the fenfe of whitenefs upon the optic nerve; or whether each ray does not make upon the organ of fight the fame impreffion, as when feparate and alone; fo that the idea of whitenefs is not excited by the impreffion from any one part of the rays, but refults from the mixture of all thofe different fenfations. And that the latter fentiment is the true one, he evinces by undeniable experiments.

20. In particular the foregoing experiment[a], wherein the convex glafs was ufed, furnifhes proofs of this: in that when the paper is brought into the fituation $\theta\gamma\eta\varkappa$, beyond N the colours, that at N difappeared, begin to emerge again; which fhews that by mingling at N they did not lofe their colorific qualities, though for fome reafon they lay concealed. This farther appears by that part of the experiment, when the paper, while in the focus, was directed to be enclined different ways; for when the paper was in fuch a fituation, that it muft of neceffity reflect the rays, which before their arrival at the point N would have given a blue colour, thofe rays in this very point itfelf by abounding in the reflected light tinged it with the fame colour; fo when the paper reflects moft copioufly the rays, which before they come to the point N exhibit rednefs, thofe fame rays tin-

[a] Ibid. exp. 10.

ſture the light reflected by the paper from that very point with their own proper colour.

21. THERE is a certain condition relating to ſight, which affords an opportunity of examining this ſtill more fully: it is this, that the impreſſions of light remain ſome ſhort ſpace upon the eye; as when a burning coal is whirl'd about in a circle, if the motion be very quick, the eye ſhall not be able to diſtinguiſh the coal, but ſhall ſee an entire circle of fire. The reaſon of which appearance is, that the impreſſion made by the coal upon the eye in any one ſituation is not worn out, before the coal returns again to the ſame place, and renews the ſenſation. This gives our author the hint to try, whether theſe colours might not be tranſmitted ſucceſſively to the eye ſo quick, that no one of the colours ſhould be diſtinctly perceived, but the mixture of the ſenſations ſhould produce a uniform whiteneſs; when the rays could not act upon each other, becauſe they never ſhould meet, but come to the eye one after another. And this thought he executed by the following expedient [a]. He made an inſtrument in ſhape like a comb, which he applied near the convex glaſs, ſo that by moving it up and down ſlowly the teeth of it might intercept ſometimes one and ſometimes another colour; and accordingly the light reflected from the paper, placed at N, ſhould change colour continually. But now when the comb-like inſtrument was moved very quick, the eye loſt all preception of the diſtinct colours, which came to it from time to time, a perfect whiteneſs reſulting from the

[a] Opt. pag. 122.

mixture

mixture of all thoſe diſtinct impreſſions in the ſenſorium. Now in this caſe there can be no ſuſpicion of the ſeveral coloured rays acting upon one another, and making any change in each other's manner of affecting the eye, ſeeing they do not ſo much as meet together there.

22. Our author farther teaches us how to view the ſpectrum of colours produced in the firſt experiment with another priſm, ſo that it ſhall appear to the eye under the ſhape of a round ſpot and perfectly white [a]. And in this caſe if the comb be uſed to intercept alternately ſome of the colours, which compoſe the ſpectrum, the round ſpot ſhall change its colour according to the colours intercepted; but if the comb be moved too ſwiftly for thoſe changes to be diſtinctly perceived, the ſpot ſhall ſeem always white, as before [b].

23. Besides this whiteneſs, which reſults from an univerſal compoſition of all ſorts of colours, our author particularly explains the effects of other leſs compounded mixtures; ſome of which compound other colours like ſome of the ſimple ones, but others produce colours different from any of them. For inſtance, a mixture of red and yellow compound a colour like in appearance to the orange, which in the ſpectrum lies between them; as a compoſition of yellow and blue is made uſe of in all dyes to make a green. But red and violet purple compounded make purples unlike to any of the priſmatic colours, and theſe joined with

[a] Opt. B. I. part 2. exp. 11. [b] Ibid prop. 4, 6.

yellow

yellow or blue make yet new colours. Besides one rule is here to be observed, that when many different colours are mixed, the colour which arises from the mixture grows languid and degenerates into whiteness. So when yellow green and blue are mixed together, the compound will be green; but if to this you add red and purple, the colour shall first grow dull and less vivid, and at length by adding more of these colours it shall turn to whiteness, or some other colour [a].

24. Only here is one thing remarkable of those compounded colours, which are like in appearance to the simple ones; that the simple ones when viewed through a prism shall still retain their colour, but the compounded colours seen through such a glass shall be parted into the simple ones of which they are the aggregate. And for this reason any body illuminated by the simple light shall appear through a prism distinctly, and have its minutest parts observable, as may easily be tried with flies, or other such little bodies, which have very small parts; but the same viewed in this manner when enlighten'd with compounded colours shall appear confused, their smallest parts not being distinguishable. How the prism separates these compounded colours, as likewise how it divides the light of the sun into its colours, has not yet been explained; but is reserved for our third chapter.

25. In the mean time what has been said, I hope, will suffice to give a taste of our author's way of arguing, and

[a] Opt. pag. 51.

in

in ſome meaſure to illuſtrate the propoſition laid down in this chapter.

26. There are methods of ſeparating the heterogeneous rays of the ſun's light by reflection, which perfectly conſpire with and confirm this reaſoning. One of which ways may be this. Let AB (in fig. 129) repreſent the window ſhutter of a darkened room; C a hole to let in the ſun's rays; DEF, GHI two priſms ſo applied together, that the ſides EF and GI be contiguous, and the ſides DF, GH parallel; by this means the light will paſs through them without any ſeparation into colours: but if it be afterwards received by a third priſm IKL, it ſhall be divided ſo as to form upon any white body PQ the uſual colours, violet at *m*, blue at *n*, green at *o*, yellow at *r*, and red at *s*. But becauſe it never happens that the two adjacent ſurfaces EF and GI perfectly touch, part only of the light incident upon the ſurface EF ſhall be tranſmitted, and part ſhall be reflected. Let now the reflected part be received by a fourth priſm ΔΘΛ, and paſſing through it paint upon a white body ZΓ the colours of the priſm, red at *t*, yellow at *u*, green at *w*, blue at *x*, violet at *y*. If the priſms DEF, GHI be ſlowly turned about while they remain contiguous, the colours upon the body PQ ſhall not ſenſibly change their ſituation, till ſuch time as the rays become pretty oblique to the ſurface EF; but then the light incident upon the ſurface EF ſhall begin to be wholly reflected. And firſt of all the violet light ſhall be wholly reflected, and thereupon will diſappear at *m*, appearing inſtead thereof at

at y, and increaſing the violet light falling there, the other colours remaining as before. If the priſms DEF, GHI be turned a little farther about, that the incident rays become yet more inclined to the ſurface EF, the blue ſhall be totally reflected, and ſhall diſappear in n, but appear at x by making the colour there more intenſe. And the ſame may be continued, till all the colours are ſucceſſively removed from the ſurface PQ to ZΓ. But in any caſe, ſuppoſe when the violet and the blue have forſaken the ſurface PQ, and appear upon the ſurface ZΓ, the green, yellow, and red only remaining upon the ſurface PQ; if the light be received upon a paper held any where in its whole paſſage between the light's coming out of the priſms DEF, GIH and its incidence upon the priſm IKL, it ſhall appear of the colour compounded of all the colours ſeen upon PQ; and the reflected ray, received upon a piece of white paper held any where between the priſms DEF and ΔΘΣ, ſhall exhibit the colour compounded of thoſe the ſurface PQ is deprived of mixed with the ſun's light: whereas before any of the light was reflected from the ſurface EF, the rays between the priſms GHI and IKL would appear white; as will likewiſe the reflected ray both before and after the total reflection, provided the difference of refraction by the ſurfaces DF and DE be inconſiderable. I call here the ſun's light white, as I have all along done; but it is more exact to aſcribe to it ſomething of a yellowiſh tincture, occaſioned by the brighter colours abounding in it; which caution is neceſſary in examining the colours of the reflected beam, when all the violet and blue are in it: for this

yellowiſh turn of the ſun's light cauſes the blue not to be quite ſo viſible in it, as it ſhould be, were the light perfectly white; but makes the beam of light incline rather towards a pale white.

CHAP. II.

Of the properties of BODIES, upon which their COLOURS depend.

AFTER having ſhewn in the laſt chapter, that the difference between the colours of bodies viewed in open day-light is only this, that ſome bodies are diſpoſed to reflect rays of one colour in the greateſt plenty, and other bodies rays of ſome other colour; order now requires us to examine more particularly into the property of bodies, which gives them this difference. But this our author ſhews to be nothing more, than the different magnitude of the particles, which compoſe each body: this I queſtion not will appear no ſmall paradox. And indeed this whole chaper will contain ſcarce any aſſertions, but what will be almoſt incredible, though the arguments for them are ſo ſtrong and convincing, that they force our aſſent. In the former chapter have been explained properties of light, not in the leaſt thought of before our author's diſcovery of them; yet are they not difficult to admit, as ſoon as experiments are known to give proof of their reality; but ſome of the propoſitions to be ſtated here will, I fear, be accounted almoſt paſt belief; notwithſtanding that the arguments, by which they

they are eſtabliſhed are unanſwerable. For it is proved by our author, that bodies are rendered tranſparent by the minuteneſs of their pores, and become opake by having them large; and more, that the moſt tranſparent body by being reduced to a great thinneſs will become leſs pervious to the light.

2. BUT whereas it had been the received opinion, and yet remains ſo among all who have not ſtudied this philoſophy, that light is reflected from bodies by its impinging againſt their ſolid parts, rebounding from them, as a tennis ball or other elaſtic ſubſtance would do, when ſtruck againſt any hard and reſiſting ſurface; it will be proper to begin with declaring our author's ſentiment concerning this, who ſhews by many arguments that reflection cannot be cauſed by any ſuch means [a]: ſome few of his proofs I ſhall ſet down, referring the reader to our author himſelf for the reſt.

3. IT is well known, that when light fails upon any tranſparent body, glaſs for inſtance, part of it is reflected and part tranſmitted; for which it is ready to account, by ſaying that part of the light enters the pores of the glaſs, and part impinges upon its ſolid parts. But when the tranſmitted light arrives at the farther ſurface of the glaſs, in paſſing out of glaſs into air there is as ſtrong a reflection cauſed, or rather ſomething ſtronger. Now it is not to be conceived, how the light ſhould find as many ſolid parts in the air to ſtrike againſt as in the glaſs, or even a greater num-

[a] Opt. Book II. prop. 8.

ber of them. And to augment the difficulty, if water be placed behind the glaſs, the reflection becomes much weaker. Can we therefore ſay, that water has fewer ſolid parts for the light to ſtrike againſt, than the air? And if we ſhould, what reaſon can be given for the reflection's being ſtronger, when the air by the air-pump is removed from behind the glaſs, than when the air receives the rays of light. Beſides the light may be ſo inclined to the hinder ſurface of the glaſs, that it ſhall wholly be reflected, which happens when the angle which the ray makes with the ſurface does not exceed about $49\frac{1}{3}$ degrees; but if the inclination be a very little increaſed, great part of the light will be tranſmitted; and how the light in one caſe ſhould meet with nothing but the ſolid parts of the air, and by ſo ſmall a change of its inclination find pores in great plenty, is wholly inconceivable. It cannot be ſaid, that the light is reflected by ſtriking againſt the ſolid parts of the ſurface of the glaſs; becauſe without making any change in that ſurface, only by placing water contiguous to it inſtead of air, great part of that light ſhall be tranſmitted, which could find no paſſage through the air. Moreover in the laſt experiment recited in the preceding chapter, when by turning the priſms DEF, GHI, the blue light became wholly reflected, while the reſt was moſtly tranſmitted, no poſſible reaſon can be aſſigned, why the blue-making rays ſhould meet with nothing but the ſolid parts of the air between the priſms, and the reſt of the light in the very ſame obliquity find pores in abundance. Nay farther, when two glaſſes touch each other, no reflection at all is made; though it

it does not in the leaſt appear, how the rays ſhould avoid the ſolid parts of glaſs, when contiguous to other glaſs, any more than when contiguous to air. But in the laſt place upon this ſuppoſition it is not to be comprehended, how the moſt poliſhed ſubſtances could reflect the light in that regular manner we find they do; for when a poliſhed looking glaſs is covered over with quick ſilver, we cannot ſuppoſe the particles of light ſo much larger than thoſe of the quickſilver, that they ſhould not be ſcattered as much in reflection, as a parcel of marbles thrown down upon a rugged pavement. The only cauſe of ſo uniform and regular a reflection muſt be ſome more ſecret cauſe, uniformly ſpread over the whole ſurface of the glaſs.

4. BUT now, ſince the reflection of light from bodies does not depend upon its impinging againſt their ſolid parts, ſome other reaſon muſt be ſought for. And firſt it is paſt doubt that the leaſt parts of almoſt all bodies are tranſparent, even the microſcope ſhewing as much [a]; beſides that it may be experienced by this method. Take any thin plate of the opakeſt body, and apply it to a ſmall hole deſigned for the admiſſion of light into a darkened room; however opake that body may ſeem in open day-light, it ſhall under theſe circumſtances ſufficiently diſcover its tranſparency, provided only the body be very thin. White metals indeed do not eaſily ſhew themſelves tranſparent in theſe trials, they reflecting almoſt all the light incident upon them at their firſt ſuperficies; the cauſe of which will appear in what

[a] Opt. Book II. par. 3. prop. 2.

follows

follows[a]. But yet these substances, when reduced into parts of extraordinary minuteness by being dissolved in aqua fortis or the like corroding liquors do also become transparent.

5. SINCE therefore the light finds free passage through the least parts of bodies, let us consider the largeness of their pores, and we shall find, that whenever a ray of light has passed through any particle of a body, and is come to its farther surface, if it finds there another particle contiguous, it will without interruption pass into that particle; just as light will pass through one piece of glass into another piece in contact with it without any impediment, or any part being reflected: but as the light in passing out of glass, or any other transparent body, shall part of it be reflected back, if it enter into air or other transparent body of a different density from that it passes out of; the same thing will happen in the light's passage through any particle of a body, whenever at its exit out of that particle it meets no other particle contiguous, but must enter into a pore, for in this case it shall not all pass through, but part of it be reflected back. Thus will the light, every time it enters a pore, be in part reflected; so that nothing more seems necessary to opacity, than that the particles, which compose any body, touch but in very few places, and that the pores of it are numerous and large, so that the light may in part be reflected from it, and the other part, which enters too deep to be returned out of the body, by numerous reflections may be stifled and lost[b]; which in all probability

[a] § 17. [b] Opt. Book II. par. 3. prop. 4.

lity happens, as often as it impinges againſt the ſolid part of the body, all the light which does ſo not being reflected back, but ſtopt, and deprived of any farther motion [a].

6. THIS notion of opacity is greatly confirmed by the obſervation, that opake bodies become tranſparent by filling up the pores with any ſubſtance of near the ſame denſity with their parts. As when paper is wet with water or oyl; when linnen cloth is either dipt in water, oyled, or varniſhed; or the oculus mundi ſtone ſteeped in water [b]. All which experiments confirm both the firſt aſſertion, that light is not reflected by ſtriking upon the ſolid parts of bodies; and alſo the ſecond, that its paſſage is obſtructed by the reflections it undergoes in the pores; ſince we find it in theſe trials to paſs in greater abundance through bodies, when the number of their ſolid parts is increaſed, only by taking away in great meaſure thoſe reflections; which filling the pores with a ſubſtance of near the ſame denſity with the parts of the body will do. Beſides as filling the pores of a dark body makes it tranſparent; ſo on the other hand evacuating the pores of a body tranſparent, or ſeparating the parts of ſuch a body, renders it opake. As ſalts or wet paper by being dried, glaſs by being reduced to powder or the ſurface made rough; and it is well known that glaſs veſſels diſcover cracks in them by their opacity. Juſt ſo water itſelf becomes impervious to the light by being formed into many ſmall bubbles, whether in froth, or by being mixed and agitated with any quantity of a liquor

[a] Opt. Book II. pag. 241. [b] Ibid. pag. 224.

with which it will not incorporate, ſuch as oyl of turpentine, or oyl olive.

7. A certain electrical experiment made by Mr. Hauksbee may not perhaps be uſeleſs to clear up the preſent ſpeculation, by ſhewing that ſomething more is neceſſary beſides mere poroſity for tranſmitting freely other fine ſubſtances. The experiment is this; that a glaſs cane rubbed till it put forth its electric quality would agitate leaf braſs incloſed under a glaſs veſſel, though not at ſo great a diſtance, as if no body had intervened; yet the ſame cane would loſe all its influence on the leaf braſs by the interpoſition of a piece of the fineſt muſlin, whoſe pores are immenſely larger and more patent than thoſe of glaſs.

8. Thus I have endeavoured to ſmooth my way, as much as I could, to the unfolding yet greater ſecrets in nature; for I ſhall now proceed to ſhew the reaſon why bodies appear of different colours. My reader no doubt will be ſufficiently ſurprized, when I inform him that the knowledge of this is deduced from that ludicrous experiment, with which children divert themſelves in blowing bubbles of water made tenacious by the ſolution of ſoap. And that theſe bubbles, as they gradually grow thinner and thinner till they break, change ſucceſſively their colours from the ſame principle, as all natural bodies preſerve theirs.

9. Our author after preparing water with ſoap, ſo as to render it very tenacious, blew it up into a bubble, and placing

ing it under a glaſs, that it might not be irregularly agitated by the air, obſerved as the water by ſubſiding changed the thickneſs of the bubble, making it gradually leſs and leſs till the bubble broke; there ſucceſſively appeared colours at the top of the bubble, which ſpread themſelves into rings ſurrounding the top and deſcending more and more, till they vaniſhed at the bottom in the ſame order in which they appeared [a]. The colours emerged in this order: firſt red, then blue; to which ſucceeded red a ſecond time, and blue immediately followed; after that red a third time, ſucceeded by blue; to which followed a fourth red, but ſucceeded by green; after this a more numerous order of colours, firſt red, then yellow, next green, and after that blue, and at laſt purple; then again red, yellow, green, blue, violet followed each other in order; and in the laſt place red, yellow, white, blue; to which ſucceeded a dark ſpot, which reflected ſcarce any light, though our author found it did make ſome very obſcure reflection, for the image of the ſun or a candle might be faintly diſcerned upon it; and this laſt ſpot ſpread itſelf more and more, till the bubble at laſt broke. Theſe colours were not ſimple and uncompounded colours, like thoſe which are exhibited by the priſm, when due care is taken to ſeparate them; but were made by a various mixture of thoſe ſimple colours, as will be ſhewn in the next chapter: whence theſe colours, to which I have given the name of blue, green, or red, were not all alike, but differed as follows. The blue, which appeared next the dark ſpot, was a pure colour, but very faint, reſembling the ſky-colour; the

[a] Ibid. Obſ. 17. &c.

white next to it a very ſtrong and intenſe white, brighter much than the white, which the bubble reflected, before any of the colours appeared. The yellow which preceded this was at firſt pretty good, but ſoon grew dilute; and the red which went before the yellow at firſt gave a tincture of ſcarlet inclining to violet, but ſoon changed into a brighter colour; the violet of the next ſeries was deep with little or no redneſs in it; the blue a briſk colour, but came much ſhort of the blue in the next order; the green was but dilute and pale; the yellow and red were very bright and full, the beſt of all the yellows which appeared among any of the colours: in the preceding orders the purple was reddiſh, but the blue, as was juſt now ſaid, the brighteſt of all; the green pretty lively better than in the order which appeared before it, though that was a good willow green; the yellow but ſmall in quantity, though bright; the red of this order not very pure: thoſe which appeared before yet more obſcure, being very dilute and dirty; as were likewiſe the three firſt blues.

10. Now it is evident, that theſe colours aroſe at the top of the bubble, as it grew by degrees thinner and thinner: but what the expreſs thickneſs of the bubble was, where each of theſe colours appeared upon it, could not be determined by theſe experiments; but was found by another means, viz. by taking the object glaſs of a long teleſcope, which is in a ſmall degree convex, and placing it upon a flat glaſs, ſo as to touch it in one point, and then water being put between them, the ſame colours appeared as in the bubble,

bubble, in the form of circles or rings ſurrounding the point where the glaſſes touched, which appeared black for want of any reflection from it, like the top of the bubble when thinneſt[a]: next to this ſpot lay a blue circle, and next without that a white one; and ſo on in the ſame order as before, reckoning from the dark ſpot. And henceforward I ſhall ſpeak of each colour, as being of the firſt, ſecond, or any following order, as it is the firſt, ſecond, or any following one, counting from the black ſpot in the center of theſe rings; which is contrary to the order in which I muſt have mentioned them, if I ſhould have reputed them the firſt, ſecond, or third, &c. in order, as they ariſe after one another upon the top of the bubble.

11. BUT now by meaſuring the diameters of each of theſe rings, and knowing the convexity of the teleſcope glaſs, the thickneſs of the water at each of thoſe rings may be determined with great exactneſs: for inſtance the thickneſs of it, where the white light of the firſt order is reflected, is about $3\frac{7}{8}$ ſuch parts, of which an inch contains 1000000[b]. And this meaſure gives the thickneſs of the bubble, where it appeared of this white colour, as well as of the water between the glaſſes; though the tranſparent body which ſurrounds the water in theſe two caſes be very different: for our author found, that the condition of the ambient body would not alter the ſpecies of the colour at all, though it might its ſtrength and brightneſs; for pieces of Muſcovy glaſs, which were ſo thin as to appear coloured by being

[a] Ibid. Obſ. 10. [b] Ibid. pag. 206.

wet with water, would have their colours faded and made leſs bright thereby; but he could not obſerve their ſpecies at all to be changed. So that the thickneſs of any tranſparent body determines its colour, whatever body the light paſſes through in coming to it [a].

12. BUT it was found that different tranſparent bodies would not under the ſame thickneſſes exhibit the ſame colours: for if the forementioned glaſſes were laid upon each other without any water between their ſurfaces, the air itſelf would afford the ſame colours as the water, but more expanded, inſomuch that each ring had a larger diameter, and all in the ſame proportion. So that the thickneſs of the air proper to each colour was in the ſame proportion larger, than the thickneſs of the water appropriated to the ſame [b].

13. IF we examine with care all the circumſtances of theſe colours, which will be enumerated in the next chapter, we ſhall not be ſurprized, that our author takes them to bear a great analogy to the colours of natural bodies [c]. For the regularity of thoſe various and ſtrange appearances relating to them, which makes the moſt myſterious part of the action between light and bodies, as the next chapter will ſhew, is ſufficient to convince us that the principle, from which they flow, is of the greateſt importance in the frame of nature; and therefore without queſtion is deſigned for no leſs a purpoſe than to give bodies their various colours, to which end it ſeems very fitly ſuited. For if any ſuch tranſ-

[a] Obſer. 21. [b] Obſerv. 5. compared with Obſerv. 10. [c] Ibid. prop. 5.

parent

parent ſubſtance of the thickneſs proper to produce any one colour ſhould be cut into ſlender threads, or broken into fragments, it does not appear but theſe ſhould retain the ſame colour; and a heap of ſuch fragments ſhould frame a body of that colour. So that this is without diſpute the cauſe why bodies are of this or the other colour, that the particles of which they are compoſed are of different ſizes. Which is farther confirmed by the analogy between the colours of thin plates, and the colours of many bodies. For example, theſe plates do not look of the ſame colour when viewed obliquely, as when ſeen direct; for if the rings and colours between a convex and plane glaſs are viewed firſt in a direct manner, and then at different degrees of obliquity, the rings will be obſerved to dilate themſelves more and more as the obliquity is increaſed [a]; which ſhews that the tranſparent ſubſtance between the glaſſes does not exhibit the ſame colour at the ſame thickneſs in all ſituations of the eye: juſt ſo the colours in the very ſame part of a peacock's tail change, as the tail changes poſture in reſpect of the ſight. Alſo the colours of ſilks, cloths, and other ſubſtances, which water or oyl can intimately penetrate, become faint and dull by the bodies being wet with ſuch fluids, and recover their brightneſs again when dry; juſt as it was before ſaid that plates of Muſcovy glaſs grew faint and dim by wetting. To this may be added, that the colours which painters uſe will be a little changed by being ground very elaborately, without queſtion by the diminution of their parts. All which particulars, and many more that

[a] Obſerv. 7.

might

might be extracted from our author, give abundant proof of the present point. I ſhall only ſubjoin one more: theſe tranſparent plates tranſmit through them all the light they do not reflect; ſo that when looked through they exhibit thoſe colours, which reſult from the depriving white light of the colour reflected. This may commodiouſly be tryed by the glaſſes ſo often mentioned; which if looked through exhibit coloured rings as by reflected light, but in a contrary order; for the middle ſpot, which in the other view appears black for want of reflected light, now looks perfectly white, oppoſite to the blue circle; next without this ſpot the light appears tinged with a yellowiſh red; where the white circle appeared before, it now ſeems dark; and ſo of the reſt [a]. Now in the ſame manner, the light tranſmitted through foliated gold into a darkened room appears greeniſh by the loſs of the yellow light, which gold reflects.

14. HENCE it follows, that the colours of bodies give a very probable ground for making conjecture concerning the magnitude of their conſtituent particles [b]. My reaſon for calling it a conjecture is, its being difficult to fix certainly the order of any colour. The green of vegetables our author judges to be of the third order, partly becauſe of the intenſeneſs of their colour; and partly from the changes they ſuffer when they wither, turning at firſt into a greeniſh or more perfect yellow, and afterwards ſome of them to an orange or red; which changes ſeem to be effected from their tinging particles growing denſer by the exhalation of their

[a] Obſerv. 9. [b] Ibid prop. 7.

moiſture, and perhaps augmented likewiſe by the accretion of the earthy and oily parts of that moiſture. How the mentioned colours ſhould ariſe from increaſing the bulk of thoſe particles, is evident; ſeeing thoſe colours lie without the ring of green between the glaſſes, and are therefore formed where the tranſparent ſubſtance which reflects them is thicker. And that the augmentation of the denſity of the colorific particles will conſpire to the production of the ſame effect, will be evident; if we remember what was ſaid of the different ſize of the rings, when air was included between the glaſſes, from their ſize when water was between them; which ſhewed that a ſubſtance of a greater denſity than another gives the ſame colour at a leſs thickneſs. Now the changes likely to be wrought in the denſity or magnitude of the parts of vegetables by withering ſeem not greater, than are ſufficient to change their colour into thoſe of the ſame order; but the yellow and red of the fourth order are not full enough to agree with thoſe, into which theſe ſubſtances change, nor is the green of the ſecond ſufficiently good to be the colour of vegetables; ſo that their colour muſt of neceſſity be of the third order.

15. THE blue colour of ſyrup of violets our author ſuppoſes to be of the third order; for acids, as vinegar, with this ſyrup change it red, and ſalt of tartar or other alcalies mixed therewith turn it green. But if the blue colour of the ſyrup were of the ſecond order, the red colour, which acids by attenuating its parts give it, muſt be of the firſt order, and the green given it by alcalies by incraſſating

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its particles ſhould be of the ſecond; whereas neither of thoſe colours is perfect enough, eſpecially the green, to anſwer thoſe produced by theſe changes; but the red may well enough be allowed to be of the ſecond order, and the green of the third; in which caſe the blue muſt be likewiſe of the third order.

16. The azure colour of the skies our author takes to be of the firſt order, which requires the ſmalleſt particles of any colour, and therefore moſt like to be exhibited by vapours, before they have ſufficiently coaleſced to produce clouds of other colours.

17. The moſt intenſe and luminous white is of the firſt order, if leſs ſtrong it is a mixture of the colours of all the orders. Of the latter ſort he takes the colour of linnen, paper, and ſuch like ſubſtances to be; but white metals to be of the former ſort. The arguments for it are theſe. The opacity of all bodies has been ſhewn to ariſe from the number and ſtrength of the reflections made within them; but all experiments ſhew, that the ſtrongeſt reflection is made at thoſe ſurfaces, which intercede tranſparent bodies differing moſt in denſity. Among other inſtances of this, the experiments before us afford one; for when air only is included between the glaſſes, the coloured rings are not only more dilated, as has before been ſaid, than when water is between them; but are likewiſe much more luminous and bright. It follows therefore, that whatever medium pervades the pores of bodies, if ſo be there

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is any, thoſe ſubſtances muſt be moſt opake, the denſity of whoſe parts differs moſt from the denſity of the medium, which fills their pores. But it has been ſufficiently proved in the former part of this tract, that there is no very denſe medium lodging in, at leaſt pervading at liberty the pores of bodies. And it is farther proved by the preſent experiments. For when air is incloſed by the denſer ſubſtance of glaſs, the rings dilate themſelves, as has been ſaid, by being viewed obliquely; this they do ſo very much, that at different obliquities the ſame thickneſs of air will exhibit all ſorts of colours. The bubble of water, though ſurrounded with the thinner ſubſtance of air, does likewiſe change its colour by being viewed obliquely; but not any thing near ſo much, as in the other caſe; for in that the ſame colour might be ſeen, when the rings were viewed moſt obliquely, at more than twelve times the thickneſs it appeared at under a direct view; whereas in this other caſe the thickneſs was never found conſiderably above half as much again. Now the colours of bodies not depending only on the light, that is incident upon them perpendicularly, but likewiſe upon that, which falls on them in all degrees of obliquity; if the medium ſurrounding their particles were denſer than thoſe particles, all ſorts of colours muſt of neceſſity be reflected from them ſo copiouſly, as would make the colours of all bodies white, or grey, or at beſt very dilute and imperfect. But on the other hand, if the medium in the pores of bodies be much rarer than their particles, the colour reflected will be ſo little changed by the obliquity of the rays, that the colour produced by the rays, which fall near the perpendicular, may

ſo much abound in the reflected light, as to give the body their colour with little allay. To this may be added, that when the difference of the contiguous tranſparent ſubſtances is the ſame, a colour reflected from the denſer ſubſtance reduced into a thin plate and ſurrounded by the rarer will be more brisk, than the ſame colour will be, when reflected from a thin plate formed of the rarer ſubſtance, and ſurrounded by the denſer; as our author experienced by blowing glaſs very thin at a lamp furnace, which exhibited in the open air more vivid colours, than the air does between two glaſſes. From theſe conſiderations it is manifeſt, that if all other circumſtances are alike, the denſeſt bodies will be moſt opake. But it was obſerved before, that theſe white metals can hardly be made ſo thin, except by being diſſolved in corroding liquors, as to be rendred tranſparent; though none of them are ſo denſe as gold, which proves their great opacity to have ſome other cauſe beſides their denſity; and none is more fit to produce this, than ſuch a ſize of their particles, as qualifies them to reflect the white of the firſt order.

18. For producing black the particles ought to be ſmaller than for exhibiting any of the colours, viz. of a ſize anſwering to the thickneſs of the bubble, where by reflecting little or no light it appears colourleſs; but yet they muſt not be too ſmall, for that will make them tranſparent through deficiency of reflections in the inward parts of the body, ſufficient to ſtop the light from going through it; but they muſt be of a ſize bordering upon that

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diſpoſed to reflect the faint blue of the firſt order, which affords an evident reaſon why blacks uſually partake a little of that colour. We ſee too, why bodies diſſolved by fire or putrefaction turn black: and why in grinding glaſſes upon copper plates the duſt of the glaſs, copper, and ſand it is ground with, become very black: and in the laſt place why theſe black ſubſtances communicate ſo eaſily to others their hue; which is, that their particles by reaſon of the great minuteneſs of them eaſily overſpread the groſſer particles of others.

19. I SHALL now finiſh this chapter with one remark of the exceeding great poroſity in bodies neceſſarily required in all that has here been ſaid; which, when duly conſidered, muſt appear very ſurprizing; but perhaps it will be matter of greater ſurprize, when I affirm that the ſagacity of our author has diſcovered a method, by which bodies may eaſily become ſo; nay how any the leaſt portion of matter may be wrought into a body of any aſſigned dimenſions how great ſo ever, and yet the pores of that body none of them greater, than any the ſmalleſt magnitude propoſed at pleaſure; notwithſtanding which the parts of the body ſhall ſo touch, that the body itſelf ſhall be hard and ſolid [a]. The manner is this: ſuppoſe the body be compounded of particles of ſuch figures, that when laid together the pores found between them may be equal in bigneſs to the particles; how this may be effected, and yet the body be hard and ſolid, is not difficult to underſtand; and the pores of ſuch a bo-

[a] Opt. pag. 243.

dy may be made of any propofed degree of fmallnefs. But the folid matter of a body fo framed will take up only half the fpace occupied by the body; and if each conftituent particle be compofed of other lefs particles according to the fame rule, the folid parts of fuch a body will be but a fourth part of its bulk; if every one of thefe leffer particles again be compounded in the fome manner, the folid parts of the whole body fhall be but one eighth of its bulk; and thus by continuing the compofition the folid parts of the body may be made to bear as fmall a proportion to the whole magnitude of the body, as fhall be defired, notwithftanding the body will be by the contiguity of its parts capable of being in any degree hard. Which fhews that this whole globe of earth, nay all the known bodies in the univerfe together, as far as we know, may be compounded of no greater a portion of folid matter, than might be reduced into a globe of one inch only in diameter, or even lefs. We fee therefore how by this means bodies may eafily be made rare enough to tranfmit light, with all that freedom pellucid bodies are found to do. Though what is the real ftructure of bodies we yet know not.

Chap. III.

Of the Refraction, Reflection, and Inflection of Light.

THUS much of the colours of natural bodies; our method now leads us to fpeculations yet greater, no lefs

leſs than to lay open the cauſes of all that has hitherto been related. For it muſt in this chapter be explained, how the priſm ſeparates the colours of the ſun's light, as we found in the firſt chapter; and why the thin tranſparent plates diſcourſed of in the laſt chapter, and conſequently the particles of coloured bodies, reflect that diverſity of colours only by being of different thickneſſes.

2. FOR the firſt it is proved by our author, that the colours of the ſun's light are manifeſted by the priſm, from the rays undergoing different degrees of refraction; that the violet-making rays, which go to the upper part of the coloured image in the firſt experiment of the firſt chapter, are the moſt refracted; that the indigo-making rays are refracted, or turned out of their courſe by paſſing through the priſm, ſomething leſs than the violet-making rays, but more than the blue-making rays; and the blue-making rays more than the green; the green-making rays more than the yellow; the yellow more than the orange; and the orange-making rays more than the red-making, which are leaſt of all refracted. The firſt proof of this, that rays of different colours are refracted unequally is this. If you take any body, and paint one half of it red and the other half blue, then upon viewing it through a priſm thoſe two parts ſhall appear ſeparated from each other; which can be cauſed no otherwiſe than by the priſm's refracting the light of one half more than the light of the other half. But the blue half will be moſt refracted; for if the body be ſeen through the priſm in ſuch a ſituation, that the body ſhall appear

lifted

lifted upwards by the refraction, as a body within a bason of water, in the experiment mentioned in the first chapter, appeared to be lifted up by the refraction of the water, so as to be seen at a greater distance than when the bason is empty, then shall the blue part appear higher than the red; but if the refraction of the prism be the contrary way, the blue part shall be depressed more than the other. Again, after laying fine threads of black silk across each of the colours, and the body well inlightened, if the rays coming from it be received upon a convex glass, so that it may by refracting the rays cast the image of the body upon a piece of white paper held beyond the glass; then it will be seen that the black threads upon the red part of the image, and those upon the blue part, do not at the same time appear distinctly in the image of the body projected by the glass; but if the paper be held so, that the threads on the blue part may distinctly appear, the threads cannot be seen distinct upon the red part; but the paper must be drawn farther off from the convex glass to make the threads on this part visible; and when the distance is great enough for the threads to be seen in this red part, they become indistinct in the other. Whence it appears that the rays proceeding from each point of the blue part of the body are sooner united again by the convex glass than the rays which come from each point of the red parts[a]. But both these experiments prove that the blue-making rays, as well in the small refraction of the convex glass, as in the greater refraction of the prism, are more bent, than the red-making rays.

[a] Newt. Opt. B. I. part. 1. prop. I.

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3. THIS seems already to explain the reason of the coloured spectrum made by refracting the sun's light with a prism; though our author proceeds to examine that in particular, and proves that the different coloured rays in that spectrum are in different degrees refracted; by shewing how to place the prism in such a posture, that if all the rays were refracted in the same manner, the spectrum should of necessity be round: whereas in that case if the angle made by the two surfaces of the prism, through which the light passes, that is the angle DFE in fig. 126, be about 63 or 64 degrees, the image instead of being round shall be near five times as long as broad; a difference enough to shew a great inequality in the refractions of the rays, which go to the opposite extremities of the image. To leave no scruple unremoved, our author is very particular in shewing by a great number of experiments, that this inequality of refraction is not casual, and that it does not depend upon any irregularities of the glass; no nor that the rays are in their passage through the prism each split and divided; but on the contrary that every ray of the sun has its own peculiar degree of refraction proper to it, according to which it is more or less refracted in passing through pellucid substances always in the same manner[a]. That the rays are not split and multiplied by the refraction of the prism, the third of the experiments related in our first chapter shews very clearly; for if they were, and the length of the spectrum in the first refraction were thereby occasioned, the breadth should be no less dilated by the cross refraction of the se-

[a] Opt. B. I. part. 1. prop. 2.

cond

cond prism; whereas the breadth is not at all increased, but the image is only thrown into an oblique posture by the upper part of the rays which were at first more refracted than the under part, being again turned farthest out of their course. But the experiment most expresly adapted to prove this regular diversity of refraction is this, which follows[a]. Two boards AB, CD (in fig. 130.) being erected in a darkened room at a proper distance, one of them AB being near the window-shutter EF, a space only being left for the prism GHI to be placed between them; so that the rays entring at the hole M of the window-shutter may after passing through the prism be trajected through a smaller hole K made in the board AB, and passing on from thence go out at another hole L made in the board CD of the same size as the hole K, and small enough to transmit the rays of one colour only at a time; let another prism NOP be placed after the board CD to receive the rays passing through the holes K and L, and after refraction by that prism let those rays fall upon the white surface QR. Suppose first the violet light to pass through the holes, and to be refracted by the prism NOP to *s*, which if the prism NOP were removed should have passed right on to W. If the prism GHI be turned slowly about, while the boards and prism NOP remain fixed, in a little time another colour will fall upon the hole L, which, if the prism NOP were taken away, would proceed like the former rays to the same point W; but the refraction of the prism NOP shall not carry these rays to *s*, but to some place less distant from W as

[a] Opt. B. I. part 1. Expec. 6.

to

to t. Suppose now the rays which go to t to be the indigo-making rays. It is manifest that the boards A B, C D, and prism N O P remaining immoveable, both the violet-making and indigo-making rays are incident alike upon the prism N O P, for they are equally inclined to its surface O P, and enter it in the same part of that surface; which shews that the indigo-making rays are less diverted out of their course by the refraction of the prism, than the violet-making rays under an exact parity of all circumstances. Farther, if the prism G H I be more turned about, 'till the blue-making rays pass through the hole L, these shall fall upon the surface Q R below I, as at v, and therefore are subjected to a less refraction than the indigo-making rays. And thus by proceeding it will be found that the green-making rays are less refracted than the blue-making rays, and so of the rest, according to the order in which they lie in the coloured spectrum.

4. THIS disposition of the different coloured rays to be refracted some more than others our author calls their respective degrees of refrangibility. And since this difference of refrangibility discovers it self to be so regular, the next step is to find the rule it observes.

5. IT is a common principle in optics, that the sine of the angle of incidence bears to the sine of the refracted angle a given proportion. If A B (in fig. 131, 132) represent the surface of any refracting substance, suppose of water or glass, and C D a ray of light incident upon that sur-

 face

face in the point D, let DE be the ray, after it has paſſed the ſurface AB; if the ray paſs out of the air into the ſubſtance whoſe ſurface is AB (as in fig. 131) it ſhall be turned from the ſurface, and if it paſs out of that ſubſtance into air it ſhall be bent towards it (as in fig. 132) But if FG be drawn through the point D perpendicular to the ſurface AB, the angle under CDF made by the incident ray and this perpendicular is called the angle of incidence; and the angle under EDG, made by this perpendicular and the ray after refraction, is called the refracted angle. And if the circle HFIG be deſcribed with any interval cutting CD in H and DE in I, then the perpendiculars HK, IL being let fall upon FG, HK is called the ſine of the angle under CDF the angle of incidence, and IL the ſine of the angle under EDG the refracted angle. The firſt of theſe ſines is called the ſine of the angle of incidence, or more briefly the ſine of incidence, the latter is the ſine of the refracted angle, or the ſine of refraction. And it has been found by numerous experiments that whatever proportion the ſine of incidence HK bears to the ſine of refraction IL in any one caſe, the ſame proportion ſhall hold in all caſes; that is, the proportion between theſe ſines will remain unalterably the ſame in the ſame refracting ſubſtance, whatever be the magnitude of the angle under CDF.

6. But now becauſe optical writers did not obſerve that every beam of white light was divided by refraction, as has been here explained, this rule collected by them can only be underſtood in the groſs of the whole beam after refraction

fraction, and not ſo much of any particular part of it, or at moſt only of the middle part of the beam. It therefore was incumbent upon our author to find by what law the rays were parted from each other; whether each ray apart obtained this property, and that the ſeparation was made by the proportion between the ſines of incidence and refraction being in each ſpecies of rays different; or whether the light was divided by ſome other rule. But he proves by a certain experiment that each ray has its ſine of incidence proportional to its ſine of refraction; and farther ſhews by mathematical reaſoning, that it muſt be ſo upon condition only that bodies refract the light by acting upon it, in a direction perpendicular to the ſurface of the refracting body, and upon the ſame ſort of rays always in an equal degree at the ſame diſtances[a].

7. OUR great author teaches in the next place how from the refraction of the moſt refrangible and leaſt refrangible rays to find the refraction of all the intermediate ones[b]. The method is this: if the ſine of incidence be to the ſine of refraction in the leaſt refrangible rays as A to B C, (in fig. 133) and to the ſine of refraction in the moſt refrangible as A to B D; if C E be taken equal to C D, and then E D be ſo divided in F, G, H, I, K, L, that E D, E F, E G, E H, E I, E K, E L, E C, ſhall be proportional to the eight lengths of muſical chords, which ſound the notes in an octave, E D being the length of the key, E F the length of the tone above

[a] Opt. pag. 67, 68, &c. [b] Ibid. B. I, par. 2. prop. 3.

that key, E G the length of the leſſer third, E H of the fourth, E I of the fifth, E K of the greater ſixth, E L of the ſeventh, and E C of the octave above that key ; that is if the lines E D, E F, E G, E H, E I, E K, E L, and E C bear the ſame proportion as the numbers, 1, $\frac{9}{8}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{9}{61}$, $\frac{1}{2}$, reſpectively then ſhall B D, BF, be the two limits of the ſines of refraction of the violet-making rays, that is the violet-making rays ſhall not all of them have preciſely the ſame ſine of refraction, but none of them ſhall have a greater ſine than B D, nor a leſs than B F, though there are violet-making rays which anſwer to any ſine of refraction that can be taken between theſe two. In the ſame manner B F and B G are the limits of the ſines of refraction of the indigo-making rays ; B G, B H are the limits belonging to the blue-making rays; B H, B I the limits pertaining to the green-making rays, B I, B K the limits for the yellow-making rays; B K, B L the limits for the orange-making rays; and laſtly, B L and B C the extreme limits of the ſines of refraction belonging to the red-making rays. Theſe are the proportions by which the heterogeneous rays of light are ſeparated from each other in refraction.

8. When light paſſes out of glaſs into air, our author found A to B C as 50 to 77, and the ſame A to B D as 50 to 78. And when it goes out of any other refracting ſubſtance into air, the exceſs of the ſine of refraction of any one ſpecies of rays above its ſine of incidence bears a conſtant proportion, which holds the ſame in each ſpecies, to the exceſs of the ſine of refraction of the ſame ſort of rays

above

above the ſine of incidence into the air out of glaſs; provided the ſines of incidence both in glaſs and the other ſubſtance are equal. This our author verified by tranſmitting the light through priſms of glaſs included within a priſmatic veſſel of water; and draws from thoſe experiments the following obſervations: that whenever the light in paſſing through ſo many ſurfaces parting diverſe tranſparent ſubſtances is by contrary refractions made to emerge into the air in a direction parallel to that of its incidence, it will appear afterwards white at any diſtance from the priſms, where you ſhall pleaſe to examine it; but if the direction of its emergence be oblique to its incidence, in receding from the place of emergence its edges ſhall appear tinged with colours: which proves that in the firſt caſe there is no inequality in the refractions of each ſpecies of rays, but that when any one ſpecies is ſo refracted as to emerge parallel to the incident rays, every ſort of rays after refraction ſhall likewiſe be parallel to the ſame incident rays, and to each other; whereas on the contrary, if the rays of any one ſort are oblique to the incident light, the ſeveral ſpecies ſhall be oblique to each other, and be gradually ſeparated by that obliquity. From hence he deduces both the forementioned theorem, and alſo this other; that in each ſort of rays the proportion of the ſine of incidence to the ſine of refraction, in the paſſage of the ray out of any refracting ſubſtance into another, is compounded of the proportion to which the ſine of incidence would have to the ſine of refraction in the paſſage of that ray out of the firſt ſubſtance into any third, and of the proportion which the

the ſine of incidence would have to the ſine of refraction in the paſſage of the ray out of that third ſubſtance into the ſecond. From ſo ſimple and plain an experiment has our moſt judicious author deduced theſe important theorems, by which we may learn how very exact and circumſpect he has been in this whole work of his optics; that notwithſtanding his great particularity in explaining his doctrine, and the numerous collection of experiments he has made to clear up every doubt which could ariſe, yet at the ſame time he has uſed the greateſt caution to make out every thing by the ſimpleſt and eaſieſt means poſſible.

9. Our author adds but one remark more upon refraction, which is, that if refraction be performed in the manner he has ſuppoſed from the light's being preſſed by the refracting power perpendicularly toward the ſurface of the refracting body, and conſequently be made to move ſwifter in the body than before its incidence; whether this power act equally at all diſtances or otherwiſe, provided only its power in the ſame body at the ſame diſtances remain without variation the ſame in one inclination of the incident rays as well as another; he obſerves that the refracting powers in different bodies will be in the duplicate proportion of the tangents of the leaſt angles, which the refracted light can make with the ſurfaces of the refracting bodies [a]. This obſervation may be explained thus. When the light paſſes into any refracting ſubſtance, it has been ſhewn above that the ſine of incidence bears a conſtant proportion to the ſine

[a] Opt. B. II. par. 3. prop. 10.

of

of refraction. Suppose the light to pass to the refracting body A B C D (in fig. 134) in the line E F, and to fall upon it at the point F, and then to proceed within the body in the line F G. Let H I be drawn through F perpendicular to the surface A B, and any circle K L M N be described to the center F. Then from the points O and P where this circle cuts the incident and refracted ray, the perpendiculars O Q, P R being drawn, the proportion of O Q to P R will remain the same in all the different obliquities, in which the same ray of light can fall on the surface A B. Now O Q is less than F L the semidiameter of the circle K L M N, but the more the ray E F is inclined down toward the surface A B, the greater will O Q be, and will approach nearer to the magnitude of F L. But the proportion of O Q to P R remaining always the same, when O Q is largest, P R will also be greatest; so that the more the incident ray E F is inclined toward the surface A B, the more the ray F G after refraction will be inclined toward the same. Now if the line F S T be so drawn, that S V being perpendicular to F I shall be to F L the semidiameter of the circle in the constant proportion of P R to O Q; then the angle under N F T is that which I meant by the least of all that can be made by the refracted ray with this surface, for the ray after refraction would proceed in this line, if it were to come to the point F lying on the very surface A B; for if the incident ray came to the point F in any line between A F and F H, the ray after refraction would proceed forward in some line between F T and F I. Here if N W be drawn perpendicular to F N, this line N W in the circle K L M N is called the

the tangent of the angle under NFS. Thus much being premised, the ſenſe of the forementioned propoſition is this. Let there be two refracting ſubſtances (in fig. 135) ABCD, and EFGH. Take a point, as I, in the ſurface AB, and to the center I with any ſemidiameter deſcribe the circle KLM. In like manner on the ſurface EF take ſome point N, as a center, and deſcribe with the ſame ſemidiameter the circle OPQ. Let the angle under BIR be the leaſt which the refracted light can make with the ſurface AB, and the angle under FNS the leaſt which the refracted light can make with the ſurface EF. Then if LT be drawn perpendicular to AB, and PV perpendicular to EF; the whole power, wherewith the ſubſtance ABCD acts on the light, will bear to the whole power wherewith the ſubſtance EFGH acts on, the light, a proportion, which is duplicate of the proportion, which LT bears to PV.

10. UPON comparing according to this rule the refractive powers of a great many bodies it is found, that unctuous bodies which abound moſt with ſulphureous parts refract the light two or three times more in proportion to their denſity than others: but that thoſe bodies, which ſeem to receive in their compoſition like proportions of ſulphureous parts, have their refractive powers proportional to their denſities; as appears beyond contradiction by comparing the refractive power of ſo rare a ſubſtance as the air with that of common glaſs or rock cryſtal, though theſe ſubſtances are 2000 times denſer than air; nay the ſame proportion

portion is found to hold without ſenſible difference in comparing air with pſeudo-topar and glaſs of antimony, though the pſeudo-topar be 3500 times denſer than air, and glaſs of antimony no leſs than 4400 times denſer. This power in other ſubſtances, as ſalts, common water, ſpirit of wine, &c. ſeems to bear a greater proportion to their denſities than theſe laſt named, according as they abound with ſulphurs more than theſe; which makes our author conclude it probable, that bodies act upon the light chiefly, if not altogether, by means of the ſulphurs in them; which kind of ſubſtances it is likely enters in ſome degree the compoſition of all bodies. Of all the ſubſtances examined by our author, none has ſo great a refractive power, in reſpect of its denſity, as a diamond.

11. OUR author finiſhes theſe remarks, and all he offers relating to refraction, with obſerving, that the action between light and bodies is mutual, ſince ſulphureous bodies, which are moſt readily ſet on fire by the ſun's light, when collected upon them with a burning glaſs, act more upon light in refracting it, than other bodies of the ſame denſity do. And farther, that the denſeſt bodies, which have been now ſhewn to act moſt upon light, contract the greateſt heat by being expoſed to the ſummer ſun.

12. HAVING thus diſpatched what relates to refraction, we muſt addreſs ourſelves to diſcourſe of the other operation of bodies upon light in reflecting it. When light paſſes through a ſurface, which divides two tranſparent bo-

dies differing in denſity, part of it only is tranſmitted, another part being reflected. And if the light paſs out of the denſer body into the rarer, by being much inclined to the foreſaid ſurface at length no part of it ſhall paſs through, but be totally reflected. Now that part of the light, which ſuffers the greateſt refraction, ſhall be wholly reflected with a leſs obliquity of the rays, than the parts of the light which undergo a leſs degree of refraction; as is evident from the laſt experiment recited in the firſt chapter; where, as the priſms D E F, G H I, (in fig. 129.) were turned about, the violet light was firſt totally reflected, and then the blue, next tothat the green, and ſo of the reſt. In conſequence of which our author lays down this propoſition; that the ſun's light differs in reflexibility, thoſe rays being moſt reflexible, which are moſt refrangible. And collects from this, in conjunction with other arguments, that the refraction and reflection of light are produced by the ſame cauſe, compaſſing thoſe different effects only by the difference of circumſtances with which it is attended. Another proof of this being taken by our author from what he has diſcovered of the paſſage of light through thin tranſparent plates, viz. that any particular ſpecies of light, ſuppoſe, for inſtance, the red-making rays, will enter and paſs out of ſuch a plate, if that plate be of ſome certain thickneſſes; but if it be of other thickneſſes, it will not break through it, but be reflected back: in which is ſeen, that the thickneſs of the plate determines whether the power, by which that plate acts upon the light, ſhall reflect it, or ſuffer it to paſs through.

13. But

13. BUT this laſt mentioned ſurpriſing property of the action between light and bodies affords the reaſon of all that has been ſaid in the preceding chapter concerning the colours of natural bodies; and muſt therefore more particularly be illuſtrated and explained, as being what will principally unfold the nature of the action of bodies upon light.

14. TO begin: The object glaſs of a long teleſcope being laid upon a plane glaſs, as propoſed in the foregoing chapter, in open day-light there will be exhibited rings of various colours, as was there related; but if in a darkened room the coloured ſpectrum be formed by the priſm, as in the firſt experiment of the firſt chapter, and the glaſſes be illuminated by a reflection from the ſpectrum, the rings ſhall not in this caſe exhibit the diverſity of colours before deſcribed, but appear all of the colour of the light which falls upon the glaſſes, having dark rings between. Which ſhews that the thin plate of air between the glaſſes at ſome thickneſſes reflects the incident light, at other places does not reflect it, but is found in thoſe places to give the light paſſage; for by holding the glaſſes in the light as it paſſes from the priſm to the ſpectrum, ſuppoſe at ſuch a diſtance from the priſm that the ſeveral ſorts of light muſt be ſufficiently ſeparated from each other, when any particular ſort of light falls on the glaſſes, you will find by holding a piece of white paper at a ſmall diſtance beyond the glaſſes, that at thoſe intervals, where the dark rings appeared upon the glaſſes, the light is ſo tranſmitted,

as to paint upon the paper rings of light having that colour which falls upon the glasses. This experiment therefore opens to us this very strange property of reflection, that in these thin plates it should bear such a relation to the thickness of the plate, as is here shewn. Farther, by carefully measuring the diameters of each ring it is found, that whereas the glasses touch where the dark spot appears in the center of the rings made by reflection, where the air is of twice the thickness at which the light of the first ring is reflected, there the light by being again transmitted makes the first dark ring; where the plate has three times that thickness which exhibits the first lucid ring, it again reflects the light forming the second lucid ring; when the thickness is four times the first, the light is again transmitted so as to make the second dark ring; where the air is five times the first thickness, the third lucid ring is made; where it has six times the thickness, the third dark ring appears, and so on: in so much that the thicknesses, at which the light is reflected, are in proportion to the numbers 1, 3, 5, 7, 9, &c. and the thicknesses, where the light is transmitted, are in the proportion of the numbers 0, 2, 4, 6, 8, &c. And these proportions between the thicknesses which reflect and transmit the light remain the same in all situations of the eye, as well when the rings are viewed obliquely, as when looked on perpendicularly. We must farther here observe, that the light, when it is reflected, as well as when it is transmitted, enters the thin plate, and is reflected from its farther surface; because, as was before remarked, the altering the transparent body behind the farther surface alters the degree

gree of reflection as when a thin piece of Muſcovy glaſs has its farther ſurface wet with water, and the colour of the glaſs made dimmer by being ſo wet; which ſhews that the light reaches to the water, otherwiſe its reflection could not be influenced by it. But yet this reflection depends upon ſome power propagated from the firſt ſurface to the ſecond; for though made at the ſecond ſurface it depends alſo upon the firſt, becauſe it depends upon the diſtance between the ſurfaces; and beſides, the body through which the light paſſes to the firſt ſurface influences the reflection: for in a plate of Muſcovy glaſs, wetting the ſurface, which firſt receives the light, diminiſhes the reflection, though not quite ſo much as wetting the farther ſurface will do. Since therefore the light in paſſing through theſe thin plates at ſome thickneſſes is reflected, but at others tranſmitted without reflection, it is evident, that this reflection is cauſed by ſome power propagated from the firſt ſurface, which intermits and returns ſucceſſively. Thus is every ray apart diſpoſed to alternate reflections and tranſmiſſions at equal intervals; the ſucceſſive returns of which diſpoſition our author calls the fits of eaſy reflection, and of eaſy tranſmiſſion. But theſe fits, which obſerve the ſame law of returning at equal intervals, whether the plates are viewed perpendicularly or obliquely, in different ſituations of the eye change their magnitude. For what was obſerved before in reſpect of thoſe rings, which appear in open day-light, holds likewiſe in theſe rings exhibited by ſimple lights; namely, that theſe two alter in bigneſs according to the different angle under which they are ſeen: and our author

lay-

lays down a rule whereby to determine the thickneſſes of the plate of air, which ſhall exhibit the ſame colour under different oblique views[a]. And the thickneſs of the aereal plate, which in different inclinations of the rays will exhibit to the eye in open day-light the ſame colour, is alſo varied by the ſame rule[b]. He contrived farther a method of comparing in the bubble of water the proportion between the thickneſs of its coat, which exhibited any colour when ſeen perpendicularly, to the thickneſs of it, where the ſame colour appeared by an oblique view; and he found the ſame rule to obtain here likewiſe[c]. But farther, if the glaſſes be enlightened ſucceſſively by all the ſeveral ſpecies of light, the rings will appear of different magnitudes; in the red light they will be larger than in the orange colour, in that larger than in the yellow, in the yellow larger than in the green, leſs in the blue, leſs yet in the indigo, and leaſt of all in the violet: which ſhew sthat the ſame thickneſs of the aereal plate is not fitted to reflect all colours, but that one colour is reflected where another would have been tranſmitted; and as the rays which are moſt ſtrongly refracted form the leaſt rings, a rule is laid down by our author for determining the relation, which the degree of refraction of each ſpecies of colour has to the thickneſſes of the plate where it is reflected.

15. FROM theſe obſervations our author ſhews the reaſon of that great variety of colours, which appears in theſe thin plates in the open white light of the day. For when this white

[a] Opt. B. II. par. 3. prop. 15. [b] Ibid. par. 1, obſerv. 7. [c] Ibid. Obſerv. 19.

light falls on the plate, each part of the light forms rings of its own colour; and the rings of the different colours not being of the ſame bigneſs are variouſly intermixed, and form a great variety of tints [a].

16. In certain experiments, which our author made with thick glaſſes, he found, that theſe fits of eaſy reflection and tranſmiſſion returned for ſome thouſands of times, and thereby farther confirmed his reaſoning concerning them [b].

17. Upon the whole, our great author concludes from ſome of the experiments made by him, that the reaſon why all tranſparent bodies refract part of the light incident upon them, and reflect another part, is, becauſe ſome of the light, when it comes to the ſurface of the body, is in a fit of eaſy tranſmiſſion, and ſome part of it in a fit of eaſy reflection; and from the durableneſs of theſe fits he thinks it probable, that the light is put into theſe fits from their firſt emiſſion out of the luminous body; and that theſe fits continue to return at equal intervals without end, unleſs thoſe intervals be changed by the light's entring into ſome refracting ſubſtance [c]. He likewiſe has taught how to determine the change which is made of the intervals of the fits of eaſy tranſmiſſion and reflection, when the light paſſes out of one tranſparent ſpace or ſubſtance into another. His rule is, that when the light paſſes perpendicularly to the ſurface, which parts any two tranſparent ſubſtances, theſe intervals in the ſubſtance, out of

[a] Opt B. II. par. 2 pag. 199. &c. [b] Ibid. par. 4 [c] Ibid. part. 3. prop. 13.

which

which the light paſſes, bear to the intervals in the ſubſtance, whereinto the light enters, the ſame proportion, as the ſine of incidence bears to the ſine of refraction [a]. It is farther to be obſerved, that though the fits of eaſy reflection return at conſtant intervals, yet the reflecting power never operates, but at or near a ſurface where the light would ſuffer refraction; and if the thickneſs of any tranſparent body ſhall be leſs than the intervals of the fits, thoſe intervals ſhall ſcarce be diſturbed by ſuch a body, but the light ſhall paſs through without any reflection [b].

18. WHAT the power in nature is, whereby this action between light and bodies is cauſed, our author has not diſcovered. But the effects, which he has diſcovered, of this power are very ſurpriſing, and altogether wide from any conjectures that had ever been framed concerning it; and from theſe diſcoveries of his no doubt this power is to be deduced, if we ever can come to the knowledge of it. Sir ISAAC NEWTON has in general hinted at his opinion concerning it; that probably it is owing to ſome very ſubtle and elaſtic ſubſtance diffuſed through the univerſe, in which ſuch vibrations may be excited by the rays of light, as they paſs through it, that ſhall occaſion it to operate ſo differently upon the light in different places as to give riſe to theſe alternate fits of reflection and tranſmiſſion, of which we have now been ſpeaking [c]. He is of opinion, that ſuch a ſubſtance may produce this, and other effects alſo in nature, though it be ſo rare as not to give any ſenſible reſiſtance to bodies in motion;

[a] Ibid. prop. 17. [b] Ibid prop. 13. [c] Opt. Qu. 18. &c.

tion[a]; and therefore not inconſiſtent with what has been ſaid above, that the planets move in ſpaces free from reſiſtance[b].

19. IN order for the more full diſcovery of this action between light and bodies, our author began another ſet of experiments, wherein he found the light to be acted on as it paſſes near the edges of ſolid bodies; in particular all ſmall bodies, ſuch as the hairs of a man's head or the like, held in a very ſmall beam of the ſun's light, caſt extremely broad ſhadows. And in one of theſe experiments the ſhadow was 35 times the breadth of the body[c]. Theſe ſhadows are alſo obſerved to be bordered with colours[d]. This our author calls the inflection of light; but as he informs us, that he was interrupted from proſecuting theſe experiments to any length, I need not detain my readers with a more particular account of them.

CHAP. IV.

Of OPTIC GLASSES.

SIR ISAAC NEWTON having deduced from his doctrine of light and colours a ſurpriſing improvement of teleſcopes, of which I intend here to give an account, I ſhall firſt premiſe ſomething in general concerning thoſe inſtruments.

[a] Sch. Concl. S, 2.

[b] B. II. Ch. 1.

[c] Opt. B. III. Obſ. 1

[d] Ibid. Obſ. 2.

2. It will be underſtood from what has been ſaid above, that when light falls upon the ſurface of glaſs obliquely, after its entrance into the glaſs it is more inclined to the line drawn through the point of incidence perpendicular to that ſurface, than before. Suppoſe a ray of light iſſuing from the point A (in fig. 136) falls on a piece of glaſs BCDE, whoſe ſurface BC, whereon the ray falls, is of a ſpherical or globular figure, the center whereof is F. Let the ray proceed in the line AG falling on the ſurface BC in the point G, anddraw FGH. Here the ray after its entrance into the glaſs will paſs on in ſome line, as GI, more inclined toward the line FGH that the line AG is inclined thereto; for the line FGH is perpendicular to the ſurface BC in the point G. By this means, if a number of rays proceeding from any one point fall on a convex ſpherical ſurface of glaſs, they ſhall be inflected (as is repreſented in fig. 137,) ſo as to be gathered pretty cloſe together about the line drawn through the center of the glaſs from the point, whence the rays proceed; which line henceforward we ſhall call the axis of the glaſs: or the point from whence the rays proceed may be ſo near the glaſs, that the rays ſhall after entring the glaſs ſtill go on to ſpread themſelves, but not ſo much as before; ſo that if the rays were to be continued backward (as in fig. 138,) they ſhould gather together about the axis at a place more remote from the glaſs, than the point is, whence they actually proceed. In theſe and the following figures A denotes the point to which the rays are related before refraction, B the point to which they are directed afterwards, and C the center of the refracting ſurface. Here we may obſerve, that it is poſſible to form the glaſs of ſuch a figure, that all the rays which proceed from one point ſhall

ſhall after refraction be reduced again exactly into one point on the axis of the glaſs. But in glaſſes of a ſpherical form though this does not happen; yet the rays, which fall within a moderate diſtance from the axis, will unite extremely near together. If the light fall on a concave ſpherical ſurface, after refraction it ſhall ſpread quicker than before (as in fig. 139,) unleſs the rays proceed from a point between the center and the ſurface of the glaſs. If we ſuppoſe the rays of light, which fall upon the glaſs, not to proceed from any point, but to move ſo as to tend all to ſome point in the axis of the glaſs beyond the ſurface; if the glaſs have a convex ſurface, the rays ſhall unite about the axis ſooner, than otherwiſe they would do (as in fig. 140,) unleſs the point to which they tended was between the ſurface and the center of that ſurface. But if the ſurface be concave, they ſhall not meet ſo ſoon: nay perhaps converge. (See fig. 141 and 142.)

3. FARTHER, becauſe the light in paſſing out of glaſs into the air is turned by the refraction farther off from the line drawn through the point of incidence perpendicular to the refracting ſurface, than it was before; the light which ſpreads from a point ſhall by paſſing through a convex ſurface of glaſs into the air be made either to ſpread leſs than before (as in fig. 143,) or to gather about the axis beyond the glaſs (as in fig. 144.) But if the rays of light were proceeding to a point in the axis of the glaſs, they ſhould by the refraction be made to unite ſooner about that axis (as in fig. 145.) If the ſurface of the glaſs be concave, rays which proceed from a point ſhall be made to ſpread faſter (as in fig. 146,) but rays which are tending to a point in the axis of the

the glaſs, ſhall be made to gather about the axis farther from the glaſs (as in fig. 147) or even to diverge (as in fig. 148,) unleſs the point, to which the rays are directed, lies between the ſurface of the glaſs and its center.

4. The rays, which ſpread themſelves from a point, are called diverging; and ſuch as move toward a point, are called converging rays. And the point in the axis of the glaſs, about which the rays gather after refraction, is called the focus of thoſe rays.

5. If a glaſs be formed of two convex ſpherical ſurfaces (as in fig. 149,) where the glaſs AB is formed of the ſurfaces ACB and ADB, the line drawn through the centers of the two ſurfaces, as the line EF, is called the axis of the glaſs; and rays, which diverge from any point of this axis, by the refraction of the glaſs will be cauſed to converge toward ſome part of the axis, or at leaſt to diverge as from a point more remote from the glaſs, than that from whence they proceeded; for the two ſurfaces both conſpire to produce this effect upon the rays. But converging rays will be cauſed by ſuch a glaſs as this to converge ſooner. If a glaſs be formed of two concave ſurfaces, as the glaſs AB (in fig. 150,) the line CD drawn through the centers, to which the two ſurfaces are formed, is called the axis of the glaſs. Such a glaſs ſhall cauſe diverging rays, which proceed from any point in the axis of the glaſs, to diverge much more, as if they came from ſome place in the axis of the glaſs nearer to it than the point, whence

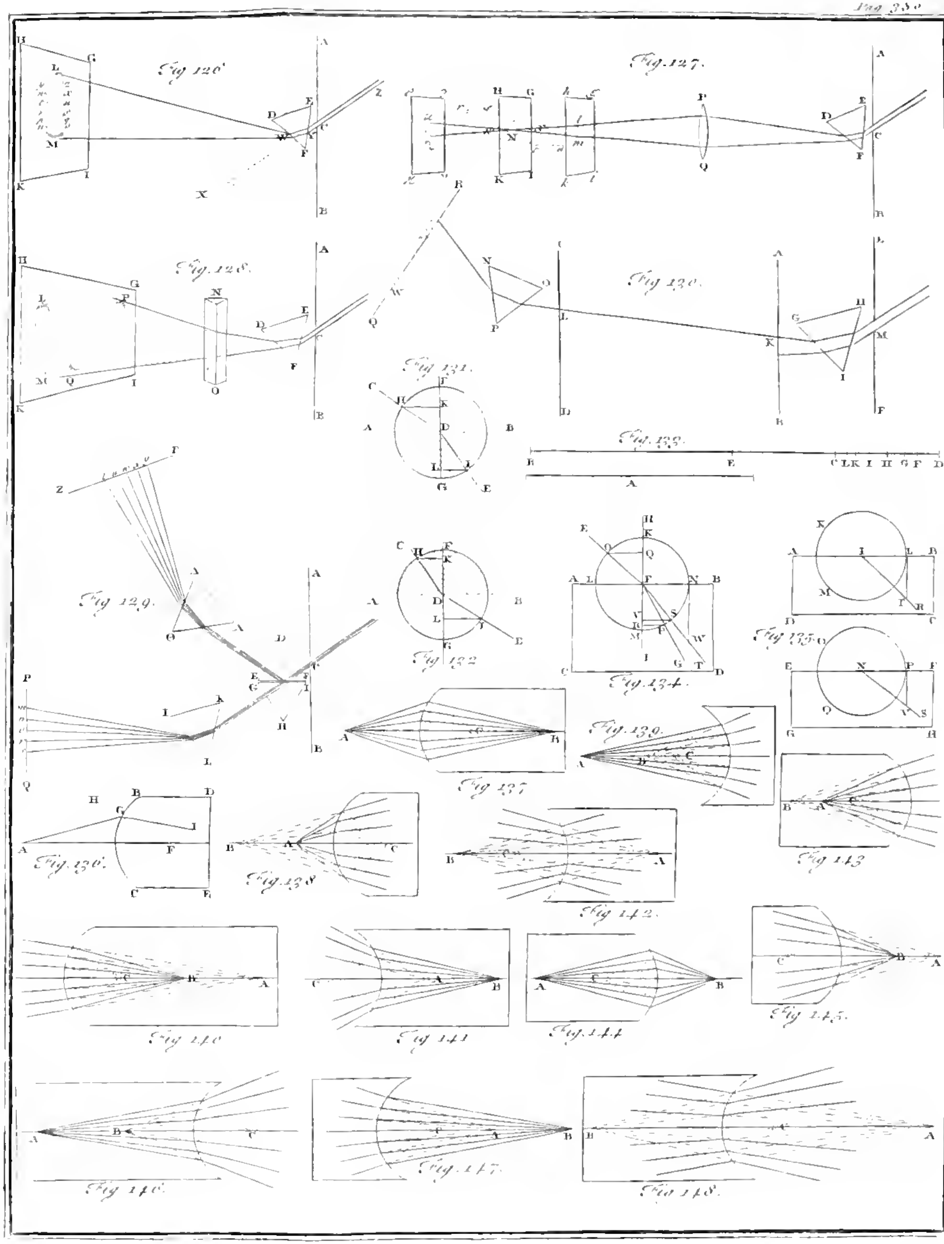


Fig. 126.
Fig. 127.
Fig. 128.
Fig. 129.
Fig. 130.
Fig. 131.
Fig. 132.
Fig. 133.
Fig. 134.
Fig. 135.
Fig. 136.
Fig. 137.
Fig. 138.
Fig. 139.
Fig. 140.
Fig. 141.
Fig. 142.
Fig. 143.
Fig. 144.
Fig. 145.
Fig. 146.
Fig. 147.
Fig. 148.

whence the rays actually proceed. But converging rays will be made either to converge leſs, or even to diverge.

6. IN theſe glaſſes rays, which proceed from any point near the axis, will be affected as it were in the ſame manner, as if they proceeded from the very axis it ſelf, and ſuch as converge toward a point at a ſmall diſtance from the axis will ſuffer much the ſame effects from the glaſs, as if they converged to ſome point in the very axis. By this means any luminous body expoſed to a convex glaſs may have an image formed upon any white body held beyond the glaſs. This may be eaſily tried with a common ſpectacle-glaſs. For if ſuch a glaſs be held between a candle and a piece of white paper, if the diſtances of the candle, glaſs, and paper be properly adjuſted, the image of the candle will appear very diſtinctly upon the paper, but be ſeen inverted; the reaſon whereof is this. Let AB (in fig. 151) be the glaſs, CD an object placed croſs the axis of the glaſs. Let the rays of light, which iſſue from the point E, where the axis of the glaſs croſſes the object, be ſo refracted by the glaſs, as to meet again about the point F. The rays, which diverge from the point C of the object, ſhall meet again almoſt at the ſame diſtance from the glaſs, but on the other ſide of the axis, as at G; for the rays at the glaſs croſs the axis. In like manner the rays, which proceed from the point D, will meet about H on the other ſide of the axis. None of theſe rays, neither thoſe which proceed from the point E in the axis, nor thoſe which iſſue from C or D, will meet again exactly in one point; but yet in one place, as is here ſuppoſed at F, G, and H, they

will

will be crouded ſo cloſe together, as to make a diſtinct image of the object upon any body proper to reflect it, which ſhall be held there.

7. If the object be too near the glaſs for the rays to converge after the refraction, the rays ſhall iſſue out of the glaſs, as if they diverged from a point more diſtant from the glaſs, than that from whence they really proceed (as in fig. 152,) where the rays coming from the point E of the object, which lies on the axis of the glaſs A B, iſſue out of the glaſs, as if they came from the point F more remote from the glaſs than E; and the rays proceeding from the point C iſſue out of the glaſs, as if they proceeded from the point G; likewiſe the rays which iſſue from the point D emerge out of the glaſs, as if they came from the point H. Here the point G is on the ſame ſide of the axis, as the point C; and the point H on the ſame ſide, as the point D. In this caſe to an eye placed beyond the glaſs the object ſhould appear, as if it were in the ſituation G F H.

8. If the glaſs A B had been concave (as in, fig. 153,) to an eye beyond the glaſs the object C D would appear in the ſituation G H, nearer to the glaſs than really it is. Here alſo the object will not be inverted; but the point G is on the ſame ſide the axe with the point C, and H on the ſame ſide as D.

9. Hence

9. Hence may be underſtood, why ſpectacles made with convex glaſſes help the ſight in old age: for the eye in that age becomes unfit to ſee objects diſtinctly, except ſuch as are remov'd to a very great diſtance; whence all men, when they firſt ſtand in need of ſpectacles, are obſerved to read at arm's length, and to hold the object at a greater diſtance, than they uſed to do before. But when an object is removed at too great a diſtance from the ſight, it cannot be ſeen clearly, by reaſon that a leſs quantity of light from the object will enter the eye, and the whole object will alſo appear ſmaller. Now by help of a convex glaſs an object may be held near, and yet the rays of light iſſuing from it will enter the eye, as if the object were farther removed.

10. After the ſame manner concave glaſſes aſſiſt ſuch, as are ſhort ſighted. For theſe require the object to be brought inconveniently near to the eye, in order to their ſeeing it diſtinctly; but by ſuch a glaſs the object may be removed to a proper diſtance, and yet the rays of light enter the eye, as if they came from a place much nearer.

11. Whence theſe defects of the ſight ariſe, that in old age objects cannot be ſeen diſtinct within a moderate diſtance, and in ſhort-ſightedneſs not without being brought too near, will be eaſily underſtood, when the manner of viſion in general ſhall be explain'd; which I ſhall now endeavour to do, in order to be better underſtood in what

follows. The eye is form'd, as is repreſented in fig. 154. It is of a globular figure, the fore part whereof ſcarce more protuberant than the reſt is tranſparent. Underneath this tranſparent part is a ſmall collection of an humour in appearance like water, and it has alſo the ſame refractive power as common water; this is called the aqueous humour, and fills the ſpace A B C D in the figure. Next beyond lies the body D E F G; this is ſolid but tranſparent, it is compoſed with two convex ſurfaces, the hinder ſurface E F G being more convex, than the anterior E D G. Between the outer membrane A B C, and this body E D G F is placed that membrane, which exhibits the colours, that are ſeen round the ſight of the eye; and the black ſpot, which is called the ſight or pupil, is a hole in this membrane, through which the light enters, whereby we ſee. This membrane is fixed only by its outward circuit, and has a muſcular power, whereby it dilates the pupil in a weak light, and contracts it in a ſtrong one. The body D E F G is called the cryſtalline humour, and has a greater refracting power than water. Behind this the bulk of the eye is filled up with what is called the vitreous humor, this has much the ſame refractive power with water. At the bottom of the eye toward the inner ſide next the noſe the optic glaſs enters, as at H, and ſpreads it ſelf all over the inſide of the eye, till within a ſmall diſtance from A and C. Now any object, as I K, being placed before the eye, the rays of light iſſuing from each point of this object are ſo refracted by the convex ſurface of the aqueous humour, as to be cauſed to converge; after this being received by the convex ſurface E D G of

of the cryſtalline humour, which has a greater refractive power than the aqueous, the rays, when they are entered into this ſurface, ſtill more converge, and at going out of the ſurface E F G into a humour of a leſs refractive power than the cryſtalline they are made to converge yet farther. By all theſe ſucceſſive refractions they are brought to converge at the bottom of the eye, ſo that a diſtinct image of the object as L M is impreſs'd on the nerve. And by this means the object is ſeen.

11. IT has been made a difficulty, that the image of the object impreſſed on the nerve is inverted, ſo that the upper part of the image is impreſſed on the lower part of the eye. But this difficulty, I think, can no longer remain, if we only conſider, that upper and lower are terms merely relative to the ordinary poſition of our bodies: and our bodies, when view'd by the eye, have their image as much inverted as other objects; ſo that the image of our own bodies, and of other objects, are impreſſed on the eye in the ſame relation to one another, as they really have.

12. THE eye can ſee objects equally diſtinct at very different diſtances, but in one diſtance only at the ſame time. That the eye may accomodate itſelf to different diſtances, ſome change in its humours is requir'd. It is my opinion, that this change is made in the figure of the cryſtalline humour, as I have indeavoured to prove in another place.

3

13. If any of the humours of the eye are too flat, they will refract the light too little; which is the case in old age. If they are too convex, they refract too much; as in those who are short-sighted.

14. The manner of direct vision being thus explained, I proceed to give some account of telescopes, by which we view more distinctly remote objects; and also of microscopes, whereby we magnify the appearance of small objects. In the first place, the most simple sort of telescope is composed of two glasses, either both convex, or one convex, and the other concave. (The first sort of these is represented in fig. 155, the latter in fig. 156.)

15. In fig. 155 let A B represent the convex glass next the object, C D the other glass more convex near the eye. Suppose the object-glass A B to form the image of the object at E F; so that if a sheet of white paper were to be held in this place, the object would appear. Now suppose the rays, which pass the glass A B, and are united about F, to proceed to the eye glass C D, and be there refracted. Three only of these rays are drawn in the figure, those which pass by the extremities of the glass A B, and that which passes its middle. If the glass C D be placed at such a distance from the image E F, that the rays, which pass by the point F, after having proceeded through the glass diverge so much, as the rays do that come from an object, which is at such a distance from the eye as

to

to be ſeen diſtinctly, theſe being received by the eye will make on the bottom of the eye a diſtinct repreſentation of the point F. In like manner the rays, which paſs through the object glaſs A B to the point E after proceeding through the eye-glaſs C D will on the bottom of the eye make a diſtinct repreſentation of the point E. But if the eye be placed where theſe rays, which proceed from E, croſs thoſe, which proceed from F, the eye will receive the diſtinct impreſſion of both theſe points at the ſame time; and conſequently will alſo receive a diſtinct impreſſion from all the intermediate parts of the image E F, that is, the eye will ſee the object, to which the teleſcope is directed, diſtinctly. The place of the eye is about the point G, where the rays H E, H F croſs, which paſs through the middle of the object-glaſs A B to the points E and F; or at the place where the focus would be formed by rays coming from the point H, and refracted by the glaſs C D. To judge how much this inſtrument magnifies any object, we muſt firſt obſerve, that the angle under E H F, in which the eye at the point H would ſee the image E F, is nearly the ſame as the angle, under which the object appears by direct viſion; but when the eye is in G, and views the object through the teleſcope, it ſees the ſame under a greater angle; for the rays, which coming from E and F croſs in G, make a greater angle than the rays, which proceed from the point H to theſe points E and F. The angle at G is greater than that at H in the proportion, as the diſtance between the glaſſes A B and C D is greater than the diſtance of the point G from the glaſs C D.

 16. THIS

16. This telescope inverts the object; for the rays, which come from the right-hand side of the object, go to the point E the left side of the image; and the rays, which come from the left side of the object, go to F the right side of the image. These rays cross again in G, so that the rays, which come from the right side of the object, go to the right side of the eye; and the rays from the left side of the object go to the left side of the eye. Therefore in this telescope the image in the eye has the same situation as the object; and seeing that in direct vision the image in the eye has an inverted situation, here, where the situation is not inverted, the object must appear so. This is no inconvenience to astronomers in celestial observations; but for objects here on the earth it is usual to add two other convex glasses, which may turn the object again (as is represented in fig. 157,) or else to use the other kind of telescope with a concave eye-glass.

17. In this other kind of telescope the effect is founded on the same principles, as in the former. The distinctness of the appearance is procured in the same manner. But here the eye-glass C D (in fig. 156) is placed between the image E F, and the object glass A B. By this means the rays, which come from the right-hand side of the object, and proceed toward E the left side of the image, being intercepted by the eye-glass are carried to the left side of the eye; and the rays, which come from the left side of the object, go to the right side of the eye; so that the impression in the eye being inverted the object appears in the same situation,

as

as when view'd by the naked eye. The eye muſt here be placed cloſe to the glaſs. The degree of magnifying in this inſtrument is thus to be found. Let the rays, which paſs through the glaſs A B at H, after the refraction of the eye-glaſs C D diverge, as if they came from the point G; then the rays, which come from the extremities of the object, enter the eye under the angle at G; ſo that here alſo the object will be magnified in the proportion of the diſtance between the glaſſes, to the diſtance of G from the eye-glaſs.

18. THE ſpace, that can be taken in at one view in this teleſcope, depends on the breadth of the pupil of the eye; for as the rays, which go to the points E, F of the image, are ſomething diſtant from each other, when they come out of the glaſs C D, if they are wider aſunder than the pupil, it is evident, that they cannot both enter the eye at once. In the other teleſcope the eye is placed in the point G, where the rays that come from the points E or F croſs each other, and therefore muſt enter the eye together. On this account the teleſcope with convex glaſſes takes in a larger view, than thoſe with concave. But in theſe alſo the extent of the view is limited, becauſe the eye-glaſs does not by the refraction towards its edges form ſo diſtinct a repreſentation of the object, as near the middle.

18. Microſcopes are of two ſorts. One kind is only a very convex glaſs, by the means of which the object may be brought very near the eye, and yet be ſeen diſtinctly.

This

This microſcope magnifies in proportion, as the object by being brought near the eye will form a broader impreſſion on the optic nerve. The other kind made with convex glaſſes produces its effects in the ſame manner as the teleſcope. Let the object A B (in fig. 158) be placed under the glaſs C D, and by this glaſs let an image be formed of this object. Above this image let the glaſs G H be placed. By this glaſs let the rays, which proceed from the points A and B, be refracted, as is expreſſed in the figure. In particular, let the rays, which from each of theſe points paſs through the middle of the glaſs C D, croſs in I, and there let the eye be placed. Here the object will appear larger, when ſeen through the microſcope, than if that inſtrument were removed, in proportion as the angle, in which theſe rays croſs in I, is greater than the angle, which the lines would make, that ſhould be drawn from I to A and B; that is, in the proportion made up of the proportion of the diſtance of the object A B from I, to the diſtance of I from the glaſs G H; and of the proportion of the diſtance between the glaſſes, to the diſtance of the object A B from the glaſs C D.

19. I SHALL now proceed to explain the imperfection in theſe inſtruments, occaſioned by the different refrangibility of the light which comes from every object. This prevents the image of the object from being formed in the focus of the object glaſs with perfect diſtinctneſs; ſo that if the eye-glaſs magnify the image overmuch, the imperfections of it muſt be viſible, and make the whole appear confuſed. Our author more fully to ſatisfy himſelf, that the different refrangibility of the

ſeveral

ſeveral ſorts of rays is ſufficient to produce this irregularity, underwent the labour of a very nice and difficult experiment, whoſe proceſs he has at large ſet down, to prove, that the rays of light are refracted as differently in the ſmall refraction of teleſcope glaſſes as in the larger of the priſm; ſo exceeding careful has he been in ſearching out the true cauſe of this effect. And he uſed, I ſuppoſe, the greater caution, becauſe another reaſon had before been generally aſſigned for it. It was the opinion of all mathematicians, that this defect in teleſcopes aroſe from the figure, in which the glaſſes were formed; a ſpherical refracting ſurface not collecting into an exact point all the rays which come from any one point of an object, as has before been ſaid [a]. But after our author has proved, that in theſe ſmall refractions, as well as in greater, the ſine of incidence into air out of glaſs, to the ſine of refraction in the red-making rays, is as 50 to 77, and in the blue-making rays 50 to 78; he proceeds to compare the inequalities of refraction ariſing from this different refrangibility of the rays, with the inequalities, which would follow from the figure of the glaſs, were light uniformly refracted. For this purpoſe he obſerves, that if rays iſſuing from a point ſo remote from the object glaſs of a teleſcope, as to be eſteemed parallel, which is the caſe of the rays, which come from the heavenly bodies; then the diſtance from the glaſs of the point, in which the leaſt refrangible rays are united, will be to the diſtance, at which the moſt refrangible rays unite, as 28 to 27; and therefore that the leaſt ſpace, into which

[a] § 2.

all

all the rays can be collected, will not be lefs than the 55th part of the breadth of the the glafs. For if A B (in fig. 159) be the glafs, C D its axis, E A, F B two rays of the light parallel to that axis entring the glafs near its edges; after refraction let the leaft refrangible part of thefe rays meet in G, the moft refrangible in H; then, as has been faid, G I will be to I H, as 28 to 27; that is, G H will be the 28th part of G I, and the 27th part of H I; whence if K L be drawn through G, and M N through H, perpendicular to C D, M N will be the 28th part of A B, the breadth of the glafs, and K L the 27th part of the fame; fo that O P the leaft fpace, into which the rays are gathered, will be about half the mean between thefe two, that is the 55th part of A B.

20. THIS is the error arifing from the different refrangibility of the rays of light, which our author finds vaftly to exceed the other, confequent upon the figure of the glafs. In particular, if the telefcope glafs be flat on one fide, and convex on the other; when the flat fide is turned towards the object, by a theorem, which he has laid down, the error from the figure comes out above 5000 times lefs than the other. This other inequality is fo great, that telefcopes could not perform fo well as they do, were it not that the light does not equally fill all the fpace O P, over which it is fcattered, but is much more denfe toward the middle of that fpace than at the extremities. And befides, all the kinds of rays affect not the fenfe equally ftrong, the yellow and orange being the ftrongeft, the

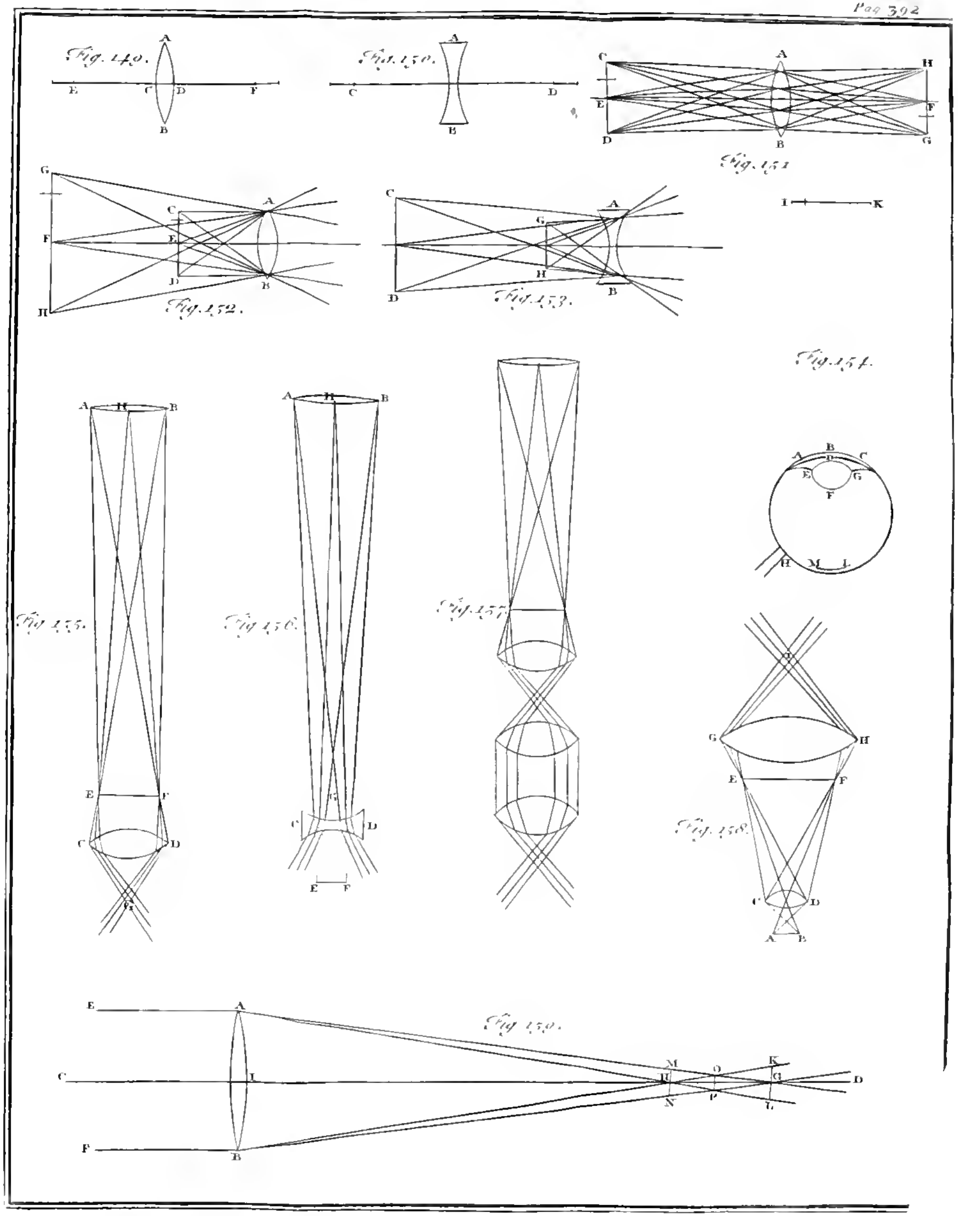


Fig. 149.
Fig. 150.
Fig. 151
Fig. 152.
Fig. 153.
Fig. 154.
Fig. 155.
Fig. 156.
Fig. 157.
Fig. 158.
Fig. 159.

the red and green next to them, the blue indigo and violet being much darker and fainter colours; and it is ſhewn that all the yellow and orange, and three fifths of the brighter half of the red next the orange, and as great a ſhare of the brighter half of the green next the yellow, will be collected into a ſpace whoſe breadth is not above the 250th part of the breadth of the glaſs. And the remaining colours, which fall without this ſpace, as they are much more dull and obſcure than theſe, ſo will they be likewiſe much more diffuſed; and therefore can hardly affect the ſenſe in compariſon of the other. And agreeable to this is the obſervation of aſtronomers, that teleſcopes between twenty and ſixty feet in length repreſent the fixed ſtars, as being about 5 or 6, at moſt about 8 or 10 ſeconds in diameter. Whereas other arguments ſhew us, that they do not really appear to us of any ſenſible magnitude any otherwiſe than as their light is dilated by refraction. One proof that the fixed ſtars do not appear to us under any ſenſible angle is, that when the moon paſſes over any of them, their light does not, like the planets on the ſame occaſion, diſappear by degrees, but vaniſhes at once.

21. OUR author being thus convinced, that teleſcopes were not capable of being brought to much greater perfection than at preſent by refractions, contrived one by reflection, in which there is no ſeparation made of the different coloured light; for in every kind of light the rays after reflection have the ſame degree of inclination to the ſurface, from whence they are reflected, as they have at their incidence, ſo

that thoſe rays which come to the ſurface in one line, will go off alſo in one line without any parting from one another. Accordingly in the attempt he ſucceeded ſo well, that a ſhort one, not much exceeding ſix inches in length, equalled an ordinary teleſcope whoſe length was four feet. Inſtruments of this kind to greater lengths, have of late been made, which fully anſwer expectation [a].

CHAP. V.

Of the RAINBOW.

I SHALL now explain the rainbow. The manner of its production was underſtood, in the general, before Sir ISAAC NEWTON had diſcovered his theory of colours; but what cauſed the diverſity of colours in it could not then be known, which obliges him to explain this appearance particularly; whom we ſhall imitate as follows. The firſt perſon, who expreſsly ſhewed the rainbow to be formed by the reflection of the ſun-beams from drops of falling rain, was ANTONIO DE DOMINIS. But this was afterwards more fully and diſtinctly explained by DES CARTES.

2. THERE appears moſt frequently two rainbows; both of which are cauſed by the foreſaid reflection of the ſun-beams from the drops of falling rain, but are not produced by all the light which falls upon and are reflected from the drops. The inner bow is produced by thoſe rays only which enter the drop, and at their entrance are ſo refracted as to unite into a point, as it were, upon the farther ſurface of the drop, as is repreſented in fig. 160; where the contiguous rays *a b*, *c d*, *e f*, coming from the ſun,

[a] Philoſ. Tranſ. No. 378.

ſun, and therefore to ſenſe parallel, upon their entrance into the drop in the points *b*, *d*, *f*, are ſo refracted as to meet together in the point *g*, upon the farther ſurface of the drop. Now theſe rays being reflected nearly from the ſame point of the ſurface, the angle of incidence of each ray upon the point *g* being equal to the angle of reflection, the rays will return in the lines *g h*, *g k*, *g l*, in the ſame manner inclined to each other, as they were before their incidence upon the point *g*, and will make the ſame angles with the ſurface of the drop at the points *h*, *k*, *l*, as at the points *b*, *d*, *f*, after their entrance; and therefore after their emergence out of the drop each ray will be inclined to the ſurface in the ſame angle, as when it firſt entered it; whence the lines *h m*, *k n*, *l o*, in which the rays emerge, muſt be parallel to each other, as well as the lines *a b*, *c d*, *e f*, in which they were incident. But theſe emerging rays being parallel will not ſpread nor diverge from each other in their paſſage from the drop, and therefore will enter the eye conveniently ſituated in ſufficient plenty to cauſe a ſenſation. Whereas all the other rays, whether thoſe nearer the center of the drop, as *p q*, *r s*, or thoſe farther off, as *t u*, *w x*, will be reflected from other points in the hinder ſurface of the drop; namely, the ray *p q* from the point *y*, *r s* from *z*, *t v* from α, and *w x* from β. And for this reaſon by their reflection and ſucceeding refraction they will be ſcattered after their emergence from the forementioned rays and from each other, and therefore cannot enter the eye placed to receive them copious enough to excite any diſtinct ſenſation.

 3. THE

3. The external rainbow is formed by two reflections made between the incidence and emergence of the rays; for it is to be noted, that the rays *g h*, *g k*, *g l*, at the points *h*, *k*, *l*, do not wholly paſs out of the drop, but are in part reflected back; though the ſecond reflection of theſe particular rays does not form the outer bow. For this bow is made by thoſe rays, which after their entrance into the drop are by the refraction of it united, before they arrive at the farther ſurface, at ſuch a diſtance from it, that when they fall upon that ſurface, they may be reflected in parallel lines, as is repreſented in fig. 161; where the rays *a b*, *c d*, *e f*, are collected by the refraction of the drop into the point *g*, and paſſing on from thence ſtrike upon the ſurface of the drop in the points *h*, *k*, *l*, and are thence reflected to *m*, *n*, *o*, paſſing from *h* to *m*, from *k* to *n*, and from *l* to *o* in parallel lines. For theſe rays after reflection at *m*, *n*, *o* will meet again in the point *p*, at the ſame diſtance from theſe points of reflection *m*, *n*, *o*, as the point *g* is from the former points of reflection *h*, *k*, *l*. Therefore theſe rays in paſſing from *p* to the ſurface of the drop will fall upon that ſurface in the points *q*, *r*, *s* in the ſame angles, as theſe rays made with the ſurface in *b*, *d*, *f*, after refraction. Conſequently, when theſe rays emerge out of the drop into the air, each ray will make with the ſurface of the drop the ſame angle, as it made at its firſt incidence; ſo that the lines *q t*, *r v*, *s w*, in which they come from the drop, will be parallel to each other, as well as the lines *a b*, *c d*, *e f*, in which they came to the drop.

drop. By this means these rays to a spectator commodiously situated will become visible. But all the other rays, as well those nearer the center of the drop $x\,y$, $z\,\alpha$, as those more remote from it $\beta\,\gamma$, $\delta\,\epsilon$, will be reflected in lines not parallel to the lines $b\,m$, $k\,n$, $l\,o$; namely, the ray $x\,y$, in the line $\zeta\,\eta$, the ray $z\,\alpha$ in the line $\theta\,\kappa$, the ray $\beta\,\gamma$ in the line $\lambda\,\mu$, and the ray $\delta\,\epsilon$ in the line $\nu\,\xi$. Whence these rays after their next reflection and subsequent refraction will be scattered from the forementioned rays, and from one another, and by that means become invisible.

4. IT is farther to be remarked, that if in the first case the incident rays $a\,b$, $c\,d$, $e\,f$, and their correspondent emergent rays $b\,m$, $k\,n$, $l\,o$, are produced till they meet, they will make with each other a greater angle, than any other incident ray will make with its corresponding emergent ray. And in the latter case, on the contrary, the emergent rays $q\,t$, $r\,v$, $s\,w$ make with the incident rays an acuter angle, than is made by any other of the emergent rays.

5. OUR author delivers a method of finding each of these extream angles from the degree of refraction being given; by which method it appears, that the first of these angles is the less, and the latter the greater, by how much the refractive power of the drop, or the refrangibility of the rays is greater. And this last consideration fully compleats the doctrine of the rainbow, and shews, why the colours of each bow are ranged in the order wherein they are seen.

6. Suppose A (in fig. 162.) to be the eye, B,C,D,E,F, drops of rain, M n, O p, Q r, S t, V w parcels of rays of the ſun, which entring the drops B, C, D, E, F after one reflection paſs out to the eye in A. Now let M n be produced to η till it meets with the emergent ray likewiſe produced, let O p produced meet its emergent ray produced in κ, let Q r meet its emergent ray in λ, let S t meet its emergent ray in μ, and let V w meet its emergent ray produced in ν. If the angle under M η A be that, which is derived from the refraction of the violet-making rays by the method we have here ſpoken of, it follows that the violet light will only enter the eye from the drop B, all the other coloured rays paſſing below it, that is, all thoſe rays which are not ſcattered, but go out parallel ſo as to cauſe a ſenſation. For the angle, which theſe parallel emergent rays makes with the incident in the moſt refrangible or violet-making rays, being leſs than this angle in any other ſort of rays, none of the rays which emerge parallel, except the violet-making, will enter the eye under the angle M η A, but the reſt making with the incident ray M η a greater angle than this will paſs below the eye. In like manner if the angle under O κ A agrees to the blue-making rays, the blue rays only ſhall enter the eye from the drop C, and all the other coloured rays will paſs by the eye, the violet-coloured rays paſſing above, the other colours below. Farther, the angle Q λ A correſponding to the green-making rays, thoſe only ſhall enter the eye from the drop D, the violet and blue-making rays paſſing above, and the other colours, that is the yellow and red,

red, below. And if the angle S μ A anſwers to the refraction of the yellow-making rays, they only ſhall come to the eye from the drop E. And in the laſt place, if the angle V ν A belongs to the red-making and leaſt refrangible rays, they only ſhall enter the eye from the drop F, all the other coloured rays paſſing above.

7. BUT now it is evident, that all the drops of water found in any of the lines A κ, A λ, A μ, A ν, whether farther from the eye, or nearer than the drops B, C, D, E, F, will give the ſame colours as theſe do, all the drops upon each line giving the ſame colour; ſo that the light reflected from a number of theſe drops will become copious enough to be viſible; whereas the reflection from one minute drop alone could not be perceived. But beſides, it is farther manifeſt, that if the line A Ξ be drawn from the ſun through the eye, that is, parallel to the lines M n, O p, Q r, S t, V ϖ, and if drops of water are placed all round this line, the ſame colour will be exhibited by all the drops at the ſame diſtance from this line. Hence it follows, that when the ſun is moderately elevated above the horizon, if it rains oppoſite to it, and the ſun ſhines upon the drops as they fall, a ſpectator with his back turned to the ſun muſt obſerve a coloured circular arch reaching to the horizon, being red without, next to that yellow, then green, blue, and on the inner edge violet; only this laſt colour appears faint by being diluted with the white light of the clouds, and from another cauſe to be mentioned hereafter [a].

[a] § 11.

8. THUS

8. Thus is cauſed the interior or primary bow. The drops of rain at ſome diſtance without this bow will cauſe the exterior or ſecondary bow by two reflections of the ſun's light. Let theſe drops be G, H, I, K, L; Xy, $Z\alpha$, $\Gamma\beta$, $\Delta\epsilon$, $\Theta\zeta$ denoting parcels of rays which enter each drop. Now it has been remarked, that theſe rays make with the viſible refracted rays the greateſt angle in thoſe rays, which are moſt refrangible. Suppoſe therefore the viſible refracted rays, which paſs out from each drop after two reflections, and enter the eye in A, to interſect the incident rays in π, ρ, σ, τ, φ reſpectively. It is manifeſt, that the angle under $\Theta\varphi A$ is the greateſt of all, next to that the angle under $\Delta\tau A$, the next in bigneſs will be the angle under $\Gamma\sigma A$, the next to this the angle under $Z\rho A$, and the leaſt of all the angle under $X\pi A$. From the drop L therefore will come to the eye the violet-making, or moſt refrangible rays, from K the blue, from I the green, from H the yellow, and from G the red-making rays; and the like will happen to all the drops in the lines $A\pi$, $A\rho$, $A\tau$, $A\varphi$, and alſo to all the drops at the ſame diſtances from the line $A\Xi$ all round that line. Whence appears the reaſon of the ſecondary bow, which is ſeen without the other, having its colours in a contrary order, violet without and red within; though the colours are fainter than in the other bow, as being made by two reflections, and two refractions; whereas the other bow is made by two refractions, and one reflection only.

9. There

9. THERE is a farther appearance in the rainbow particularly described about five years ago [a], which is, that under the upper part of the inner bow there appears often two or three orders of very faint colours, making alternate arches of green, and a reddish purple. At the time this appearance was taken notice of, I gave my thoughts concerning the cause of it [b], which I shall here repeat. Sir ISAAC NEWTON has observed, that in glass, which is polished and quicksilvered, there is an irregular refraction made, whereby some small quantity of light is scattered from the principal reflected beam [c]. If we allow the same thing to happen in the reflection whereby the rainbow is caused, it seems sufficient to produce the appearance now mentioned.

10. LET AB (in fig. 162.) represent a globule of water, B the point from whence the rays of any determinate species being reflected to C, and afterwards emerging in the line CD, would proceed to the eye, and cause the appearance of that colour in the rainbow, which appertains to this species. Here suppose, that besides what is reflected regularly, some small part of the light is irregularly scattered every way; so that from the point B, besides the rays that are regularly reflected from B to C, some scattered rays will return in other lines, as in BE, BF, BG, BH, on each side the line BC. Now it has been observed above [d], that the rays of light in their passage from one superficies of a refracting body to the other undergo alternate fits of

[a] Philos. Transact. No. 375. [b] Ibid. [c] Opt B. II part 4. [d] Ch. 3. § 14.

eaſy tranſmiſſion and reflection, ſucceeding each other at equal intervals; inſomuch that if they reach the farther ſuperficies in one ſort of thoſe fits, they ſhall be tranſmitted; if in the other kind of them, they ſhall rather be reflected back. Whence the rays that proceed from B to C, and emerge in the line CD, being in a fit of eaſy tranſmiſſion, the ſcattered rays, that fall at a ſmall diſtance without theſe on either ſide (ſuppoſe the rays that paſs in the lines BE, BG) ſhall fall on the ſurface in a fit of eaſy reflection, and ſhall not emerge; but the ſcattered rays, that paſs at ſome diſtance without theſe laſt, ſhall arrive at the ſurface of the globule in a fit of eaſy tranſmiſſion, and break through that ſurface. Suppoſe theſe rays to paſs in the lines BF, BH; the former of which rays ſhall have had one fit more of eaſy tranſmiſſion, and the latter one fit leſs, than the rays that paſs from B to C. Now both theſe rays, when they go out of the globule, will proceed by the refraction of the water in the lines FI, HK, that will be inclined almoſt equally to the rays incident on the globule, which come from the ſun; but the angles of their inclination will be leſs than the angle, in which the rays emerging in the line CD are inclined to thoſe incident rays. And after the ſame manner rays ſcattered from the point B at a certain diſtance without theſe will emerge out of the globule, while the intermediate rays are intercepted; and theſe emergent rays will be inclined to the rays incident on the globule in angles ſtill leſs than the angles, in which the rays FI and HK are inclined to them; and without theſe rays will emerge other rays, that ſhall be inclined to the incident rays in angles yet leſs. Now

by

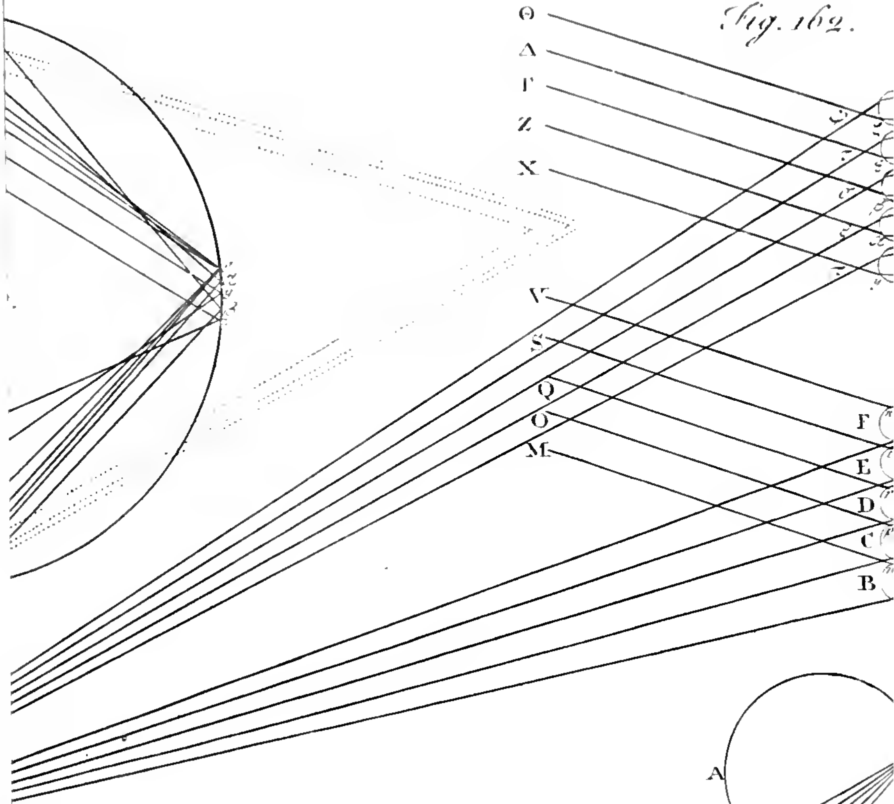
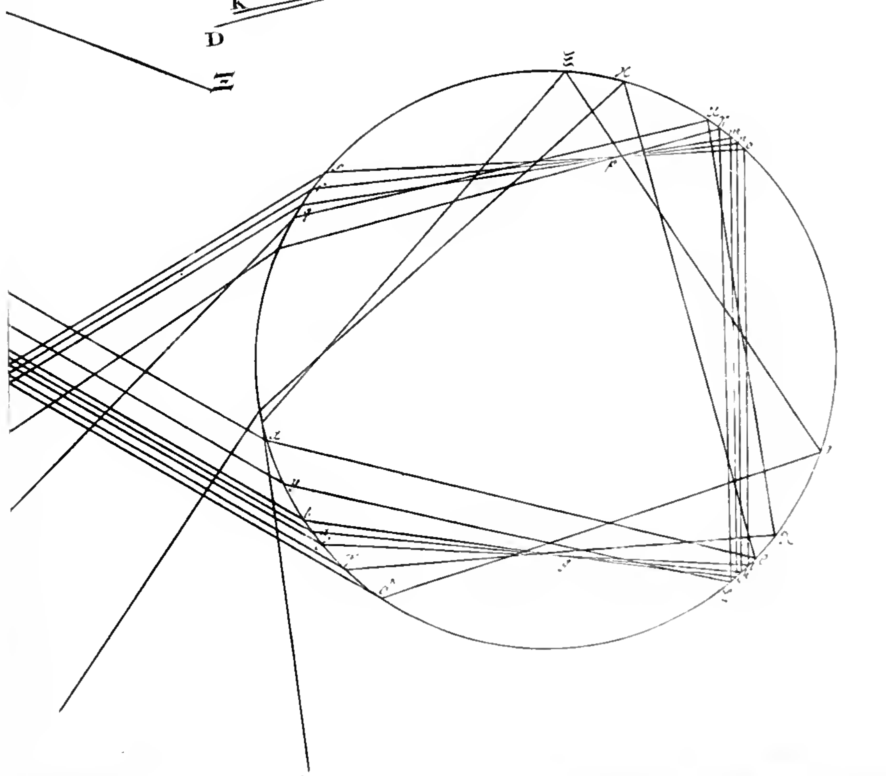


Fig. 162.

Fig. 163.



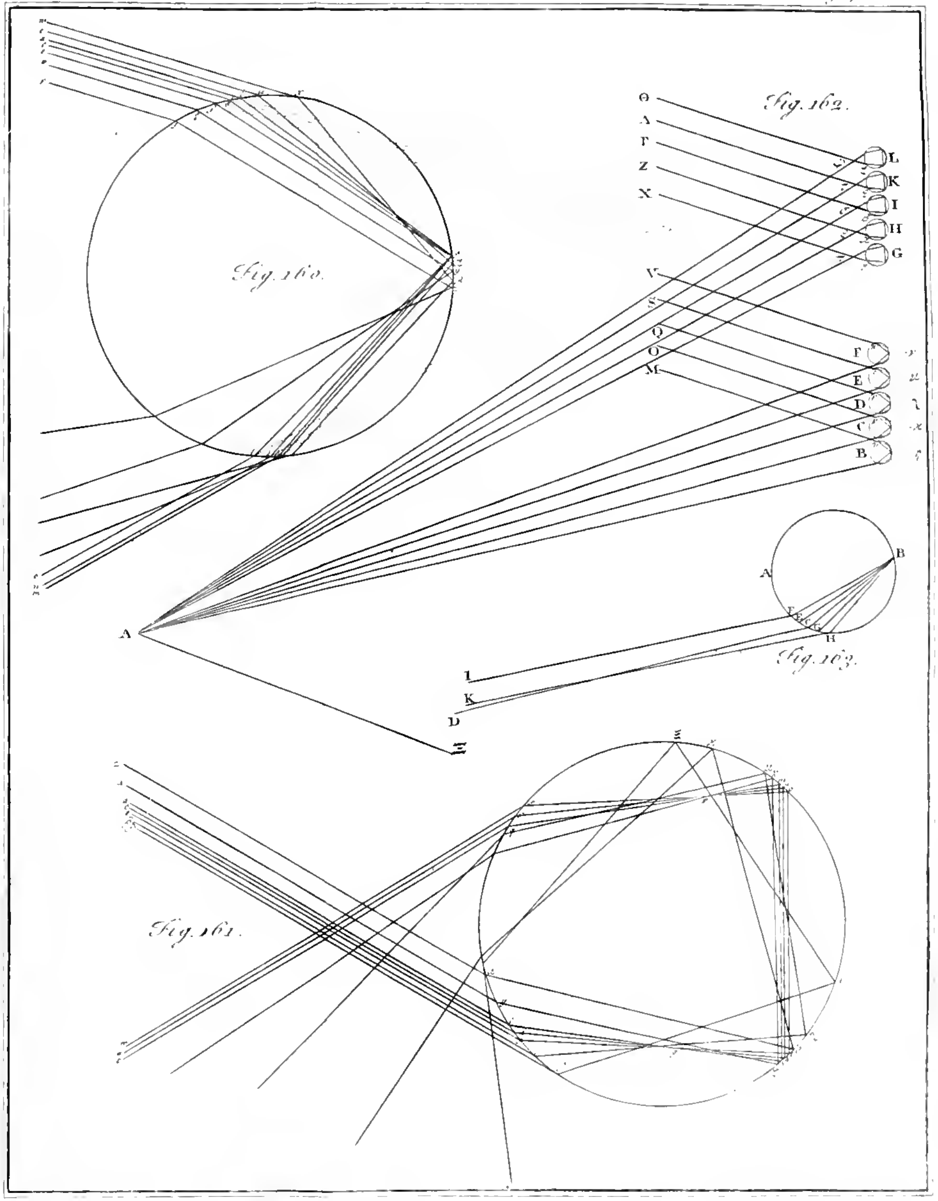


Fig. 160.
Fig. 161.
Fig. 162.
Fig. 163.
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by this means may be formed of every kind of rays, besides the principal arch, which goes to the formation of the rainbow, other arches within every one of the principal of the same colour, though much more faint; and this for divers successions, as long as these weak lights, which in every arch grow more and more obscure, shall continue visible. Now as the arches produced by each colour will be variously mixed together, the diversity of colours observ'd in these secondary arches may very possibly arise from them.

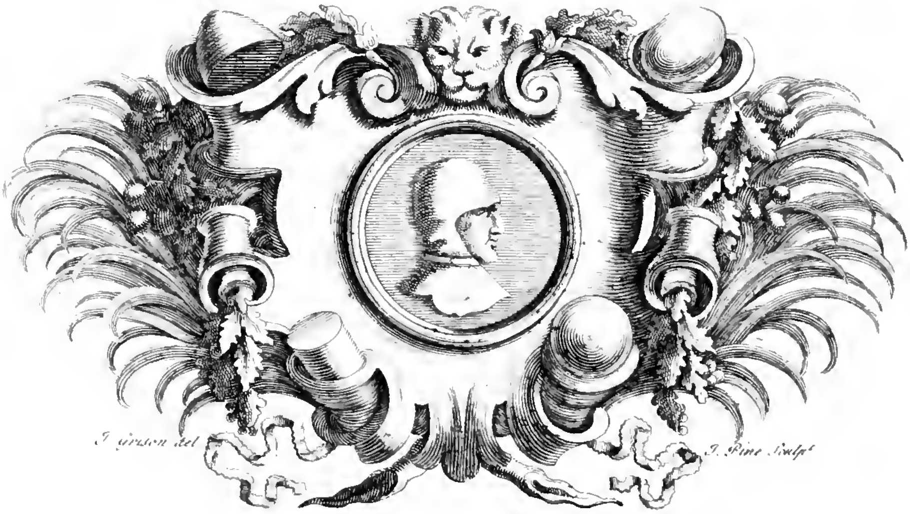
11. IN the darker colours these arches may reach below the bow, and be seen distinct. In the brighter colours these arches are lost in the inferior part of the principal light of the rainbow; but in all probability they contribute to the red tincture, which the purple of the rainbow usually has, and is most remarkable when these secondary colours appear strongest. However these secondary arches in the brightest colours may possibly extend with a very faint light below the bow, and tinge the purple of the secondary arches with a reddish hue.

12. THE precise distances between the principal arch and these fainter arches depend on the magnitude of the drops, wherein they are formed. To make them any degree separate it is necessary the drop be exceeding small. It is most likely, that they are formed in the vapour of the cloud, which the air being put in motion by the fall of the rain may carry down along with the larger drops; and this may be the reason, why these colours appear under the upper

part of the bow only, this vapour not deſcending very low. As a farther confirmation of this, theſe colours are ſeen ſtrongeſt, when the rain falls from very black clouds, which cauſe the fierceſt rains, by the fall whereof the air will be moſt agitated.

13. TO the like alternate return of the fits of eaſy tranſmiſſion and reflection in the paſſage of light through the globules of water, which compoſe the clouds, Sir ISAAC NEWTON aſcribes ſome of thoſe coloured circles, which at times appear about the ſun and moon [a].

[a] Opt. B. II. part 4. obſ. 13.



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CONCLUSION.

SIR Isaac Newton having concluded each of his philoſophical treatiſes with ſome general reflections, I ſhall now take leave of my readers with a ſhort account of what he has there delivered. At the end of his mathematical principles of natural philoſophy he has given us his thoughts concerning the Deity. Wherein he firſt obſerves, that the ſimilitude found in all parts of the univerſe makes it undoubted, that the whole is governed by one ſupreme being, to whom the original is owing of the frame of nature, which evidently is the effect of choice and deſign. He then proceeds briefly to ſtate the beſt metaphyſical notions concerning God. In ſhort, we cannot conceive either of ſpace or time otherwiſe than as neceſſarily

ſarily exiſting; this Being therefore, on whom all others depend, muſt certainly exiſt by the ſame neceſſity of nature. Conſequently wherever ſpace and time is found, there God muſt alſo be. And as it appears impoſſible to us, that ſpace ſhould be limited, or that time ſhould have had a beginning, the Deity muſt be both immenſe and eternal.

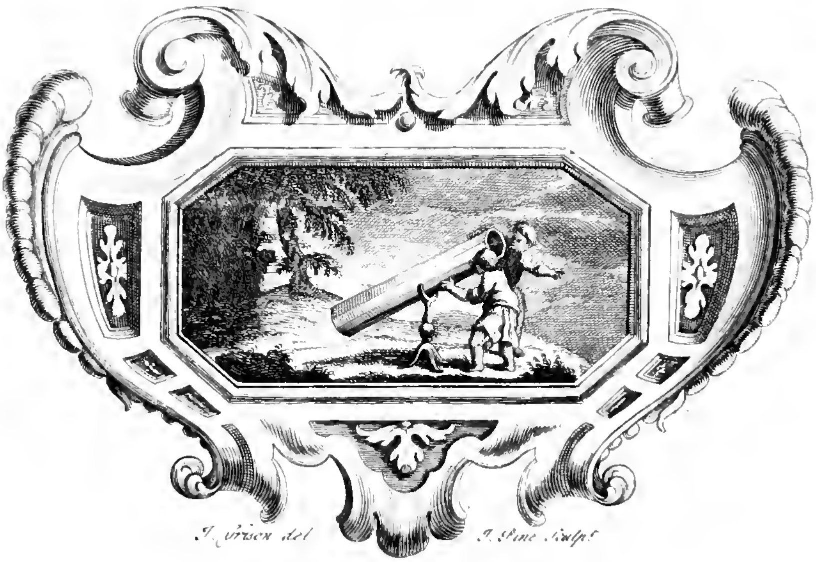
2. At the end of his treatiſe of optics he has propoſed ſome thoughts concerning other parts of nature, which he had not diſtinctly ſearched into. He begins with ſome farther reflections concerning light, which he had not fully examined. In particular he declares his ſentiments at large concerning the power, whereby bodies and light act on each other. In ſome parts of his book he had given ſhort hints at his opinion concerning this[a], but here he expreſly declares his conjecture, which we have already mentioned[b], that this power is lodged in a very ſubtle ſpirit of a great elaſtic force diffuſed thro' the univerſe, producing not only this, but many other natural operations. He thinks it not impoſſible, that the power of gravity it ſelf ſhould be owing to it. On this occaſion he enumerates many natural appearances, the chief of which are produced by chymical experiments. From numerous obſervations of this kind he makes no doubt, that the ſmalleſt parts of matter, when near contact, act ſtrongly on each other, ſometimes being mutually attracted, at other times repelled.

3. The attractive power is more manifeſt than the other, for the parts of all bodies adhere by this principle. And the name

[a] Opt. pag. 255. [b] Ch. 3. § 18.

name of attraction, which our author has given to it, has been very freely made use of by many writers, and as much objected to by others. He has often complained to me of having been misunderstood in this matter. What he says upon this head was not intended by him as a philosophical explanation of any appearances, but only to point out a power in nature not hitherto distinctly observed, the cause of which, and the manner of its acting, he thought was worthy of a diligent enquiry. To acquiesce in the explanation of any appearance by asserting it to be a general power of attraction, is not to improve our knowledge in philosophy, but rather to put a stop to our farther search.

FINIS.



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